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PROBABILITY

See also A7930, A8079, A8184, A8285, 8806, 8840, 9086, 9322, 9324.

8538:

Fréchet, Maurice. Généralisations d'une inégalité de Minkowski. Bull. Inst. Internat. Statist. **35** (1957), no. 2, 99–103.

Summary of the results established and discussed in the author's paper in Ann. Inst. H. Poincaré **15** (1956), 1–33 [MR **19**, 400]. *M. Loève* (Berkeley, Calif.)

8539:

Castoldi, Luigi. Stazionarietà e reversibilità in processi a catena di Markov. Rend. Sem. Fac. Sci. Univ. Cagliari **28** (1958), 169–174. (English summary)

Die Übergangsmatrix einer Markoffschen Kette mit endlich vielen Zuständen und eine Wahrscheinlichkeitsverteilung über dem Zustandsraum sind gleichzeitig durch spezielle Parameterdarstellungen gegeben. Es werden diejenigen Parameterwerte ausgerechnet, für die die zugehörige Verteilung hinsichtlich der zugehörigen Kette stationär ist, und eine Anwendung auf die Genetik (Hardysches Gesetz) wird beschrieben.

K. Krickeberg (Heidelberg)

8540:

Castoldi, Luigi. Variante di un problema probabilistico di Banach. Rend. Sem. Fac. Sci. Univ. Cagliari **28** (1958), 175–180. (French summary)

Das Banachsche Streichholzschachtelproblem in der Formulierung von W. Feller, *An introduction to probability theory and its applications* [Wiley, New York, 1950; MR **12**, 424; p. 108] wird etwas modifiziert. Insbesondere werden Erwartung und Varianz der entsprechenden Verteilung berechnet und asymptotisch bei großer Streichholzzahl dargestellt. *K. Krickeberg* (Heidelberg)

8541:

Castoldi, Luigi. La distribuzione del numero dei nati e connessi problemi di probabilità. Rend. Sem. Fac. Sci. Univ. Cagliari **29** (1959), 14–25. (English summary)

Der Verf. denkt sich die Nachkommenverteilung eines Elternpaares in der folgenden Weise beschrieben: gegeben seien Urnen $U_1, U_2, \cdots$, von denen U_i weiße und schwarze Kugeln im Verhältnis r_i und s_i mit $r_i+s_i=1$ enthält. Das Paar hat n Kinder, wenn man bei einer Folge unabhängiger Ziehungen aus $U_1, \cdots, U_n$ weiße und aus U_{n+1} eine schwarze Kugel bekommt. Die Wahrscheinlichkeiten für mindestens n order genau n Nachkommen werden aus den r_i abgeleitet und untersucht, insbesondere mit verschiedenen speziellen Werten der r_i.

K. Krickeberg (Heidelberg)

8542:

Castoldi, Luigi. A property of Poissonian distributions. Rend. Sem. Fac. Sci. Univ. Cagliari **29** (1959), 220–222. (Italian summary)

The reviewer once proposed as a problem a certain characterization of the Poisson distribution [Amer. Math. Monthly **66** (1959), 317]. Solutions have been published by L. Daboni [Boll. Un. Mat. Ital. (3) **14** (1959), 318–320; MR **22** #6002] and W. Kruskal [Amer. Math. Monthly **67** (1960), 297–298] and the problem was solved by others as well. The present author here offers (independently) essentially the solution found by Kruskal.

J. Lamperti (Stanford, Calif.)

8543:

Mallows, C. L. On the probability integral transformation. Biometrika **46** (1959), 481–483.

Let $x_1, x_2, \cdots, x_n$ be n random variables with common probability density function $f(x, \theta_1, \theta_2, \cdots, \theta_s)$ depending on s parameters, where $f(x, \theta_1, \theta_2, \cdots, \theta_s)>0$ $(a<x<b)$. If the parameters are known, the probability integral transformation defined by $z_i=\int_a^{x_i} f(t, \theta_1, \theta_2, \cdots, \theta_s)dt$ defines n random variables which are independent and rectangularly distributed. However, if θ_i is estimated by $F_i(x_1, x_2, \cdots, x_n)$, the same cannot be said for the corresponding variables $y_i=\int_a^{x_i} f(t, F_1, F_2, \cdots, F_s)dt$. The author studies relations between the estimators F_i, the density f, and the rank of Jacobian $|\partial(y_1, \cdots, y_n)/\partial(x_1, \cdots, x_n)|$, which is generally between $n-s$ and n.

H. Chernoff (Stanford, Calif.)

8544:

Laha, R. G. On a class of distribution functions where the quotient follows the Cauchy law. Trans. Amer. Math. Soc. **93** (1959), 205–215.

Let x and y be two independently and identically distributed normal random variables with zero mean. It is well known that the quotient x/y is distributed according to Cauchy's law. The author [Proc. Nat. Acad. Sci. U.S.A. **44** (1958), 222–223; MR **19**, 1201], G. P. Steck [Ann. Math. Statist. **29** (1958), 604–606; MR **20** #4901] and J. G. Mauldon [Quart. J. Math. Oxford Ser. (2) **7** (1956), 155–160; MR **20** #4884] constructed examples which show that the converse statement is not true. In the present paper the author studies the class of distribution functions $F(x)$ which have the following property: (*) If x and y are two independently and identically distributed random variables with common distribution function $F(x)$, then the quotient x/y has a Cauchy distribution with zero

median. Some theorems concerning this family were obtained by the author in an earlier paper [Ann. Math. Statist. **30** (1959), 1165–1174; MR **21** #7556]. These are used now to construct, in two different ways, families of distribution functions which have the property (*) and have also finite moments up to a preassigned even order.

E. Lukacs (Berkeley, Calif.)

8545:

Ėĭdel'nant, M. I. Approximate formulas for hypergeometric distribution. Izv. Akad. Nauk UzSSR. Ser. Fiz.-Mat. **1958**, no. 5, 79–92. (Russian. Uzbek summary)

Some of the approximations are obtained by an obvious use of Stirling's formula, others from the theorem on harmonic, geometric and arithmetic means. Numerical examples illustrate the accuracy of the formulas, but no thorough error analysis is given.

Walter Gautschi (Oak Ridge, Tenn.)

8546:

Turin, G. L. The characteristic function of Hermitian quadratic forms in complex normal variables. Biometrika **47** (1960), 199–201.

Consider complex-valued stochastic variables $v_n = x_n + iy_n$, where it is assumed that $\mathrm{Cov}(x_m, x_n) = \mathrm{Cov}(y_m, y_n)$ and $\mathrm{Cov}(x_m, y_n) = -\mathrm{Cov}(x_n, y_m)$. An explicit expression is obtained for the characteristic function $\varphi(z)$ of the real-valued quadratic form v^*Qv, where v is the column vector $(v_1, v_2, \cdots v_N)$ and Q is a Hermitian matrix. If the mean values vanish, $Ev_n = 0$, then $\varphi(z)$ is a rational function, so that explicit Fourier inversion is possible.

U. Grenander (Stockholm)

8547:

Spitzer, F. L.; Stone, C. J. A class of Toeplitz forms and their application to probability theory. Illinois J. Math. **4** (1960), 253–277.

Let c_k ($k = 0, \pm 1, \pm 2, \cdots$) be a symmetric probability sequence, $c_k = c_{-k} \geq 0$, $\sum c_k = 1$, with $\sigma^2 = \sum k^2 c_k < \infty$. Introduce the characteristic function $\phi(z) = \sum c_k z^k$, $z = e^{i\theta}$, and the orthonormal polynomials $p_n(z)$ introduced in the customary manner corresponding to the non-negative weight function $1 - \phi(z)$. A great number of results are derived for these polynomials and strong use is made of probabilistic analogies to the theory of Toeplitz forms. The main result of the paper is the following: Let $p_n(z) = \sum p_{nk} z^k$; then

$$\frac{\sigma}{\sqrt{2}} p_{nk} = \frac{k}{n} + o(1) \quad (0 \leq k \leq n < \infty),$$

where $o(1) \to 0$ as $n - k \to \infty$ uniformly in k and n.

U. Grenander (Stockholm)

8548:

Sarmanov, O. V.; Zaharov, V. K. Maximum coefficients of multiple correlation. Dokl. Akad. Nauk SSSR **130** (1960), 269–271 (Russian); translated as Soviet Math. Dokl. **1**, 51–53.

A concept of maximum coefficient of correlation between random vectors is defined and used to characterize mutual independence of a set of random variables.

J. G. Wendel (Ann Arbor, Mich.)

8549:

Bahadur, R. R.; Ranga Rao, R. On deviations of the sample mean. Ann. Math. Statist. **31** (1960), 1015–1027.

Let $X_1, \cdots, X_n, \cdots$ be independent and identically distributed, with a moment-generating function $\varphi(t)$ that exists at least on a nondegenerate open interval T. Estimates are obtained for $p_n = \Pr\{n^{-1}(X_1 + \cdots + X_n) \geq a\}$, where a is a number such that the equation $(d/dt)\log\varphi(t) = a$ has a unique solution τ in T. With $\rho = \varphi(\tau)\exp(-a\tau)$, the estimates state that $(2\pi n)^{1/2}\rho^{-n}p_n$ is bounded away from zero and infinity, and is even asymptotically constant in the nonlattice case.

J. G. Wendel (Ann Arbor, Mich.)

8550:

Geiringer, Hilda. On a limit theorem leading to a compound Poisson distribution. Math. Z. **72** (1959/60), 229–234.

Consider discrete distributions $\{p_\nu(\cdot), \nu \geq 1\}$ on $\{0, 1, \cdots, m\}$ such that $\sum_{r=1}^{n} p_r(0) = a$, $\max_{1 \leq r \leq n} p_r(0) = O(n^{-1})$. Then the convolution $p_1 * \cdots * p_n$ tends as $n \to \infty$ to a compound Poisson distribution with specified parameters.

K. L. Chung (Syracuse, N.Y.)

8551:

Lubenow, Klaus. Verallgemeinerung eines Satzes von Prochorov auf mehrere Dimensionen. Wiss. Z. Humboldt-Univ. Berlin. Math.-Nat. Reihe **8** (1958/59), 535–547. (Russian, English and French summaries)

Let $x_{n,k} = (x_{n,k}^{(1)}, x_{n,k}^{(2)}, \cdots, x_{n,k}^{(m)})$, $n = 1, 2, \cdots, k = 1, 2, \cdots, k_n$, be a scheme of m-component random vectors. For fixed n the vectors $(x_{n,j})$, $j = 1, 2, \cdots, k_n$, are independent. Define $S_{n,k} = \sum_{j=1}^{k} x_{n,j}$, $S_n = S_{n,k_n}$, and suppose Var $(x_{n,k}^{(j)}) > 0$ and that the $x_{n,k}$ are linearly transformed so that $E(x_{n,k}) = 0$ and $E(S_n' S_n) = I$, the identity matrix. Suppose they satisfy further the m-dimensional Lindeberg condition: for every $\varepsilon > 0$

$$\max_{1 \leq j \leq m} \sum_{i=1}^{k_n} \int_{|x_{n,i}^{(j)}| > \varepsilon} (x_{n,i}^{(j)})^2 \, dP \to 0, \quad n \to \infty.$$

Denote by (C_m, B) the measurable space in which C_m consists of m-tuples of continuous functions $(f_j(t))$, $0 \leq t \leq 1$, $j = 1, 2, \cdots, m$, and B is the usual Borel field of subsets of C_m. Construct a sequence of continuous processes $X_n(t)$ by putting $t_{n,i} = m^{-1}E(\|S_{n,i}\|^2)$, $X_n(t_{n,i}) = S_{n,i}$ and $X_n(t)$ linear in $t_{n,i} \leq t \leq t_{n,i+1}$, $i = 0, 1, \cdots, k_n - 1$, where $t_{n,0} = X_n^{(j)}(0) = 0$. Let P_n be the measure over (C_m, B) induced by $X_n(t)$ and let W_n be the measure induced by the m-dimensional Gaussian process $Y_n(t)$ with independent increments whose mean and covariance matrix for each t agree with that of $X_n(t)$, $0 \leq t \leq 1$. Such a process exists, though it may not have stationary increments.

The author proves that $Y_n(t)$ is continuous with probability one, and the main theorem asserts that P_n and W_n are conditionally compact relative to weak convergence and that $P_n - W_n$ converges (weakly) to the null measure over (C_m, B). Consequently, if W_n converges weakly to W, so does P_n.

For $m = 1$ this latter condition is always realized. Then W is the usual Wiener measure and the theorem of Prohorov [Teor. Veroyatnost. i Primenen. **1** (1956), 177–238;

MR 18, 943] is recovered. For $m>1$ there can be a kind of interference between the components and no result as simple as this obtains.

D. A. Darling (Ann Arbor, Mich.)

8552:

Blom, Gunnar. On the asymptotic distribution of linear combinations of interchangeable random variables. Math. Scand. **7** (1959), 321–332.

A sequence of random variables is said to be interchangeable if any finite subsequence has a joint distribution function that is symmetric with respect to its arguments. The paper derives conditions under which linear combinations $T_n=\sum_{i=1}^{k_n} h_{in}x_{in}$ of interchangeable random variables x_{in} ($n=1, 2, \cdots$; $i=1, 2, \cdots, k_n\to\infty$) satisfying either $\sum_{i=1}^{k_n} h_{in}=0$ or $\sum_{i=1}^{k_n} x_{in}=C_n=\text{const}$ are asymptotically normally distributed. The theorem generalizes one of the reviewer [see Ann. Math. Statist. **20** (1949), 455–458; MR **11**, 188] dealing with the asymptotic normality of linear combinations of variables under permutations. The new theorem is applied to linear combinations of random intervals from a uniform distribution.

G. E. Noether (Boston, Mass.)

8553:

Bühlmann, Hans. Austauschbare stochastische Variablen und ihre Grenzwertsaetze. Univ. California Publ. Statist. **3**, 1–35 (1960).

Es handelt sich um eine erweiterte mit Beweisen versehene Darstellung teilweise bereits angekündigter Resultate [Bühlmann, C. R. Acad. Sci. Paris **246** (1958), 534–536; MR **19**, 1088]. Der Autor beweist nun auch Sätze über Prozesse mit austauschbaren Zuwächsen. Ein auf $[a, b]$ definierter stochastischer Prozeß x_t ist ein Prozeß mit austauschbaren Zuwächsen, wenn für jedes n die zufälligen Variablen $x_{t_{2^n,k}}-x_{t_{2^n,k-1}}$, $1\leq k\leq 2^n$, $t_{2^n,k}=a+(b-a)k/2^n$ Glieder einer unendlichen Folge von austauschbaren Variablen sind. Zu dieser Definition bemerkt der Autor, daß es endliche Mengen austauschbarer zufälliger Variablen gibt, die sich nicht zu einer unendlichen Folge solcher Variablen erweitern lassen. Ein separabler Prozeß x_t mit austauschbaren Zuwächsen ist stetig mit Wahrscheinlichkeit 1. Da nach einem Satz von de Finetti eine unendliche Folge austauschbarer zufälliger Variablen bedingt unabhängig ist der (Autor präzisiert diese Aussage), ergibt sich, daß die Verteilung separabler Prozesse mit austauschbaren Zuwächsen zur Klasse der gewichteten unendlich oft teilbaren Verteilungen gehört. Insbesondere wird der Fall der Prozesse mit gewichteter Normalverteilung und gewichteter Poisson-Verteilungen studiert.

L. Schmetterer (Hamburg)

8554:

Kano, Seigo. On the convergence of generalized discrete martingale processes. Sci. Rep. Kagoshima Univ. No. 7 (1958), 11–13.

Ein stochastischer Prozeß $\{x_1, x_2, \cdots\}$ wird vom Verf. als verallgemeinertes Martingal bezeichnet, wenn $E\{|x_n|\}<+\infty$, $n=1, 2, \cdots$, und wenn es zu jedem n eine Bairesche Funktion f_n im n-dimensionalen Raum R_n gibt, so daß $E\{x_n|x_1, \cdots, x_m\}=f_m(x_1, \cdots, x_m)$ für alle $n>m$. Es sei nun $y_0=0$ und $y_m=E\{x_{m+1}|x_1, \cdots, x_m\}$, $m=1, 2, \cdots$, und es seien weiter die folgenden Voraussetzungen erfüllt: bei beliebigem n ist die Abbildung $v_k=f_k(u_1, \cdots, u_k)$ ($k=1, \cdots, n$) von R_n in sich umkehrbar eindeutig, $\limsup_{n\to\infty} E\{|x_n|\}<+\infty$ und $E\{\sup_{j\geq 1}(x_{j+1}-y_j)\}<+\infty$. Dann existieren die Grenzwerte $\lim_{n\to\infty} x_n(\omega)$ und $\lim_{n\to\infty} y_n(\omega)$ und sind endlich und einander gleich für fast alle ω, für die $\limsup_{n\to\infty} z_n(\omega)<+\infty$, wobei $z_n=\sum_{j=0}^{n}(x_{j+1}-y_j)$. Der Beweis beruht auf der Anwendung des Doobschen Konvergenzsatzes über Martingale auf das Martingal $\{y_0, y_1, \cdots\}$ und auf Martingale, die aus dem Martingal $\{z_0, z_1, \cdots\}$ durch "optional stopping" hervorgehen. {Anmerkung des Ref.: Der Gebrauch des Martingals $\{z_0, z_1, \cdots\}$ zur Ausdehnung der Konvergenzsätze wurde unter der Bezeichnung $x'_{n+1}=z_n$ bereits von Doob vorgeschlagen [*Stochastic processes*, Wiley, New York, 1953; MR **15**, 445; p. 296, (1.5)].

K. Krickeberg (Heidelberg)

8555:

Krickeberg, Klaus. Absteigende Semimartingale mit filtrierendem Parameterbereich. Abh. Math. Sem. Univ. Hamburg **24** (1960), 109–125.

Continuation of the author's study [Trans. Amer. Math. Soc. **83** (1956), 313–337; Math. Z. **66** (1957), 470–486; MR **19**, 947, 948] of submartingales $(f_s, \mathfrak{B}_s)$ with partially ordered parameter, turning now to the case of decreasing martingale basis: $\mathfrak{B}_s\supset\mathfrak{B}_t$ $(s<t)$. $E(f_s)<+\infty$ is found to imply stochastic convergence of f_s. If the basic probability space has a kind of "compactness" relative to the martingale basis $(\mathfrak{B}_s)$, then $E(f_s)<+\infty$ is found to imply essential convergence of f_s; an example is presented of a martingale basis which lacks this compactness.

H. P. McKean, Jr. (Cambridge, Mass.)

8556:

Jerison, Meyer. Martingale formulation of ergodic theorems. Proc. Amer. Math. Soc. **10** (1959), 531–539.

Let $(N, \mathfrak{N}, \nu)$ be the space of non-negative integers, the measure of each set being the number of its elements, and let $\mathfrak{G}_{-n}$ be the smallest σ-algebra that contains the set $\{0, 1, \cdots, n-1\}$ and all one-element sets $\{k\}$ with $k\geq n$. Let $(S, \mathfrak{S}, \sigma)$ be a totally finite measure space and consider $(S\times N, \mathfrak{S}\times\mathfrak{N}, \sigma\times\nu)$. For $n\geq 1$, let $\mathfrak{F}_{-n}$ be the σ-algebra $\mathfrak{S}\times\mathfrak{G}_{-n}$. Then $\mathfrak{F}_{-n}\subseteq\mathfrak{F}_{-m}$ if $n>m$. Let T be a linear transformation on $L_1(S)$ into itself such that $\|Tx\|_1\leq\|x\|_1$ and $\operatorname{ess\,sup}|Tx(s)|\leq\operatorname{ess\,sup}|x(s)|$. Then, for $f(s, k)=T^k x(s)$, the conditional expectation $E\{f|\mathfrak{F}_{-n}\}$ exists and is equal to f_{-n} where $f_{-n}(s, k)=n^{-1}\sum_{k=0}^{n-1} T^k x(s)$ or $=T^k x(s)$ according as $k<n$ or $k\geq n$, respectively. In this way, the Dunford-Schwartz individual ergodic theorem [see their book, #8302; p. 668] is exhibited as a statement of the convergence a.e. of certain martingales on "infinite measure space" $(S\times N, \mathfrak{S}\times\mathfrak{N}, \sigma\times\nu)$. It is also proved that the usual martingale convergence theorem on "finite measure space" does yield the convergence a.e. of certain subsequences of the averages of the ergodic theorems. Finally it is proved that a theorem of mean convergence of martingales in L_p is derived from a general mean ergodic theorem due to L. Alaoglu and G. Birkhoff [Ann. of Math. (2) **41** (1940), 293–309; MR **1**, 339].

K. Yosida (Tokyo)

8557:

Kiefer, J. A functional equation technique for obtaining Wiener process probabilities associated with theorems of Kolmogorov-Smirnov type. Proc. Cambridge Philos. Soc. **55** (1959), 328–332.

Let $X(t), t \geq 0$, be the usual one-dimensional Wiener process with $X(0)=0$. The author considers the probability that $X(t)$ will, for all $t \geq 0$, lie inside certain regions bounded by linear functions of t. Simple partial differential equations, in terms of the coefficients of the linear functions, are deduced which are satisfied by these probabilities. The uniqueness of the solutions is assured by imposing suitable boundary conditions dictated by superficial probabilistic considerations. Corresponding results are obtained in the k-dimensional case; i.e., if $X_k(t)$ is the k-dimensional Wiener process, the author considers the probability that $X_k(t)$ will always lie within a sphere whose radius increases linearly with t.

D. A. Darling (Ann Arbor, Mich.)

8558:

Takacs, L. On a sojourn time problem. Teor. Veroyatnost. i Primenen. **3** (1958), 61–69. (Russian summary)

The author continues his earlier studies [Acta Math. Acad. Sci. Hungar. **8** (1957), 169–191, 279–294; MR **19**, 467; **20** #2799]. A new derivation of the exact distribution of $\beta(t)$ [for the notation see the reviews quoted] is presented and a systematic survey of the asymptotic distributions of $\beta(t)$ under a variety of assumptions is given.

E. Lukacs (Washington, D.C.)

8559:

Lamperti, John. The first-passage moments and the invariant measure of a Markov chain. Ann. Math. Statist. **31** (1960), 515–517.

Consider a recurrent Markov chain and let $\{\pi_i\}$ be the unique positive solution of the equation $\pi_j = \sum_i \pi_i p_{ij}$, where (p_{ij}) is the one-step transition matrix. Let α_{ij} be the first recurrence or entrance time from i to j, and let

$$\mu_{ij}^{(k)} = E\{\alpha_{ij}(\alpha_{ij}-1)\cdots(\alpha_{ij}-k+1)\}.$$

Then for every positive integer k,

$$\pi_j \mu_{jj}^{(k+1)} = (k+1) \sum_{i \neq j} \pi_i \mu_{ij}^{(k)}$$

(both members may be infinite). For $k=1$ this reduces to a result of the reviewer [Trans. Amer. Math. Soc. **76** (1954), 397–419; MR **16**, 149].

K. L. Chung (Syracuse, N.Y.)

8560:

Breiman, Leo. The strong law of large numbers for a class of Markov chains. Ann. Math. Statist. **31** (1960), 801–803.

Let $\{X_n\}$ be a Markov process with the unique stationary measure π, $P_x(dy)$ the transition measure, C the class of continuous functions on the state space such that $f \in C$ implies $\int f(y)P_x(dy) \in C$. Theorem: If $f \in C$, then for each x, $N^{-1}\sum_1^N f(X_n) \to \int f(y)\pi(dy)$ a.e. (P_x). This is better than "a. e. (π)", especially when π is singular with respect to some basic measure.

K. L. Chung (Syracuse, N.Y.)

8561:

Katz, Melvin, Jr.; Thomasian, A. J. An exponential bound for functions of a Markov chain. Ann. Math. Statist. **31** (1960), 470–474.

An explicit and simple exponential bound is obtained for $P\{|n^{-1}\sum_{k=1}^n f(X_k) - \mu| \geq \varepsilon$ for some $n \geq m\}$, where $\{X_k, k \geq 1\}$ is a finite Markov chain with stationary probabilities which is ergodic (positive-recurrent), f is any real-valued function, and μ is the expectation of $f(X_1)$ under the stationary measure. Bounds for one-sided inequalities are also given.

K. L. Chung (Syracuse, N.Y.)

8562:

Neveu, Jacques. Processus sous-markoviens stationnaires. C. R. Acad. Sci. Paris **249** (1959), 2477–2479.

Soit P un processus sous-markovien stationnaire défini sur un ensemble dénombrable Ω et $\bar{P}$ un processus du même type qui domine P absolument (c-à-d., essentiellement tel que $P \subset \bar{P}$). L'auteur donne un procédé pour construire $\bar{P}$ à partir de P sous certaines hypothèses qui sont précisées.

A. Fuchs (Strasbourg)

8563:

Akaike, Hirotugu. On a limiting process which asymptotically produces f^{-2} spectral density. Ann. Inst. Statist. Math. Tokyo **12** (1960), 7–11.

The author points out that, if a one-dimensional stationary process repeatedly suffers the alternated operations of smoothing and of the injection of random variation, then the power spectrum of the process will tend to a form which is $O(f^{-2})$ for small f, f representing frequency. Some care is necessary in the interpretation of the limiting process, and of the result, since no finite variance process could have a spectrum of this type.

P. Whittle (Cambridge, England)

8564:

Parthasarathy, K. R. On the estimation of the spectrum of a stationary stochastic process. Ann. Math. Statist. **31** (1960), 568–573.

Let X_t $(t=0, \pm 1, \cdots)$ be a weakly stationary stochastic process with means 0, covariance sequences ρ_ν and spectral distribution function $F(\lambda)$. Two results are given in the paper. First the author introduces a class of estimates of the spectral distribution function whose bias can be made to decrease (with increasing sample size) at any preassigned rate. Second, under the assumption that the sample covariances converge almost surely and in L_1 mean to the population covariances, and that $\sum |\rho_\nu| < \infty$, he proves the existence of consistent estimates of the spectral density function.

E. Parzen (Stanford, Calif.)

8565:

Rozanov, Yu. A. Spectral analysis of abstract functions. Teor. Veroyatnost. i Primenen. **4** (1959), 291–310. (Russian. English summary)

Der Verf. behandelt einleitend additive Maße Φ mit Werten in einem Hilbertschen Raum H, definiert im Bereich der rechts offenen Intervalle der Zahlengeraden, und die ihnen vermöge $F(\Delta, \Delta') = (\Phi(\Delta), \Phi(\Delta'))$ entsprechenden komplexwertigen Maße, insbesondere Begriffe wie Stetigkeit, schwache Stetigkeit, beschränkte Variation und Integration hinsichtlich dieser Maße. Sodann werden Sätze aus der Theorie der stationären Prozesse $x(t)$, $-\infty < t < +\infty$, $x(t) \in H$, in zwei Richtungen verallgemeinert. Die erste Verallgemeinerung führt auf die "harmonisierbaren" Funktionen x, d.h. solche, die sich in

der Gestalt $x(t)=\int_{-\infty}^{\infty} e^{i\lambda t}\Phi(d\lambda)$ darstellen lassen, wobei aber das zugehörige Maß $F(\Delta, \Delta')$ nicht notwendig, wie bei stationären Prozessen, die Form $F_0(\Delta \cap \Delta')$, mit dem Spektralmaß F_0, haben muß. Zur Berechnung von Φ und F mit Hilfe von x gibt es Umkehrformeln. Innerhalb einer Reihe von Beispielen wird u.a. das Verhalten von x beim Übergang von $\Phi(d\lambda)$ zu $\varphi(\lambda)\Phi(d\lambda)$ in einigen Fällen studiert; darunter fällt die Differentiation von x, nämlich mit $\varphi(\lambda)=i\lambda$. Die zweite Verallgemeinerung besteht in der Betrachtung von Funktionen x, bei denen zwar $(x(t+\tau), x(t))$ nicht notwendig, wie bei stationären Prozessen, nur von τ abhängt, aber doch der Grenzwert $B(\tau)=\lim_{T\to\infty} T^{-1}\int_0^T (x(t+\tau), x(t))dt$ existiert und stetig ist. B kann dann in der Form $B(\tau)=\int_{-\infty}^{\infty} e^{i\lambda\tau}F_0(d\lambda)$ dargestellt werden, und F_0 heißt das Spektralmaß von x. Jede harmonisierbare Funktion hat ein Spektralmaß in diesem Sinn, nämlich $F_0(\Delta)=\int_\Delta F(d\lambda, d\lambda)$. Der Übergang von x zu Funktionen der Form $y(t)=\int_{-\infty}^{\infty}\tilde{\varphi}(\tau-t)x(\tau)d\tau$ nebst anderen Beispielen wird untersucht. Als Anwendung behandelt der Verf. die lineare Extrapolation des Prozesses x auf Grund seines Verhaltens in einem Zeitabschnitt $t\leqq\tau$ und Eigenschaften der minimalen mittleren quadratischen Abweichung zwischen x und den extrapolierten Prozessen. *K. Krickeberg* (Heidelberg)

8566:

Schneider, B. Beitrag zur Theorie der Verzweigungsprozesse. Metrika **3** (1960), 128–142. (English summary)

The author points out that difficulties arise in the treatment of branching processes where, instead of there being only a finite number of types, each object is described by a parameter x, where x is a vector in n-dimensional Euclidean space. Bartlett [*An introduction to stochastic processes*, Cambridge Univ. Press, 1955; MR **16**, 939; and in prior publications] has treated such processes as an example of "point stochastic processes". The author also discusses the method of product densities [Bhabha and Ramakrishnan, Proc. Indian Acad. Sci. Sect. A **32** (1950), 141–153; MR **14**, 62] and age-dependent branching processes where the system is described by $Z(x, t)$, the number of objects of age $\leqq x$ at time t [Harris, Proc. 2nd Berkeley Sympos. Math. Statist. Prob., 1950, pp. 305–328, Univ. Calif. Press, Berkeley, Calif., 1951; MR **13**, 567]. The author states that his idea is exemplified by introducing a process $z(x, t)$ which will be a density (at x) for $Z(x, t)$. However, it seems to the reviewer that the author does not do this. Rather, he shows that a random vector set function on n-dimensional sets can be replaced by a random vector point function, which, when differenced over an n-dimensional interval, gives the value of the set function for the interval, with an appropriate relation between covariances (the statement of the theorem on p. 136 seems incomplete; perhaps the words "with a covariance function ρ of bounded variation" should be inserted after "$z(x)$"). The statements (a) and (b) on p. 130, concerning simple branching processes, contain errors.

T. E. Harris (Santa Monica, Calif.)

8567:

Jewell, William S. The properties of recurrent-event processes. Operations Res. **8** (1960), 446–472.

This paper is largely expository and concerned with certain aspects of renewal theory. The emphasis is upon the computation of formulae (of which many are given) rather than upon questions of mathematical rigour. Special attention is given to the special cases when renewal lifetimes have exponential, gamma, or the so-called geometric-exponential distributions.

W. L. Smith (Chapel Hill, N.C.)

8568:

Anselone, P. M.; Porcelli, P. Oscillatory limiting behavior of a random sequence. Amer. Math. Monthly **67** (1960), 565–566.

Let $\{(R_k, W_k): k\geqq 0\}$ be a sequence of independent, identically distributed random vectors with non-negative integer components. Define the sequence of random variables $\{X_n\}$ by $X_n=R_k$ if $\sum_0^{k-1} W_h\leqq n\leqq\sum_0^k W_h$. Formulae, in terms of generating functions, are given for computing $\lim_{n\to\infty}\Pr\{X_n=i\}$ when it is known that this limit exists. It is also shown that it is possible to have $\liminf\Pr\{X_n=i\}\leqq\alpha\leqq\beta\leqq\limsup\Pr\{X_n=i\}$ for any prescribed α and β, $0\leqq\alpha\leqq\beta\leqq 1$.

W. L. Smith (Chapel Hill, N.C.)

8569:

Roberts, P. H.; Ursell, H. D. Random walk on a sphere and on a Riemannian manifold. Philos. Trans. Roy. Soc. London. Ser. A **252** (1960), 317–356.

Choose an angle $0\leqq\alpha\leqq\pi$ and consider a particle performing a random walk on the surface $|\mathfrak{r}|=1$ of the unit 3-dimensional sphere according to the following rule: having arrived after $n-1$ steps at the point $\mathfrak{a}$, the particle jumps at time n out onto the circle $\mathfrak{b}\cdot\mathfrak{a}=\cos\alpha$ with a uniform angular distribution. A notable feature of this motion is that the eigenfunctions of the spherical Laplace operator (spherical harmonics) are simultaneous eigenfunctions of all these random walks, i.e., if $\mathfrak{a}$ is the initial position of the particle and if $\mathfrak{r}_1$ is its position at time 1, then $E[S_n^l(\mathfrak{r}_1)]=\gamma S_n^l(\mathfrak{a})$, $\gamma=P_n(\cos\alpha)$, $|l|\leqq n$; in this circumstance,

$$P[\mathfrak{r}_t\in d\mathfrak{b}]=\sum_{n=0}^{+\infty}\frac{2n+1}{4\pi}P_n(\cos\alpha)^t P_n(\mathfrak{a}\cdot\mathfrak{b})d\mathfrak{b},$$

where $\mathfrak{r}_t$ is the position of the wandering particle t units of time after leaving $\mathfrak{a}$ and $d\mathfrak{b}$ is the element of surface area.

Consider now a closed Riemannian space M; about each point $\mathfrak{a}\in M$ draw a (geodesic) sphere $\widehat{\mathfrak{a}\mathfrak{b}}=\alpha$ of radius α; think of a random walk in which the particle waiting at $\mathfrak{a}$ jumps at time 1 out onto the spherical surface with the uniform angular distribution do on the directions of the geodesics issuing from $\mathfrak{a}$; and let us call M a commutative space if these motions have common eigenfunctions f spanning $L^2(M, \sqrt{g(\mathfrak{r})}\,d\mathfrak{r})$ as in the spherical case:

$$\int_{\widehat{\mathfrak{a}\mathfrak{b}}=\alpha} f(\mathfrak{b})do=\gamma f(\mathfrak{a}),\quad \gamma=\gamma(\alpha).$$

A space is said to be harmonic if the surface area on $\widehat{\mathfrak{a}\mathfrak{b}}=\alpha$ can be expressed as do times a factor depending upon the radius alone; a harmonic space is commutative, but the converse is false. A closed analytic Riemannian manifold is commutative if and only if certain differential operators commute, but the general problem of characterizing commutative spaces is not solved, though several necessary conditions are deduced. Other topics touched upon include the convergence of the eigenfunction expansion for $P[\mathfrak{r}_t\in d\mathfrak{b}]$ and its convergence as $t\uparrow+\infty$ to the uniform distribution $(\int\sqrt{g}\,d\mathfrak{r})^{-1}\sqrt{g}\,d\mathfrak{r}$.

A brief list of related papers would include S. Bochner [Proc. Nat. Acad. Sci. U.S.A. **40** (1954), 1141–1147; MR **16**, 834] and G. A. Hunt [Trans. Amer. Math. Soc. **81** (1956), 264–293; MR **18**, 54] on differential motions on Lie groups and homogeneous spaces, and S. Helgason [#8457] on mean-value operators associated with invariant differential operators on homogeneous spaces.

H. P. McKean, Jr. (Cambridge, Mass.)

8570:

Sirao, Tunekiti. On the continuity of Brownian motion with a multidimensional parameter. Nagoya Math. J. **16** (1960), 135–156.

By Brownian motion with an N-dimensional parameter the author means a stochastic process $\{X(A), A \in E_N\}$, where E_N is N-dimensional Euclidean space, satisfying the conditions: (1) $X(A_1), \cdots, X(A_n)$ are jointly normally distributed with means zero for any n points $A_1, \cdots, A_n$ in E_N; (2) $2E[X(A_i)X(A_j)] = \text{dis}\,(O, A_i) + \text{dis}\,(O, A_j) - \text{dis}\,(A_i, A_j)$, where O is the origin of E_N, and dis (A, B) is the Euclidean distance between A and B; (3) $X(O) = 0$ and $X(A)$ is a continuous function of A, for almost all sample paths. The author defines notions of upper and lower classes with regard to the behavior of the process $\{X(A), A \in E_N\}$ at O, and with regard to the uniform continuity of the process. Using a method of Chung and Erdös [Trans. Amer. Math. Soc. **72** (1952), 179–186; MR **13**, 567] the author proves two theorems giving conditions for a function to belong to these classes.

E. Parzen (Stanford, Calif.)

8571:

Mercer, A. A queueing problem in which the arrival times of the customers are scheduled. J. Roy. Statist. Soc. Ser. B **22** (1960), 108–113.

Author's summary: "The queueing problem considered is that in which (i) the customers are scheduled to arrive at equal time intervals but a customer may arrive at any time after the start of the interval during which he was scheduled to arrive, or may not even arrive at all, the lateness distribution being perfectly general; (ii) the total service-time of a customer is supposed to consist of a finite number of stages, such that the times spent in each of the stages are independent and similarly distributed, and the probability of leaving a stage at any time depends on the time since the current scheduling interval began. A method is discussed which enables the non-equilibrium distribution of the queue-length at any time to be derived, and results for the equilibrium distribution are given. Most of the results relate to the case in which there is a single server but extensions of the argument for more than one server are outlined."

W. L. Smith (Chapel Hill, N.C.)

8572:

Conolly, B. W. Queueing at a single serving point with group arrival. J. Roy. Statist. Soc. Ser. B **22** (1960), 285–298.

The single server queueing system examined is as follows: the service time has an exponential distribution function, and the arrivals are in groups of C in a recurrent process, that is, the interarrival intervals are independent and have the same general distribution function. The groups are served in order of arrival, the order of service within a group is a matter of indifference, and there are no defections. The results of chief interest are the steady state values of the size of the queue, the waiting time, and the server's idle and busy period. As the author notes, the system is formally similar to a system with single arrivals and a service time distribution function of Erlang-C type; each service consists of C consecutive phases, each with the same exponential distribution function), previously studied by David M. G. Wishart [Ann. Math. Statist. **27** (1956), 768–779; MR **18**, 157]. The treatments differ, the present one employing a supplementary variable, the time since the last arrival. All results are expressed in terms of the C roots (less than unity) of the equation $x^C = a^*[(1-x)b^{-1}]$ where $a^*(x)$ is the Laplace-Stieltjes transform of the interarrival distribution function, b the average service time. Indeed, the author limits himself to those interarrival distribution functions for which all roots are distinct. Numerical results are given for the special case of determinate arrivals (successive groups arrive at fixed intervals). It may be noticed that the system has a realization in the discharge of ships arriving at a port in convoy.

J. Riordan (New York)

8573:

Gnedenko, B. W. Über einige Aspekte der Entwicklung der Theorie der Warteschlangen. Math.-Tech.-Wirtschaft **7** (1960), 162–166.

The author gives a general survey of the importance of and some problems involved in queueing theory. The author and his students (Bródi, Farinitš, Jarošenko and Kowalenko) have achieved several new results in the theory of queues, of which he gives a short account. These results are concerned with telephone loss systems, waiting systems, queues with limited waiting times, the stochastic law of occupation time, etc. They contain the generalizations of certain results of D. Y. Barrer [Operations Res. **5** (1957), 644–649, 650–656; MR **19**, 779] and the reviewer [Acta Math. Acad. Sci. Hungar. **6** (1955), 101–129; Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. **8** (1958), 151–210; MR **17**, 51; **20** #2803]. The papers in which these results will appear are in process of publication.

L. Takács (New York)

8574:

Prabhu, N. U. Application of storage theory to queues with Poisson arrivals. Ann. Math. Statist. **31** (1960), 475–482.

The waiting time distribution of the queueing system $M/G/1$ is considered. The Pollaczek-Khintchine formula for the transform of the distribution $W(t)$ when the process is stationary is studied by identifying it with a formula in the theory of continuous time storage processes and the transition distribution function of the waiting time obtained.

P. A. P. Moran (Canberra)

8575:

Ghosal, A. Emptiness in the finite dam. Ann. Math. Statist. **31** (1960), 803–808.

The author considers the storage system of the reviewer in which the store (e.g., a dam) has a finite capacity and at discrete intervals of time receives a series of independent random inputs. With suitable overflow and release rules the author considers the probability that, starting with some initial content, the dam empties before it fills up.

An integral equation is obtained for this probability and is shown to be equivalent to the integral equation for the stationary distribution. Solutions for particular distributions of input are discussed but uniqueness of the solution is not considered. *P. A. P. Moran* (Canberra)

8576:

Gani, J. First emptiness of two dams in parallel. Ann. Math. Statist. **32** (1961), 219–229.

Two dams with continuous release are fed by an input of unit amount occurring at points determined by a Poisson process, and fed into the dam with smaller content. The determination of the first emptiness distribution is shown to be equivalent to that of a single dam with inputs which are alternately of two different sizes. A recurrence relation for the probabilities is found, and from this an equation for a generating function, and a formal solution. The problem is then reformulated as an occupancy problem with restrictions, and a method of computation for the solution given. A generalisation to the case where the two ordered inputs have probability distributions is considered. *P. A. P. Moran* (Canberra)

8577:

Finch, P. D. The transient behaviour of a coincidence variate in telephone traffic. Ann. Math. Statist. **32** (1961), 230–234.

The author investigates a telephone loss system with m lines for recurrent input and exponential distribution of holding times. The interarrival times of the calls are identically distributed, mutually independent, positive random variables with distribution function $A(x)$, the holding times are mutually independent random variables with distribution function $B(x)=1-e^{-\mu x}$ $(x\geq 0)$ and independent of the arrival times. If an arriving call finds every line busy then it is lost. Denote by $P_{k,n}$ the probability that the nth arriving call finds k lines busy. The author determines the generating function $\sum_{n=1}^{\infty} P_{k,n}w^{n-1}$ $(|w|<1)$ and, in the case $m=\infty$, the probabilities $P_{k,n}$.

L. Takács (New York)

8578:

Howie, A. J.; Shenton, L. R. The efficiency of automatic winding machines with constant patrolling time. J. Roy. Statist. Soc. Ser. B **21** (1959), 381–395.

Authors' summary: "The problem is that of maximizing the average efficiency of a machine which (i) stops when its total running time has reached a fixed value a, (ii) has accidental stops, and (iii) is patrolled for servicing (i.e., restarting if necessary) at a fixed interval P which may be less than a, efficiency being defined as the ratio of the running time to the total time under consideration. The probability distribution of the number of patrols required to exhaust the total running time of the machine is derived in the case where the accidental stops have uniform probability, and is found to be expressible in terms of Laguerre polynomials. The properties of the distribution are illustrated by an example. Its asymptotic normality as a increases is established. The properties of the average efficiency are discussed."

M. Rosenblatt (Providence, R.I.)

STATISTICS

See also A8287, 8543, 8644, 8697, 8698, 8699, 8700, 8710, 8712, 8963, 9291, 9359, 9364, 9365, 9366.

8579:

Huntsberger, David V. ★Elements of statistical inference. Allyn and Bacon, Inc., Boston, Mass., 1961. xi+291 pp. $7.25.

This book is intended as an introduction to some of the basic concepts and techniques of statistical inference. Mathematical prerequisites have been kept to a minimum, a knowledge of arithmetic and some algebra being sufficient.

8580:

Quenouille, M. H. ★Rapid statistical calculations: A collection of distribution-free and easy methods of estimation and testing. Hafner Publishing Co., New York, 1959. xv+81 pp. $1.65.

A pocket-manual, presenting the practical details of rapid methods in a brief, straightforward fashion. There are methods involving mean level or scatter of groups of observations, methods comparing frequencies or proportions and methods involving association.

8581:

Gini, Corrado. Sulla frequenza delle cifre iniziali dei numeri osservati. Bull. Inst. Internat. Statist. **35** (1957), no. 2, 57–76. (English and French summaries)

8582:

Ros-Jimeno, José. The statistical mentality. Bull. Inst. Internat. Statist. **35** (1957), no. 2, 85–90. (Spanish. French and English summaries)

Expository. *L. J. Savage* (Ann Arbor, Mich.)

8583:

Goodman, Leo A. On the exact variance of products. J. Amer. Statist. Assoc. **55** (1960), 708–713.

An exact formula for the variance of a product of random variables is given and compared with the usual approximation. A comparison is made between the variances of some estimates of the product of the two means of a bivariate population, and estimation of these variances is discussed. *J. W. Pratt* (Cambridge, Mass.)

8584:

Lieberman, Gerald J.; Owen, Donald B. ★Tables of the hypergeometric probability distribution. Stanford Studies in Mathematics and Statistics, III. Stanford University Press, Stanford, Calif., 1961. vii+726 pp. $15.00.

This volume gives tables of both point and cumulative probabilities of the hypergeometric distribution

$$p(N, n, k, x) = \binom{n}{x}\binom{N-n}{k-x}\Big/\binom{N}{N-k},$$

$$\max[0, n-k-N] \leq x \leq \min[n, k]$$

to six decimals for $N=1(1)49$, $50(10)100$, and $100(100)2000$. For $N \leqq 25$ the tabulation is for all x, k, n such that $k \leqq n$; for $25 \leqq N \leqq 100$ these probabilities are shown for values of the random variable x and the parameters n, k, restricted by the inequality $x \leqq k \leqq n \leqq \frac{1}{2}N$. Other probabilities for these N can be obtained by various symmetries which are noted.

The tabulation for $N=1000$ shows the various probabilities for $n=500$ and all x, k satisfying this restriction. For other N from 100 to 2000 the probabilities are tabulated for the parameter values $n=\frac{1}{2}N$, $k=n-1$, n. Also tabulated is $\log N!$ to fifteen places for $N=1(1)2000$.

The introduction includes a series of applications to sequential procedures, 2×2 tables, distribution of the number of exceedances, Bayesian prediction with a binomial distribution and to sampling inspection. Various approximations to the hypergeometric distribution as well as procedures of interpolation in the tables (for other N) are also treated. The final section of the introduction lists a number of useful formulas on sums of combinatorials.

D. G. Chapman (Seattle, Wash.)

8585:

Zia-ud-Din, M. On expression of the k-statistic k_{11} in terms of power sums and sample moments. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 102–106. (French summary)

This paper is a continuation of Ann. Math. Statist. **25** (1954), 800–803 [MR **16**, 381] and gives an explicit expression for the eleventh k-statistic in terms of power sums and and the sample size. *I. Olkin* (Minneapolis, Minn.)

8586:

Hodges, J. L., Jr.; Le Cam, Lucien. The Poisson approximation to the Poisson binomial distribution. Ann. Math. Statist. **31** (1960), 737–740.

Let $x_1, \cdots, x_n$ be independent binomial random variables such that $P(x_i=1)=p_i$, $P(x_0=0)=1-p_i$. Let $y_1, \cdots, y_n$ be independent random variables such that

$$\begin{aligned} P(x_i = y_i = 1) &= p_i e^{-p_i}, \\ P(x_i = 1, y_i = 0) &= p_i(1-e^{-p_i}), \\ P(x_i = 0, y_i = 0) &= e^{-p_i} - p_i(1-e^{-p_i}), \\ P(x_i = 0, y_i = y) &= p_i^y e^{-p_i}/y! \quad (y = 2, 3, \cdots). \end{aligned}$$

Let $t=\sum y_i$ and $s=\sum x_i$ and $D=\sup_u |P(s \leqq u) - P(t \leqq u)|$. The authors show that $D \leqq 3\alpha^{1/2}$, where

$$\alpha = \max\{p_1, \cdots, p_n\}.$$

S. S. Wilks (Princeton, N.J.)

8587:

Pachares, James. Tables of confidence limits for the binomial distribution. J. Amer. Statist. Assoc. **55** (1960), 521–533.

Author's summary: "Values of p (times 10,000) satisfying the equation $\alpha=\sum_{r=0}^{x}\binom{n}{r}p^r(1-p)^{n-r}$ are given to four figures for $n=55(5)100$, $x=0(1)n-1$ and $\alpha=.005$, .010, .025, .050." *T. N. E. Greville* (Kensington, Md.)

8588:

Bush, K. A.; Olkin, I. Extrema of quadratic forms with applications to statistics. Biometrika **46** (1959), 483–486.

The authors discuss the problems (i) find min wAw' when $wB=a$, where w is an unknown row-vector with p components, A is a $(p\times p)$ matrix and positive-definite, and B is a $(p\times k)$ matrix $(p \geqq k)$ and is of rank k; (ii) find $\max (wAw')/(wBw')$ where w is as in (i), A is a $(p\times p)$ positive-semi-definite matrix, and B is a $(p\times p)$ positive-definite matrix. There are applications to (a) stratified sampling, (b) a moment-problem, (c) discrimination, (d) canonical correlations, and (e) the problem discussed by W. Karush and N. Z. Wolfsohn in Proc. Amer. Math. Soc. **6** (1955), 323–332 [MR **16**, 1047].

Unfortunately the paper contains an error, which will be eliminated in a further note to be submitted to Biometrika. *D. G. Kendall* (Oxford)

8589:

Laubscher, Nico F. Normalizing the noncentral t and F distributions. Ann. Math. Statist. **31** (1960), 1105–1112.

The noncentral t and "topside" noncentral F distributions are subjected to variance stabilizing transformations. Numerical comparisons are made between the resulting distributions and the standard normal distribution.

J. G. Wendel (Ann Arbor, Mich.)

8590:

Bose, P. K.; Sen, P. K. On U-shaped curves. Calcutta Statist. Assoc. Bull. **10** (1960), 54–57.

8591:

Perayya Sastry, M. Estimation of the parameters of censored truncated (CT-) chi-distributed radial error. Calcutta Statist. Assoc. Bull. **10** (1960), 41–47.

8592:

Birnbaum, Allan; Healy, William C., Jr. Estimates with prescribed variance based on two-stage sampling. Ann. Math. Statist. **31** (1960), 662–676.

Let $S=\{x\}$ be the sample space for a single random observation X on which a density or discrete elementary probability function $f(x, \theta)$ is defined for each θ in a parameter space Ω. Let $\rho(\theta)$ be a real-valued function of θ. It is assumed that (i) for each non-sequential sample size n $(\geqq n_0)$ there exists an unbiased estimator $t(X_n)$ of ρ, (ii) for each non-sequential sample size m $(\geqq m_0)$ there exists a measurable integer-valued function $n(X)$ such that $E\sigma^2(\theta, n(X_m)) \leqq B(\theta)$, where $\sigma^2(\theta, n)$ is the variance of t for sample size n. Under these assumptions it is shown that the following procedure yields an unbiased estimate of ρ with variance $\leqq B(\theta)$: (1) take a sample of m observations $(m \geqq m_0)$ and compute $n(X_m)$; (2) take a second independent sample of size $n(X_m)$; (3) estimate ρ by $t(X_{m+1}, \cdots, X_{m+n})$. The method is applied to estimation of the means of binomial, Poisson and hypergeometric distributions; scale parameters in general and of the Gamma distribution in particular; the variance of a normal distribution and a component of variance. Optimum sampling rules are discussed for some of these estimators and some tables are given to facilitate their use. It is also shown that the efficiency of the method is high despite the fact that the final estimator is not based on $X_1, \cdots, X_m$. *D. G. Chapman* (Seattle, Wash.)

8593:

Des Raj. On matching lists by samples. J. Amer. Statist. Assoc. **56** (1961), 151–155.

In the present note the author considers the problem of estimating the number of names common to two lists by sampling methods investigated earlier by L. A. Goodman [Ann. Math. Statist. **23** (1952), 632–634; MR **14**, 390] and W. E. Deming and G. J. Glasser [J. Amer. Statist. Assoc. **54** (1959), 403–415; MR **21** #4504]. He discusses the case when two lists are merged and sampled as one and compares it with the case when two lists are sampled separately. *R. G. Laha* (Calcutta)

8594:

Richter, Donald. Two-stage experiments for estimating a common mean. Ann. Math. Statist. **31** (1960), 1164–1173.

Given two normal populations of common mean μ and variances, σ_1^2, $\theta\sigma_1^2$, respectively (all parameters unknown), a sample of n independent observations is taken as follows: m ($\leq n/2$) observations are taken from each population. Sample variances are computed. $n-2m$ final observations are drawn from the population which has the smaller sample variance. The problem is to estimate μ. Estimates are restricted to weighted averages of the two sample means, respectively computed from all the observations on each population, where the weights are non-negative random variables (independent of the sample means) which add to one. These estimates are all unbiased. The criterion of goodness chosen (called the risk) is $R_n(\theta, m) = \text{var estimate}/\sigma_1^2 \min(1, \theta)$, shown to be always ≥ 1. m, dependent on n ($=m(n)$, say), is called uniformly consistent relative to a given estimate in the above class, if $\lim_{n\to\infty} \sup_\theta R_n(\theta, m) = 1$. It is called minimax relative to the estimate, if the above sup is finite and minimum among all other possible choices for m. An asymptotically minimax solution is exhibited relative to the mean, $\hat{\mu}_1$, say, of the total sample from the population producing the smaller first sample variance. Uniform consistency is characterized for all estimates which have the property that

$$\sup_\theta R_n(\theta, m) \leq \max_\theta R_{1n}(\theta, m) + o(1), \quad \text{as } n \to \infty,$$

where $R_{1n}(\theta, m)$ is the risk associated with $\hat{\mu}_1$. Minimax solutions are shown to be asymptotically the same for all estimates satisfying

$$R_n(\theta, m) \leq R_{2n}(\theta, m) + o(n^{-2/3}), \quad \text{as } n \to \infty, \text{ all } \theta > 0,$$

where $R_{2n}(\theta, m)$ is a specified lower bound for the risk of all estimators in the class considered. An estimate with this property is exhibited.

M. Skibinsky (W. Lafayette, Ind.)

8595:

Durbin, J. Sampling theory for estimates based on fewer individuals than the number selected. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 113–119. (French summary)

The author gives the variance of sample mean which is usually based on fewer individuals than the number originally selected, because of, for example, non-response.

He makes $\bar{y}$ = (total value estimated by sample observations; they are taken to be zero for non-respondents in sample)/(total size estimated by sample size—size of non-respondents) as an estimator of $\bar{Y}$ = (total value of typical population observations; they are taken to be zero for non-respondents)/(population size—size of non-respondents in population). He derives the mean square error (variance as the first order of approximation) of the estimators in stratified and multistage random sampling.

C. Hayashi (Tokyo)

8596:

Stuart, Alan. The measurement of estimation and test efficiency. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 79–86. (French summary)

8597:

Berkson, Joseph. Problems recently discussed regarding estimating the logistic curve. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 207–211.

Further remarks on the sufficiency and consistency (or lack of them) of minimum logit χ^2 and maximum likelihood estimates of the logistic parameters.

J. W. Pratt (Cambridge, Mass.)

8598:

David, H. A. A conservative property of binomial tests. Ann. Math. Statist. **31** (1960), 1205–1207.

Let S_n denote the number of successes in n independent trials, the probability of success in the ith trial being π_i ($i=1, 2, \cdots, n$). It has been shown by the reviewer [same Ann. **27** (1956), 713–721; MR **18**, 240] that if $\pi_1 + \cdots + \pi_n = n\pi_0$ is fixed, then for any integer a such that $n\pi_0 + 1 \leq a \leq n$ the probability of $S_n \geq a$ is maximized for $\pi_1 = \cdots = \pi_n = \pi_0$. In the present note an alternative argument to prove this result is presented. However, no justification is given for the conclusion following equation (4). *W. Hoeffding* (Chapel Hill, N.C.)

8599:

Harter, H. Leon. Critical values for Duncan's new multiple range test. Biometrics **16** (1960), 671–685.

Limited tables to facilitate the application of his test were included in Duncan's article [Biometrics **11** (1955), 1–42; MR **16**, 842]. The author has corrected these results, giving answers to four decimal places, for significance levels $\alpha = 0.001, 0.005, 0.01, 0.05$ and 0.10, sample sizes $n = 2(1)20(2)40(10)100$ and d.f. $\nu = 1(1)20,24,30,40,60,120,\infty$.

C. J. Maloney (Frederick, Md.)

8600:

Lowe, J. R. A table of the integral of the bivariate normal distribution over an offset circle. J. Roy. Statist. Soc. Ser. B **22** (1960), 177–187.

A table is given containing values rounded to three decimal places of the integral $I = \iint f(x, y)dxdy$ over a circle with arbitrary center and radius, where $f(x, y) = (1/2\pi\sigma_1\sigma_2) \exp\{-x^2/2\sigma_1^2 - y^2/2\sigma_2^2\}$. Taking the circle as of radius a with a center (u, v), ranges of circle size, $a/\sigma_2 \leq 64$, and of distribution ellipticity, $1 \leq \sigma_1/\sigma_2 \leq 8$, are covered, and the standardized biases u/σ_1, v/σ_2 are printed as column and line headings, respectively. The biases are given exactly without rounding, and are chosen to cover the u, v region in which the variation of I is appreciable.

T. N. E. Greville (Kensington, Md.)

8601:

Kshirsagar, A. M. Some extensions of the multivariate generalization t-distribution and the multivariate generalization of the distribution of the regression coefficient. Proc. Cambridge Philos. Soc. **57** (1961), 80–85.

This paper is concerned with three noncentral distributions: (i) multivariate t-distribution; (ii) a special studentized Wishart distribution; (iii) distribution of regression coefficients. The essential feature is exemplified by (i). If $x \sim N(\mu, \sigma^2 I)$, and $s^2/\tau^2 \sim \chi_m^2$ are independently distributed, the null distribution of $t = x/s$, i.e., $\tau^2 = \sigma^2$, $\mu = 0$, was obtained by Bechhofer, Dunnett, and Sobel [Biometrika **41** (1954), 170–176; MR **15**, 885]. The author now obtains the noncentral distribution of t. In each of the three cases, the denominator introduces additional noncentral parameters. *I. Olkin* (Minneapolis, Minn.)

8602:

Ruben, Harold. Probability content of regions under spherical normal distributions. I. Ann. Math. Statist. **31** (1960), 598–618.

If $x_1, \cdots, x_N$ are independent random variables, each having the normal distribution $N(0, 1)$, $(x_1, \cdots, x_N)$ has an N-dimensional spherical distribution. In this paper the author gives a systematic presentation of the problem of determining the probability that $(x_1, \cdots, x_N) \in R$ for various N-dimensional regions R in the sample space which occur in statistical problems. In the early part of the paper he considers the familiar cases where R is a half space of form $\sum_1^N a_i x_i \leq K$; a sphere centered [not centered] at the origin (resulting in the central [noncentral] chi-square distribution); a spherical half-cone with vertex at [not at] the origin (resulting in the central [non-central] Student t-distribution). In the latter part of the paper he determines the probability content of R in the cases where R is (i) the region bounded by a cylinder with axis through the origin, (ii) the region bounded by a cylinder with axis not through the origin, (iii) the region bounded by the surface of revolution for which

$$\left[\frac{x_1^2 + \cdots + x_{N-p-1}^2}{N-p-1}\right] \Big/ \left[\frac{x_{N-p}^2 + \cdots + x_N^2}{(p+1)}\right] \geq K > 0,$$

(iv) an ellipsoid with center at the origin, (v) a regular simplex with center at the origin. The distribution of R as a function of edge length in case (v) is a new result.

S. S. Wilks (Princeton, N.J.)

8603:

Ruben, Harold. Probability content of regions under spherical normal distributions. II. The distribution of the range in normal samples. Ann. Math. Statist. **31** (1960), 1113–1121.

Suppose $x_1, \cdots, x_n$ are independent random variables all having identical normal distributions $N(0, 1)$. Let $w = \max(x_1, \cdots, x_n) - \min(x_1, \cdots, x_n)$. It has long been known that the cumulative distribution function (c.d.f.) of w is given by

$$G_n(w) = n \int_{-\infty}^{\infty} f(x)[F(x+w) - F(x)]^{n-1}\, dx,$$

where $F(x)$ is the c.d.f. of $N(0, 1)$ and $f(x) = dF(x)/dx$. The author shows that $G_n(w)$ can be expressed as the product of the sample size n and the probability content of a certain $(n-1)$-dimensional parallellotope as evaluated from a spherical normal distribution in $n-1$ dimensions.

S. S. Wilks (Princeton, N.J.)

8604:

Ruben, Harold. Probability content of regions under spherical normal distributions. III. The bivariate normal integral. Ann. Math. Statist. **32** (1961), 171–186.

Let $f(x, y)$ be the density function of a bivariate normal distribution with zero means, unit variances, and correlation coefficient ρ. Let

$$L(x_0, y_0; \rho) = \int_{y_0}^{\infty} \int_{x_0}^{\infty} f(x, y)\, dx\, dy.$$

$L(x_0, y_0; \rho)$ has been throughly investigated by many authors. In the present paper the author develops an infinite series form for $L(x_0, y_0; \rho)$ which converges more rapidly for values of $|\rho|$ near 1 than previously obtained series for $L(x_0, y_0; \rho)$. *S. S. Wilks* (Princeton, N.J.)

8605:

Linhart, H. A measure of predictive precision in regression analysis. Ann. Math. Statist. **31** (1960), 399–404.

Suppose $(x_{0\alpha}, x_{1\alpha}, \cdots, x_{k\alpha})$, $\alpha = 1, \cdots, n$, is a sample of size n from a $(k+1)$-dimensional normal distribution, and let $(y_0, y_1, \cdots, y_k)$ be a further independent element from the same distribution. If the sample of size n is used to estimate the means and covariance matrix of the population distribution and if $y_1, \cdots, y_k$ are known but not y_0, let $\hat{y}_0$ be the usual least squares estimator for y_0 in terms of $y_1, \cdots, y_k$ and the estimators for the means and covariance matrix of the $(k+1)$-dimensional normal distribution from which the sample elements are drawn. Let $(\bar{x}_0, \bar{x}_1, \cdots, \bar{x}_k)$ be the estimator for the vector of population means and L_0 the estimator for the population covariance matrix, using only the initial sample of size n. Let L be L_0 reduced by taking $i, j = 1, \cdots, k$. Let $|L_0|$ and $|L|$ be the determinants of L_0 and L. Let $L_0^{-1} = \|l_0^{ij}\|$, and

$$T = \sum_{i,j=1}^{k} (y_i - \bar{x}_i)(y_j - \bar{x}_j) l_0^{ij}.$$

It is shown that $(\hat{y}_0 - t_{\alpha/2, n-k-1}\sqrt{R}, \hat{y}_0 + t_{\alpha/2, n-k-1}\sqrt{R})$ is a $100(1-\alpha)\%$ confidence interval l for y_0, where $t_{\alpha/2, n-k-1}$ is the upper $100\alpha/2\%$ point on the Student t-distribution with $n-k-1$ degrees of freedom and

$$R = [|L|(1+n+T)]/[|L_0|(n-k-1)].$$

The author examines the precision of the interval estimator l for $\hat{y}_0$ by two methods: first, by using the Neyman "shortness" $S(\delta)$, that is, the mean value (with respect to T) of the conditional probability that l covers $y_0 + \delta$, given T; and second, by finding the distribution of l and its mean value. He concludes that the first method has some advantages over the second. *S. S. Wilks* (Princeton, N.J.)

8606:

Buck, S. F. A method of estimation of missing values in multivariate data suitable for use with an electronic computer. J. Roy. Statist. Soc. Ser. B **22** (1960), 302–306.

In this note the author describes the use of standard linear regression for estimating missing components in

random samples of vector observations from multivariate distributions. As pointed out by the author this method is particularly suitable for electronic computors.

S. S. Wilks (Princeton, N.J.)

8607:

Kshirsagar, A. M. The non-central multivariate beta distribution. Ann. Math. Statist. **32** (1961), 104–111.

If $\mathbf{A}$ and $\mathbf{B}$ are symmetric matrices of order p, with independent Wishart distributions with degrees of freedom f_1 and f_2, if $\mathbf{C}$ is a lower triangular matrix such that $\mathbf{A}+\mathbf{B}=\mathbf{CC}'$, and if $\mathbf{L}$ is given by $\mathbf{A}=\mathbf{CLC}'$, then $\mathbf{L}$ has a multivariate beta distribution with density function $k|\mathbf{L}|^{(f_1-p-1)/2}|\mathbf{I}-\mathbf{L}|^{(f_2-p-1)/2}$ when $\mathbf{L}$ and $\mathbf{I}-\mathbf{L}$ are positive definite, 0 otherwise (k depends only on p, f_1 and f_2). The author derives the distribution (non-central multivariate beta) when $\mathbf{B}$ has a non-central Wishart distribution of the Anderson "linear" type [same Ann. **17** (1946), 409–431; MR **8**, 394] the "planar" case seems intractable.

Wilk's likelihood-ratio criterion for testing the independence of two multivariate normal distributions is identified in the central case as $|\mathbf{L}|=\prod_{i=1}^{p} t_{ii}^2$, where $\mathbf{T}$ is a lower triangular matrix (t_{ij}) such that $\mathbf{L}=\mathbf{TT}'$; the t_i^2 are independent beta variates with parameters $\frac{1}{2}(f_1+1-i)$ and $\frac{1}{2}f_2$. In the "linear" non-central case, the t_{ii} for $i=2, 3, \cdots, p$ are still beta variates but t_{11} has a non-central distribution depending on the correlation between x_1 and y_1.

The role of $|\mathbf{L}|$ in discriminant analysis is discussed.

E. S. Keeping (Edmonton, Alta.)

8608:

Uzawa, Hirofumi. Locally most powerful rank tests for two-sample problems. Ann. Math. Statist. **31** (1960), 685–702.

Let $(X_1, \cdots, X_{n_1})$ be a sample from the continuous distribution $F(x)$ and $(X_{n_1+1}, \cdots, X_n)$ be a sample from the continuous distribution $G(x)$. Define $Z_i=O(1)$ if the ith smallest X is from $F(G)$. A locally most powerful rank test is based on a function of $(Z_1, \cdots, Z_n)$ and is most powerful for $0<\theta\leqq\theta_0$ for some "uniformly differentiable in θ" pair $F=F(x, \theta)$ and $G=G(x, \theta)$ such that $F(x, 0)=G(x, 0)$.

The geometry of moment spaces is used to characterize locally most powerful rank tests for different classes of alternatives, e.g., (1) $F(x)\neq G(x)$, (2) $F(x)\leqq G(x)$, (3) $F(x)\neq G(x)$ and $\int F(x)dG(x)\geqq\frac{1}{2}$, (4) $F(x)\neq G(x)$ and $F(x)+F(-x)=G(x)+G(-x)=1$. In each case a necessary condition is that the test procedure is to reject for large values of a statistic of the form $\sum a_iZ_i$. In case (1) this is also sufficient; in case (2) a sufficient condition is given in terms of a Hankel determinant of a linear transformation of the $(a_1, \cdots, a_n)$; in case (3) a sufficient condition is $\sum^*(\frac{1}{2}(n+1)-s)(a_{n-s}-a_s)\geqq 0$, where $\sum^*$ ranges over $s=1, \cdots, [\frac{1}{2}(n+1)]$; and in case (4) a sufficient condition is $a_s=a_{n-s+1}$ for $s=1, \cdots, n$.

I. R. Savage (Cambridge, Mass.)

8609:

Ansari, A. R.; Bradley, R. A. Rank-sum tests for dispersions. Ann. Math. Statist. **31** (1960), 1174–1189.

Authors' summary: "This paper deals with non-parametric two-sample tests on dispersions. Two samples, X- and Y-samples of m and n independent observations from populations with continuous cumulative distribution functions $F(u)$ and $G(u)$ respectively, are considered. It is required for the basic test that the difference in locations (medians) of the two populations be known and, when this is so, the two samples may be adjusted to have equal locations. Taking these location parameters to be zero without loss of generality, we test the hypothesis that $G(u)\equiv F(u)$ against alternatives of the form $G(u)\equiv F(\theta u)$, $\theta\neq 1$. The two samples are ordered in a single joint array and ranks are assigned from each end of the joint array towards the middle. The statistic used is W, the sum of ranks for the X-sample." Tables of significance levels, some asymptotic results, a review of other tests, and comparisons of efficiency are also presented.

I. Olkin (Minneapolis, Minn.)

8610:

Uhlmann, W. Zu einem nichtparametrischen Test von E. L. Lehmann. Metrika **2** (1959), 169–185. (English summary)

The author rederives the rank test proposed by Lehmann [Ann. Math. Statist. **24** (1953), 23–43; MR **14**, 888] for the two-sided two-sample problem. The purpose of the paper is to give tables of the critical values of this test for sample sizes $m=n=4, 5, \cdots, 9$ and representative significance levels less than or equal to .06. "Then some theorems are proved which are useful for the calculation of the tables and their possible extension. They moreover show the close connection between Lehmann's test and the two-sided Wilcoxon test." (From the author's summary)

R. Pyke (Seattle, Wash.)

8611:

Birnbaum, Z. W.; Hall, R. A. Small sample distributions for multi-sample statistics of the Smirnov type. Ann. Math. Statist. **31** (1960), 710–720.

Various k-sample analogues of the Kolmogorov-Smirnov test have been given by H. T. David [same Ann. **29** (1958), 842–851; MR **20** #391] when $k=3$ and by the reviewer [ibid. **30** (1959), 420–447; MR **21** #1668] for all k; exact as well as limiting results were given in the former case, while limiting results were proved and tabulated in the latter. If F_j, $1\leqq j\leqq k$, are k univariate sample d.f.'s, the authors consider the statistics $U=\sup_{x,i,j}|F_i(x)-F_j(x)|$ and $V=\sup_{x,i<j}[F_i(x)-F_j(x)]$, which were mentioned but not treated in detail in the above papers. By counting paths, a simple difference equation is derived for a function from which the probability laws of these statistics can be obtained numerically. The probability law of the statistic U is tabulated for $k=3$ for selected equal sample sizes $\leqq 40$. Tables are also given for the case $k=2$, extending previous ones obtained by various other authors (especially F. J. Massey) using the same difference equation. The limiting law, which for $k=3$ can be obtained by solving the diffusion equation with boundary a linearly expanding hexagon (as with a triangle and circle in the above mentioned papers), is not given. The authors warn against the use of such limiting laws for moderate sample sizes, and suggest for $k\geqq 3$ using standard inequalities relating the exact probabilities to ones tabled for the case $k=2$.

J. Kiefer (Ithaca, N.Y.)

8612:

Ross, Alan. Variance estimates in "optimum" sample designs. J. Amer. Statist. Assoc. **56** (1961), 135–142.

The author observes that the optimum allocation of sampling units for estimation of variances may not be identical with the optimum allocation for estimating means and totals, which is usually the principal consideration. Simple stratified and (a special) two-stage sampling scheme with and without cost function considerations are examined, and the loss in efficiency for estimating variances when a sampling allocation, optimum for estimating means, is used, is examined numerically for each design. The author appears to deprecate the importance of his own work. *C. J. Maloney* (Frederick, Md.)

8613:

Shah, K. R. Optimality criteria for incomplete block designs. Ann. Math. Statist. **31** (1960), 791–794.

Let C be Bose's (singular) information matrix for the treatment effects in an incomplete block design, and let λ_i be the eigenvalues of C, excluding one zero value. Optimality criteria which have been considered previously for comparing incomplete block designs include minimizing $\prod \lambda_i^{-1}$ or $\sum \lambda_i^{-1}$ or $\max_i \lambda_i^{-1}$, etc. The meanings of these and other criteria were discussed by the reviewer [same Ann. **29** (1958), 675–699; MR **20** #4910], who also showed that the optimality of the BIBD in any of these senses, as well as of other appropriately symmetrical designs in other settings, is a simple consequence of the symmetry of C for such a design and the maximization of $\operatorname{tr} C$ by it. In the incomplete block design setting with a fixed number of blocks of fixed equal size, $\operatorname{tr} C$ is a constant. If no BIBD exists, the various criteria need not lead to the same design. In this case the author suggests the minimization of $\sum (\lambda_i - \bar{\lambda})^2$ (where $\bar{\lambda}$ is the average of the λ_i's), hence of $\sum \lambda_i^2 = \operatorname{tr} C^2$. The computability of this last form is the reason for the author's suggestion, no precise statistical properties being given.

J. Kiefer (Ithaca, N.Y.)

8614:

Raghavarao, Damaraju. Some aspects of weighing designs. Ann. Math. Statist. **31** (1960), 878–884.

The author continues his study [same Ann. **30** (1959), 295–303; MR **21** #3077] of $N \times N$ design matrices X (subject to least squares estimated weights having equal variances and being equally correlated) for determining N weights in N weighings on an unbiased chemical balance. An X is BMS [BES], i.e., best in Mood's [Ehrenfeld's] sense, if the determinant [least latent root] of $X'X$ is greatest possible. Let $X = X_{r;\lambda}$ be an $N \times N$ matrix such that $X'X = (a_{ij})$, where $a_{ij} = r$ $(i = j)$, $a_{ij} = \lambda$ $(i \neq j)$. Then (1) $P_N = X_{N;1}$ is BMS, and BES if N is odd; (2) $X_{N;2}$ is BMS and $S_N = X_{N-1;0}$ is BES if $2 < N \equiv 2 \pmod 4$. Necessary conditions that $X_{N;2}$ and S_N exist are found, and applied for $N < 200$ and 80, respectively. An $X_{6;2}$ $(N = 6)$ is constructed. *P. H. Diananda* (Singapore)

8615:

Box, G. E. P. Use of statistical methods in the elucidation of basic mechanisms. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 215–225. (French summary)

This paper is an expository paper on some work by the author and colleagues on exploring experimentally a possible functional relationship $\eta = f(\xi, \theta)$, where ξ is a vector representing variables under control of the experimenter and θ is a vector representing the parameters of the relationship. The author discusses examples and methods of obtaining theoretical mechanisms, obtaining least squares estimates numerically, and the problem of design. The last corresponds to the appropriate choice of successive values of ξ to yield good estimates of θ and data which may be used to test the model efficiently. The author remarks on the necessity of preliminary estimates of θ in aiding in the design problem for models which are not the traditional regression model. The author remarks in passing that one possible criterion for a good design involves minimizing the Wilks generalized variance for the estimate of θ. This is the determinant of the covariance matrix of the estimates. {This reviewer would like to take advantage of the opportunity to repeat his previously expressed lack of enthusiasm for this criterion in spite of its elegant mathematical properties.}

H. Chernoff (Stanford, Calif.)

8616:

Kiefer, J.; Wolfowitz, J. The equivalence of two extremum problems. Canad. J. Math. **12** (1960), 363–366.

Let $f_1, f_2, \cdots, f_k$ be linearly independent real functions on a space X such that the range R of $f = (f_1, f_2, \cdots, f_k)$ is a compact set. Let S be any Borel field of subsets of X which includes X and all sets which consist of a finite number of points, and let $C = \{\xi\}$ be any class of probability measures on S which includes all probability distributions concentrated on a finite set. Let $M(\xi)$ be the $k \times k$ matrix whose (i, j) element is $m_{ij}(\xi) = \int_X f_i(x) f_j(x) \xi(dx)$. The authors prove that the problems of selecting ξ_0 (1) to maximize $\det M(\xi)$, (2) to minimize $\max_x d(x, \xi)$, where $d(x, \xi) = f(x)'[M(\xi)]^{-1} f(x)$, and (3) to obtain $\max_x d(x, \xi) = k$ are equivalent. The proof implicitly makes use of the fact that the determinant of a symmetric positive definite matrix is quasi-concave and that in the above problem the range of $M(\xi)$ is convex. This result is of considerable relevance to the problem of optimum designs in regression problems [see Kiefer and Wolfowitz, Ann. Math. Statist. **30** (1959), 271–294; MR **21** #3079] and shows that two criteria of optimality are equivalent. One of these criteria is that of minimizing the generalized variance of the estimates of the coefficients. The other is that of minimizing the variance of the estimate of the regression at the "worst value of the independent variable".

H. Chernoff (Stanford, Calif.)

8617:

Shah, B. V. A matrix substitution method of constructing partially balanced designs. Ann. Math. Statist. **31** (1960), 34–42.

Definitions are given for incidence, canonical, and association matrices of experimental designs. In turn a partially balanced matrix A is defined in terms of the incidence matrices of partially balanced incomplete block (PBIB) designs satisfying certain conditions. The s designs $N_1, N_2, \cdots, N_s$ each in v treatments and b blocks are called associable if there exists an orthogonal canonical matrix of association L such that $L'N_iN_j'L$ is diagonal for all subscript pairs. Then a new PBIB design is obtained from the matrix A of s associable designs N_i by replacing each element by the appropriate incidence matrix N_i (Kronecker product). Application is made to the construction of factorial designs. A number of minor misprints were noted. *C. J. Maloney* (Frederick, Md.)

8618:

Raiffa, Howard; Schlaifer, Robert. ★Applied statistical decision theory. Studies in Managerial Economics. Division of Research, Graduate School of Business Administration, Harvard University, Boston, Mass., 1961. xxviii + 356 pp. (2 inserts) $9.50.

This is an original and stimulating book. It is concerned with statistical decision theory when a prior distribution of the parameters is admitted and the criterion of optimality is to take that action which maximizes the expected utility, the expectation being taken with respect to this prior distribution: the so-called "Bayesian" analysis. Both choices of experimentation and terminal actions are considered. It is in three parts. The first part presents important new developments in the theory (despite the book's title). Almost all these stem from the use of the prior distribution as an essential part of the analysis. They include a new definition of sufficiency; a discussion of the irrelevance of the rule for stopping experimentation; and the notion of a class of prior distributions which are conjugate to a family of distributions possessing a fixed number of sufficient statistics for all sample sizes (so permitting a parametrization of posterior distributions). This last specialization is maintained throughout the rest of the book.

The second part is devoted to the special case where the utility function is the sum of two terms: one due to the terminal action and one to the choice of experiment. The following topics are discussed: terminal analysis when the utility is linear in the parameter; optimal sample size in two action problems for normal and binomial observations; choice of the best of several normal processes; estimation problems (when the action and state spaces coincide). The third part provides the necessary distributional theory that is needed in dealing with conjugate prior distributions. There are few direct applications and the book will mainly be of interest to theoreticians, or at least to persons well equipped with the necessary ability to master the theory. This feature is emphasized by the fact that the book begins with very general theory and proceeds by increasing amounts of specialization to particular cases. Several potentially applicable results are obtained. There is no index, but there is a very full introduction and preface and a good list of contents. The authors get involved in some horrible notation, which they attempt to defend. Even if the defence is admitted, the result is still difficult to read. Statisticians will not be pleased to see different notation and nomenclature from the standard. It is to be hoped that these difficulties will not prevent a book, which makes very significant contributions to the theory of decision making under uncertainty, from being widely read. There is no comparable book. It should have an important effect on the future development of the subject. *D. V. Lindley* (Aberystwyth)

8619:

Birnbaum, A.; Maxwell, A. E. Classification procedures based on Bayes's formula. Appl. Statist. **9** (1960), 152–169.

A lucid exposition of the methods of the title, with an application to psychological diagnosis.

D. V. Lindley (Aberystwyth)

8620:

Burkholder, D. L. Effect on the minimal complete class of tests of changes in the testing problem. Ann. Math. Statist. **31** (1960), 325–331.

It is shown that, with respect to the discrete exponential family of distributions,

$$\Pr(X \in A) = \sum_{x \in A} k(\theta_1, \theta_2) h(x) \exp(\theta_1 s(x) + \theta_2 t(x)),$$

the minimal complete class (MCC) of tests for a standard testing problem is minimal complete for a wide variety of related problems. Thus, under mild restrictions, the MCC is the same (i) for testing $\theta_1 \leq r$ against $\theta_1 > r$ and (ii) for testing $\theta_1 = a$ against $\theta_1 = b$ $(a < b)$. (The risk is taken as the probability of a wrong decision.) An example is given showing that with respect to a continuous exponential family the MCC for a problem of type (i) is not necessarily the same as for a problem of type (ii).

W. Hoeffding (Chapel Hill, N.C.)

8621:

Nandi, H. K. A complete class of compound decision procedures. Calcutta Statist. Assoc. Bull. **9** (1960), 157–162.

Conditions are given under which a decision problem involving several parameters has a complete class of decision functions provided by considering each of the parameters separately. *D. V. Lindley* (Aberystwyth)

8622:

Hoeffding, Wassily. Lower bounds for the expected sample size of a sequential test. Information and decision processes, pp. 53–61. McGraw-Hill, New York, 1960.

Consider a sequential test of the hypothesis that the density of a random variable is $f = f_1$ versus the alternative $f = f_2$. Suppose that the error probabilities are α_1 and α_2 and that the true distribution is f_0 which may be a third distribution. This paper is concerned with three lower bounds for $E_0(n)$ which is the expectation of the sample size when the distribution is f_0. These are

$$E_0(n) \geq \sup_{0 \leq c \leq 1} \{-\log[\alpha_1^c(1-\alpha_2)^{1-c} + (1-\alpha_1)^c\alpha_2^{1-c}]\}/\{c\int f_0 \log[f_0/f_1]\,dx + (1-c)\int f_0 \log[f_0/f_2]\,dx\},$$

$$E_0(N) \geq [1-\alpha_1-\alpha_2]/[1-\int \min(f_0, f_1, f_2)\,dx],$$

$$E_0(N) \geq \{[(\tau/4)^2 - \zeta \log(\alpha_1+\alpha_2)]^{1/2} - \tau/4\}^2/\zeta^2,$$

where $\zeta = \max(\zeta_1, \zeta_2)$, $\zeta_i = \int f_0 \log(f_0/f_i)dx$, and $\tau^2 = \int (\log[f_2/f_1] - \zeta_1 + \zeta_2)^2 f_0 dx$.

The author indicates circumstances under which the middle bound is attained. For the sequential test for the mean of a normal distribution, using straight line boundaries in the $(\sum X_i, n)$ plane, computations of Anderson indicate that the third bound is almost attained.

H. Chernoff (Stanford, Calif.)

8623:

Ghosh, Jayanta Kumar. On the monotonicity of the *OC* of a class of sequential probability ratio tests. Calcutta Statist. Assoc. Bull. **9** (1960), 139–144.

It is shown that a (generalized) sequential probability ratio test for testing a simple hypothesis against a simple alternative is unbiased. Hence it follows that if the distribution of the sequence of random variables depends on a real parameter and the probability densities possess a monotone likelihood ratio, then the operating characteristic function of any sequential probability ratio test is monotone. There are many misprints.

W. Hoeffding (Chapel Hill, N.C.)

8624:

Ghosh, M. N. Bounds for the expected sample size in a sequential probability ratio rest. J. Roy. Statist. Soc. Ser. B **22** (1960), 360–367.

Wald's sequential probability ratio test which is based on the random variable $Z=\log\{f(X,\theta)/f(X,\theta_0)\}$ has the property that the test can be easily constructed to have approximately prescribed levels of first and second bounds of error. Wald also gave an approximate formula for the expected sample size and a bound on the error of approximation. Wald evaluated this bound for the normal and binomial distributions and suggested that in general it would be small if $E(Z)$, $\sigma^2(Z)$ were small. An example is given showing this conjecture is not true, and a new formula is developed for a bound on the error in Wald's approximate formula. *D. G. Chapman* (Seattle, Wash.)

8625:

Richter, Hans. Die Einflussgrössenrechnung und ihre praktische Nutzanwendung im Betrieb. Math.-Tech.-Wirtschaft **8** (1961), 18–24.

8626:

Davidson, J. H. Statistical methods in life testing. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 543–544. (French summary)

8627:

Zelen, M.; Dannemiller, Mary C. The robustness of life testing procedures derived from the exponential distribution. Technometrics **3** (1961), 29–49.

Authors' summary: "This paper studies the behavior of several statistical life testing procedures based on the exponential failure law if the true failure law is the Weibull distribution. It is found that these statistical techniques, which are widely used, are very sensitive to departures from initial assumptions. Applying these techniques to life test data when the exponential failure law is not satisfied may result in substantially increasing the probability of accepting components or equipment having poor mean-time-to-failure. This paper also develops convenient analytic techniques for approximating (i) the distribution of sums of independent random variables, and (ii) the characteristics of sequential procedures, for non-negative random variables. These techniques are based on a general life distribution which consists of an expansion of generalized Laguerre polynomials. Application of these methods to the Weibull distribution yields excellent results." *J. Kiefer* (Ithaca, N.Y.)

8628:

Ekman, Gunnar. A limit theorem in connection with stratified sampling. I. Skand. Aktuarietidskr. **1959**, 208–223 (1960).

Suppose x is a random variable having probability density function $f(x)$ where $f(x)\equiv 0$ for x outside some finite interval (a, b). Let $(x_0, x_1), (x_1, x_2), \cdots, (x_{n-1}, x_n)$ be intervals into which (a, b) is divided, where $x_0=a$, and $x_n=b$. Let $c(x)$ be a non-negative "cost" function defined on (a, b). Let $P_h=P(x_{h-1}<x\leqq x_h)$, σ_h^2 be the conditional variance of x, and c_h be the conditional mean value of $c(x)$, given that x lies in (x_{h-1}, x_h). The author investigates the problem of choosing $x_1, \cdots, x_{n-1}$ so as to minimize $\sum_{h=1}^{n} c_h^{1/2}\sigma_h P_h$, a problem which arises in a stratified sampling situation. Observing the computational difficulties involved for a fixed n, he obtains, under some regularity conditions on $f(x)$ and $c(x)$, an asymptotic solution for large n, including an expression for the minimum value of the sum mentioned above. The discussion of the adequacy of the asymptotic solution for finite n and a practical application are promised in part II of the paper to appear later.

S. S. Wilks (Princeton, N.J.)

8629:

Banerjee, K. S. Calculation of sampling errors for index numbers. Sankhyā **22** (1960), 119–130.

The author comes to the conclusion that the variance of the random sampling error of index numbers can be calculated by means of the usual formula for stratified random sampling and demonstrates by means of numerical examples the size of this type of error for some methods of constructing index numbers. *L. Törnqvist* (Helsinki)

8630:

Gayen, A. K.; Roy, G. C. Some aspects of prediction problems in time-series. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 285–292. Indian Soc. Theoret. Appl. Mech., Kharagpur.

Expository paper. *J. Durbin* (London)

8631:

Janos, William A. Analytic inversion of a class of covariance matrices. Trans. IRE **IT-6** (1960), 477–484.

Author's summary: "The sample covariance matrix arising out of finite memory linear least squares estimation over a set of equally spaced time points, is inverted by spectral methods (operationally referred to as the z transform). It is shown that the complexity of the problem depends only upon the complexity of the input correlation function. The final solution is shown to reduce to the inversion of a triangular system of linear equations of an order less than half the degree of the denominator of the input power spectral density function."

S. Kullback (Washington, D.C.)

8632:

Edgington, Eugene S. Probability table for number of runs of signs of first differences in ordered series. J. Amer. Statist. Assoc. **56** (1961), 156–159.

A table, appropriate for either one-tailed or two-tailed tests, is provided for any number of observations from 2 to 25 and any number of runs from 1 to 24. Formulae

for applying a normal approximation for more than 25 observations are given. The lack of an estimate of the power of the tests is noted and some not too convincing remarks on the properties of the tests substituted.

C. J. Maloney (Frederick, Md.)

8633:

Goodman, N. R. Measuring amplitude and phase. J. Franklin Inst. **270** (1960), 437–450.

Let $x_1, x_2, \cdots, x_n$ be a random sample from a weakly stationary, zero mean, discrete parameter process X_k ($k=-\infty, \cdots, \infty$), and let Y_k be the Hilbert transform of X_k. Writing $X_k+iY_k=R_k\exp\{i\phi_k\}$, R_k is defined as the instantaneous amplitude and ϕ_k as the instantaneous phase. Considering the finite moving average processes $\tilde{X}_k=\sum_{s=-m}^{m}a_sX_{k+s}$ and $\tilde{Y}_k=\sum_{s=-m}^{m}b_sX_{k+s}$, and defining $\tilde{R}_n$ and $\tilde{\phi}_k$ by $\tilde{R}_k\exp\{i\tilde{\phi}_k\}=\tilde{X}_k+i\tilde{Y}_k$, the paper discusses methods of determining a_s and b_s so that $\tilde{R}_k$ and $\tilde{\phi}_k$ approximate R_k and ϕ_k, respectively. They were developed for determining amplitude and phase of radio star scintillation data.

W. Freiberger (Providence, R.I.)

8634:

Akaike, Hirotugu. Effect of timing-error on the power spectrum of sampled-data. Ann. Inst. Statist. Math. Tokyo **11** (1960), 145–165.

This paper discusses a problem in analog-digital conversion. When a continuous process is sampled at discrete time points, the influence of aliasing on spectral estimation is well known; another problem is the influence of timing errors which is here discussed for the case where the timing is independent of the original process and the intervals between sampling-time points form a stationary process. Two special types of time-sampling are treated in more detail: where the deviations of the sampling-time points from the preassigned ones form a random process, and where the interval lengths between successive sampling-time points form such a process. The effect on the power spectral distribution function of the sampled data is analysed, and a numerical example given.

W. Freiberger (Providence, R.I.)

8635:

Brudno, A. L.; Lunc, A. L. On filtration of random sequences. Dokl. Akad. Nauk SSSR **131** (1960), 485–488 (Russian); translated as Soviet Math. Dokl. **1**, 265–268.

Let the stochastic process v_k ($k=0, 1, \cdots$) be the sum of a quantity u_k which we wish to estimate and an error term ξ_k with $E\{\xi_k\}=0$, $E\{\xi_k^2\}=\sigma$. u_0 is an unknown constant. The problem is to find a linear function $f(v_0, v_1, \cdots)$ such that (a) $E\{f(v_0)\}=u_0$ and (b) $E\{(f(v_0, v_1, \cdots)-v_0)^2\}$ is minimal.

(1) If $u_i-u_{i+1}=\zeta_i$ with the ζ_i independent, $E\{\zeta_i\}=0$, $E\{\zeta_i^2\}=\rho$, then the solution is

$$f(v_0, v_1, \cdots) = (\lambda/(1-\lambda)^2)\sum_{i=0}^{\infty}\lambda^iv_i,$$

where $\lambda/(1-\lambda)^2=\sigma/\rho$.

(2) If $u_i=h_i+k_i$, where k_i has independent increments ζ_i and h_i has independent second differences η_i, and $E\{\zeta_i\}=E\{\eta_i\}=0$, $E\{\zeta_i^2\}=\rho$, $E\{\eta_i^2\}=\tau$, then a solution exists (and an expression is given) provided either $\sigma=0$ (trivial: $f=v_0$) or $\rho^2-4\tau^2\neq 0$.

(3) If in case 1 it were assumed only that $E\{\zeta_i\}=c$, c some unknown constant, then there would exist no solution. No proofs are given.

J. Feldman (Princeton, N.J.)

NUMERICAL METHODS

See also A7914, A8222, 8584, 8680, 8681, 8682, 8683, 8684, 8685, 8686, 8687, 8688, 8689, 8690, 8691, 8692, 8693, 8694, 8695, 8696, 8701, 8702, 8703, 8704, 8705, 9256.

8636;

★Modern computing methods. 2nd ed. Philosophical Library, New York, 1961. vi+170 pp. $8.75.

American edition of the book reviewed below.

8637:

★Modern computing methods. 2nd ed. National Physical Laboratory. Notes on Applied Science, No. 16. Her Majesty's Stationery Office, London, 1961. vi+170 pp. £1 1s.

Whereas the first edition [MR **19**, 579] was completely anonymous, the present edition contains an introduction by E. T. Goodwin which lists contributors: himself, C. W. Clenshaw, D. W. Martin, G. F. Miller, F. W. J. Olver, and J. H. Wilkinson. For the first edition the authors are listed as Goodwin, Olver, and Wilkinson, along with L. Fox and J. G. L. Michel. The computing world is indebted to them.

There are fifteen chapters: the first five on matrices, one on zeros of polynomials, the finite difference methods, Chebyshev series, ordinary and partial differential equations (two for each), limits, integrals, and mathematical functions. There follows a selected list of references (217 items), with brief comments on each.

Although much has been redone and some chapters added in this edition, the approach is the same as that of the first: for each topic a few selected methods are described, succinctly but clearly. Comments are made on adaptation to desk and to automatic computation; a few numerical examples are given; and careful attention is paid to the assessment of computational error. While the book is not concerned with theory, a reader with reasonable mathematical maturity should have little trouble filling in. It can be recommended highly.

A. S. Householder (Oak Ridge, Tenn.)

8638:

Stanton, Ralph G. ★Numerical methods for science and engineering. Prentice-Hall Applied Mathematics Series. International Series in Engineering. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1961. xii+266 pp. $9.00.

This book was written to serve as an introductory textbook for undergraduate students of science and engineering at about the sophomore or junior level. Correspondingly, the background assumed on the part of the reader is about that of these students after they have taken the usual courses in calculus and differential equations for engineers.

In several places in the book the author stresses that "the book is addressed to an audience whose concern is with solving problems, not with talking about solving

them"; that the author is "not interested in a puristic approach to numerical analysis", and that, in his opinion, "numerical analysis is like engineering, partly a science, partly an art". It is therefore clear that the book belongs to the category of textbooks for engineering students which replace precise statements—not to mention proofs—with heuristic arguments and manipulative recipes, thereby making it difficult for the students to gain a genuine understanding of the underlying mathematical concepts. But nevertheless, these same students could be taught to understand and appreciate precise statements about the limitations of a result, convergence of a procedure, etc., without being "subjected" to a "puristic approach".

The selection and presentation of the material has been guided by the author's conviction that students should start to learn numerical analysis from the standpoint of hand and desk calculator techniques, rather than with their eyes on an electronic computer. There are certainly some good arguments for such an approach. However, in stressing desk calculator techniques one easily runs the risk of neglecting the modern methods and developments that have changed numerical analysis following the advent of electronic computers. The author does not succeed in avoiding this pitfall. For example, he places considerable stress on the relaxation procedure in Southwell's basic form while barely mentioning its unsuitability for computer application and without at least pointing to the more modern techniques that are based on these same ideas.

In general, it can be said that the selection of material in this book is quite conservative and therefore similar to several other books in the field. A list of chapter headings supports this: Chapter 1, Ordinary finite differences; Chapter 2, Divided differences; Chapter 3, Central differences; Chapter 4, Inverse interpolation and the solution of equations; Chapter 5, Computation with series and integrals; Chapter 6, Numerical solution of differential equations; Chapter 7, Linear systems and matrices; Chapter 8, Solution of linear equations; Chapter 9, Difference equations; Chapter 10, Solution of differential equations by difference equation methods; Chapter 11, The principles of automatic computation.

Several of these chapters include helpful reviews of material from calculus, algebra, etc. Chapter 7 contains an introduction into matrix notation and chapter 8 ends in a nice discussion of ill-conditioned matrices. Chapters 9 and 10 are little more than a "brief introduction to the basic ideas of difference equations and partial differential equations". Chapter 11 points out some basic facts about computers in a rather summary fashion. Throughout the book a great number of examples are worked out in detail and each paragraph ends in a list of problems.

In conclusion it might be said that there is certainly a great need for a good textbook on numerical analysis at the level of this book and addressed to the same audience. Whether this particular volume can fill this need is an open question. *W. C. Rheinboldt* (Syracuse, N.Y.)

8639:

Sarafyan, Diran. Divisionless computation of square roots through continued squaring. Comm. ACM **3** (1960), 319–321.

A convergent iterative procedure is given in this note for the computation of square roots of real numbers. In order to determine $r = n^{1/2}$, one first chooses $m > 0$ such that m is "close" to n, then puts $r_0 = (1/2m)[m^2 + n]$, and computes $r_{i+1} = r_i + (1/2m)[n - r_i^2]$, $i = 0, 1, 2, \cdots$. It is shown that the r_i converge to $n^{1/2}$, and that the order of convergence is about the same as that of the second order Newton-Raphson iteration process. Illustrative examples are provided; they indicate the rapidity of convergence.

M. Lotkin (Merchantville, N.J.)

8640:

Douglas, Jim, Jr. A numerical method for analytic continuation. Boundary problems in differential equations, pp. 179–189. Univ. of Wisconsin Press, Madison, 1960.

The present paper is concerned with the following problem. Let $f(z)$ be an analytic function on the closed unit disk and suppose that bounds of f and its first and second derivatives are known. Suppose that approximate values of f are known at p equally distributed points on a circle $|z| = \rho < 1$ and are given with an error of at most ε. These values are used to determine an approximation $F(z, n, p, \varepsilon)$ of $f(z)$ on $|z| < 1$. A priori and a posteriori estimates of the error on $\rho < |z| = R < 1$ are given. It is shown that the numerical computations can be carried out by a linear programming procedure.

M. R. Hestenes (Los Angeles, Calif.)

8641:

Kublanovskaya, V. N. Application of analytic continuation by means of change of variables in numerical analysis. Trudy Mat. Inst. Steklov. **53** (1959), 145–185. (Russian)

An analytic function $F(z)$ defined by its Maclaurin expansion in $|z| \leq R$ can be analytically continued with the aid of a substitution $z = f(\zeta)$, where $f(\zeta)$ is also analytic. This method of analytic continuation is applied by the author to various problems of numerical analysis, the solution of which reduces to the computation of $F(z)$ for a prescribed value of z, $|z| > R$ or $< R$, or to the localization of poles of $F(z)$.

The first part of the paper gives a description of the procedure with estimates of accuracy as well as very useful indications on how to choose the substitution $z = f(\zeta)$ in order to obtain the best results. All necessary proofs are given.

In the second part the method is applied to particular problems such as, for instance, solution of systems of linear equations, determination of eigenvalues and eigenvectors of a matrix, integration of differential equations by series, solution of Dirichlet problem by finite differences, solution of integral equations, etc. For all these cases, well-chosen numerical examples are analyzed and the solutions are tabulated. These examples not only prove the practical importance of the method studied in this paper, but also illustrate the high accuracy attainable—provided the substitution $z = f(\zeta)$ was chosen judiciously—with much less computational work than usual. Seven pages of tables annexed at the end of the paper facilitate the numerical work with the particular substitutions used in the examples and they exemplify how such a table should be constructed when a new substitution is chosen. This paper is an important contribution to numerical analysis. *E. Kogbetliantz* (New York)

8642:
Rotenberg, A. A new pseudo-random number generator. J. Assoc. Comput. Mach. **7** (1960), 75–77.

For binary and decimal digital computers a new and efficient method is given for generating pseudo-random numbers; namely,

$$x_{t+1} = (2^a+1)x_t+c \pmod{2^k},$$

$$x_{t+1} = (10^a+1)x_t+c \pmod{10^k},$$

where in both instances, $a \geqq 2$, c is odd, and k is any integer. Since a multiplication involving a number which is a power of the number base of the computer can be accomplished by a shift operation within the computer, the method presented is essentially an additive one involving a shift and three additions. The use of a number such as (2^a+1) or (10^a+1) makes it possible to replace the multiplication of x_t by such a constant, with an operation involving a series of shifts. This type of technique has been used by earlier workers in this field. However, prior methods had $c=0$. The method is shown to generate a sequence of full period, namely, 2^k or 10^k, numbers. 4096 numbers were generated on a binary computer, for $a=7$, $c=1$, and $k=35$, and were subjected to some statistical tests. *M. Muller* (New York)

8643:
Coveyou, R. R. Serial correlation in the generation of of pseudo-random numbers. J. Assoc. Comput. Mach. **7** (1960), 72–74.

Given the following method of generating pseudo-random numbers,

$$x_{n+1} \equiv \lambda x_n+\mu \pmod{P},$$

where $x_0=a$, $0 \leqq x_n < P$, an approximation is provided, in terms of λ, μ, and P, for the serial correlation between numbers of the sequence generated by the method.
M. Muller (New York)

8644:
Hammersley, J. M. Monte Carlo methods for solving multivariable problems. Ann. New York Acad. Sci. **86**, 844–874 (1960).

The paper is chiefly expository in spirit but contains a number of original contributions to Monte Carlo error analysis and to the more esoteric aspects of numerical analysis. The chief topics dealt with are as follows: (1) the traveling-salesman problem; (2) interpolation to functions of many variables; (3) matrix multiplication by sampling of elements; (4) representation of functions of many independent variables by sums of products of functions of a single variable; (5) Monte Carlo integration; and (6) the behavior of quasi-random (deterministically equidistributed) numbers. The apparently new results stated and proved here include an upper bound for the variance of a certain random linear interpolation process discussed in (2), a generalization in (4) of a technique previously available for two variables, and in (6) the lower bound $-1+1/2(\log_e 2)^2$ for $\limsup nV_n$, where V_n is the variance of the lengths of the n intervals created by the first $n-1$ points of an ordered infinite sequence of points on the unit interval. New avenues for research are freely indicated. *J. H. Curtiss* (Coral Gables, Fla.)

8645:
Thacher, Henry C., Jr.; Milne, W. E. Interpolation in several variables. J. Soc. Indust. Appl. Math. **8** (1960), 33–42.

The authors give algorithms to carry out practically various interpolation operations for functions of several variables. The presentation using matrix notation gives a simple view of the problems as well as generating new algorithms which simplify the application in some directions. *P. C. Hammer* (Madison, Wis.)

8646:
Barbašin, E. A. Estimating the mean square deviation from a given trajectory. Avtomat. i Telemeh. **21** (1960), 941–950 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 661–667.

Author's summary: "New methods for estimating the mean square deviation from a given trajectory are given. The estimates obtained provide a method of choosing the external influences so that this mean square value will be small. Conditions are given for the existence of motion exactly over a given trajectory."

8647:
de Witte, Leendert. Least squares fitting of a great circle through points on a sphere. Comm. ACM **3** (1960), 611–613.

8648:
Laurenti, Fernando. Sul teorema di Maclaurin relativo alle serie. Archimede **12** (1960), 38–42.

$f(x)$ étant une fonction positive dans l'intervalle $(1, \infty)$, l'auteur, généralisant une formule de Maclaurin, démontre l'inegalité suivante, dont il donne des applications:

$$f\left(\frac{2u+1}{2n}\right)+f\left(\frac{2u+3}{2n}\right)+\cdots < n\int_1^\infty f(x)\,dx.$$

R. Campbell (Caen)

8649:
Laasonen, Pentti. A Ritz method for simultaneous determination of several eigenvalues and eigenvectors of a big matrix. Ann. Acad. Sci. Fenn. Ser. A I No. 265 (1959), 16 pp.

An integer $m<n$ is said to be "normal" with respect to an $(n \times n)$ matrix A, if there are m eigenvalues of A greater in absolute value than the $n-m$ others. Let Y and V be two $(n \times m)$ matrices of rank m, for each of which the leading $(m \times m)$ submatrix is nonsingular. Let $Y^{(k)}=A^kY$, $V^{(k)}=A'^kV$, $Q^{(k+1)}=V^{(k)'}Y^{(k)}$. If m is normal with respect to A, the author proves that the roots of $\det(Q^{(k+1)}-\mu Q^{(k)})=0$ converge (as $k \to \infty$) to the m dominant eigenvalues of A. The direction of the convergence is studied for Hermitian matrices, when $V=\overline{Y}$. The author concludes from a perturbation analysis that his method is essentially comparable in regard to round-off error with related biorthogonalization methods of F. L. Bauer [#8650]. *G. E. Forsythe* (Stanford, Calif.)

8650:
Bauer, F. L. On modern matrix iteration processes of Bernoulli and Graeffe type. J. Assoc. Comput. Mach. **5** (1958), 246–257.

A Bernoulli process converges linearly, a Graeffe process converges quadratically. Modern methods of these types are used to find intermediate eigenvalues and eigenvectors, with less work than the classical techniques suggested by Perron and used by Aitken, and with no progressive loss of accurate significant figures. Bernoulli processes are illustrated by the methods of biorthogonalised iteration and other variants of simultaneous iteration, and include also the LR and QD algorithms of Rutishauser. Their relations and properties are discussed and illustrated numerically by test examples. The main Graeffe method mentioned, which avoids matrix squaring, is an adaptation of the LR method. The papers by Rutishauser and Bauer giving original descriptions of the various techniques of this paper are itemised in its bibliography.

L. Fox (Oxford)

8651:

Cuthill, Elizabeth H.; Varga, Richard S. A method of normalized block iteration. J. Assoc. Comput. Mach. **6** (1959), 236–244.

The authors consider a method of block iteration for solving systems of linear algebraic equations. The method proposed is a modification of the successive block over-relaxation procedure which was introduced by Gates, Arms, and Zondek [J. Soc. Indust. Appl. Math. **4** (1956), 220–229; MR **22** #10167] and which under certain conditions converges faster than the successive (point) overrelaxation procedure. For the usual discrete analogue of the Dirichlet problem the successive block over-relaxation method is asymptotically faster by a factor of $2^{1/2}$. However, when the method was originally proposed, there appeared to be little overall gain since one would be required to solve at each iteration a number of systems of linear equations with tri-diagonal matrices. If this were done in the usual way, then the number of operations would be several times as large as with point over-relaxation. The authors, however, solve the systems of equations by means of a normalization and factorization of the tri-diagonal matrices involved. As a result, the number of operations per point is the same as for point overrelaxation. Hence one can gain the full advantage of the block relaxation procedure at no increase in the number of operations required per iteration.

The authors show for a large class of self-adjoint elliptic partial differential equations with two and three independent variables that block overrelaxation is better than point overrelaxation. They also give a procedure based on the Perron-Frobenius theory for non-negative matrices for estimating the optimum relaxation factors both for point and for block overrelaxation. In each case they determine a sequence of upper and lower bounds for the optimum value which can be made as accurate as desired.

D. M. Young, Jr. (Austin, Tex.)

8652:

Egerváry, Eugen. On rank-diminishing operations and their applications to the solution of linear equations. Z. Angew. Math. Phys. **11** (1960), 376–386. (German summary)

The present paper describes the most general rank diminishing operations of a matrix. This enables one to give a unified treatment for various iterative procedures occurring in the solution of linear equations. The general method includes the triangular factorization of Gauss-Banachewicz, the recursive method for inverting a matrix, the method of conjugate directions and conjugate gradients, general projection methods and the author's method for solving diophantine equations.

M. R. Hestenes (Los Angeles, Calif.)

8653:

Andrus, Jan F. Note on eigenvalue computation. Comm. ACM **3** (1960), 617.

8654:

Ostrowski, A. M. On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors. VI. (Usual Rayleigh quotient for non-linear elementary divisors). Arch. Rational Mech. Anal. **4**, 153–165 (1959).

[For part V see same Arch. **3** (1959), 472–481; MR **21** #6691.] This is the sixth paper concerned with the computation of characteristic roots and vectors. The present paper is concerned with a non-Hermitian matrix A having a non-linear elementary divisor. Starting with λ_0 and a column vector η, the sequence λ_ν is defined by

$$\xi_\nu = (A-\lambda_\nu I)^{-1}\eta, \quad \lambda_{\nu+1} = \frac{\xi_\nu^* A \xi_\nu}{|\xi_\nu|^2} = \varphi(\lambda_\nu).$$

The convergence of this iteration, together with accelerating procedures of Steffensen and of Householder, is studied in detail. It is pointed out that the classical Rayleigh iteration has a distinct advantage over the generalized Rayleigh iteration.

M. R. Hestenes (Los Angeles, Calif.)

8655:

Ellenberger, Kenneth W. On programming the numerical solution of polynomial equations. Comm. ACM **3** (1960), 644–647.

This paper describes standard methods for the solution of a polynomial equation and how they may be programmed for a floating decimal computer. The method given for scaling the coefficients in the equation, will not work when some of the coefficients are nearly zero.

C. B. Haselgrove (Manchester)

8656:

Salzer, Herbert E. A note on the solution of quartic equations. Math. Comput. **14** (1960), 279–281.

The author outlines in detail the algebraic method of solution of quartic equations with real coefficients for every possible case with the aid of a table for the solution of cubic equations. *I. A. Stegun* (Washington, D.C.)

8657:

Hunter, D. B. An iterative method of numerical differentiation. Comput. J. **3** (1960/61), 270–271.

A method for numerical differentiation based on Aitken's interpolation method [A. C. Aitken, Proc. Edinburgh Math. Soc. (2) **3** (1932), 56–76], suited for unequally-spaced ordinates. *F. Stallmann* (Washington, D.C.)

8658:

Salzer, Herbert E. Optimal points for numerical differentiation. Numer. Math. **2** (1960), 214–227.

The author investigates the possibility of finding optimal evaluation points for a function when the values at these points are to be used for estimating derivatives of various orders. He considers the problem for finding the derivative values at a fixed point and for finding derivatives throughout an interval. The latter problem is related to Tschebyscheff polynomials.

P. C. Hammer (Madison, Wis.)

8659:

Clenshaw, C. W.; Curtis, A. R. A method for numerical integration on an automatic computer. Numer. Math. **2** (1960), 197–205.

A method is described for the numerical integration of a "well-behaved" function over a finite interval. Essentially the method consists of expanding the integrand in a series of Chebyshev polynomials and integrating this series term by term. The method has the advantage that the accuracy can be easily checked before the integration is completed. The authors make some comparisons of this method with Simpson's rule and the Gauss formulas. Their method shares the advantage of Simpson's rule that if the number of points is doubled the previous calculations are not wasted and in some cases the method may give accuracy comparable to the Gauss formulas. The method will also give, with only little extra complication, the values of the indefinite integral throughout the range of integration. Several examples are given.

A. H. Stroud (Madison, Wis.)

8660:

Törnig, W. Zur Konvergenz der Differenzenschemaverfahren. Z. Angew. Math. Mech. **40** (1960), 423–424.

The convergence of Adam's method of numerical solution of $y'=(x, y)$ has been studied by D. Gaier [same Z. **36** (1956), 230; MR **18**, 154]. This paper consists of an extension of the procedure to a differential equation of arbitrary order. An upper bound for the error is given as a function of the step size. Numerical stability is not discussed.

J. A. Ward (Washington, D.C.)

8661:

Mikeladze, Š. E. A general method for numerical solution of differential equations. Soobšč. Akad. Nauk Gruzin. SSR **22** (1959), 513–518. (Russian)

This report treats a new method for the numerical solution of ordinary differential equations which reduces the problem to one of solving a system of equations (algebraic or transcendental) by means of special formulae of numerical differentiation, which may then be solved by one of known general methods.

The method is based on the author's formula for a derivative of the function $f(x)$:

$$(1)\quad f'(x)=\sum_{\nu=0}^{n}\frac{\prod_{k=0}^{n}(x-a_k)}{(x-a_\nu)^2\prod_{k=0,\,k\neq\nu}^{n}(a_\nu-a_k)}[f(x)-f(a_\nu)]+\prod_{k=0}^{n}(x-a_k)\frac{f^{(n+2)}(\xi)}{(n+2)!},$$

where ξ is a certain value of argument contained between the largest and smallest values of the numbers $x, a_0, a_1, \cdots, a_n$, where x may obtain every value, except $a_0, a_1, \cdots, a_n$; for values $x=a_\mu$ $(\mu=0, 1, \cdots, n)$ the formula (1) is also right, but the interpolation knot a_μ must then be calculated. In this case

$$(2)\quad f'(a_\mu)=\sum_{\nu=0}^{n}A_\nu f(a_\nu)+\prod_{\substack{k=0\\k\neq\mu}}^{n}(a_\mu-a_k)\frac{f^{(n+1)}(\xi)}{(n+1)!},$$

where A_ν means the coefficients which in this case may be presented as

$$A_\nu=\frac{1}{a_\nu-a_\mu}\prod_{\substack{k=0\\k\neq\nu\\k\neq\mu}}^{n}\frac{a_\mu-a_k}{a_\nu-a_k}\quad(\nu\neq\mu),\qquad A_\mu=\sum_{\substack{\nu=0\\\nu\neq\mu}}^{n}\frac{1}{a_\mu-a_\nu}.$$

When interpolation knots are at equal distances h, i.e., $a_0=a$, $a_\mu=a+\mu h$, $a_\nu=a+\nu h$, we obtain

$$(3)\quad hf'(a+\mu h)=\sum_{\nu=0}^{n}A_\nu f(a+\nu h)+h^{n+1}\prod_{\substack{k=0\\k\neq\mu}}^{n}(\mu-k)\frac{f^{(n+1)}(\xi)}{(n+1)!}.$$

Formulae (2) and (3) have a larger number of applications. The Cauchy problem may be solved, as well as border problems and eigenvalue problems for ordinary differential equations. Formula (2) may be taken in account when the interpolation poles are not equidistant.

These formulae permit one to solve the Cauchy problem for an equation of the type $y'=\varphi(x, y)$ with the initial condition $y(a)=y_0$ for $x=a$. Putting in (3) the number of interpolation knots $n=3$, and $f(x)\equiv y(x)$, and dispensing with the second term in (3), one obtains a system of equations

$$(4)\quad\begin{aligned}18y_1-9y_2+2y_3&=11y_0+6h\varphi_0,\\-3y_1+6y_2-y_3&=2y_0+6h\varphi_1,\\-6y_1+3y_2+2y_3&=-y_0+6h\varphi_2,\end{aligned}$$

for $x=a+h$, $a+2h$, $a+3h$. Here one has $\varphi_k=y_k'\equiv\varphi(a+kh, y_k)$, $y_k=y(a+kh)$, $k=0, 1, 2, 3$. For $n>3$, we obtain systems similar to (4).

It is shown further that simple operations permit to reduce the general case of integrating differential equations to similar equation systems. It is further shown that equations of higher order may be also reduced to a system of equations, e.g., $y''=\varphi(x, y, y')$, which may be solved at the points $x=a+h$, $a+2h$, $a+3h$, with the initial conditions $[a, y(a), y'(a)]$, and then apply (3) twice with $n=3$: the first time with $f(x)=y(x)$ and afterwards with $f(x)=y'(x)$. Disregarding the second term in (3), and taking $(\mu=0, 1, 2, 3)$, one obtains for $y(a+kh)$ and $y'(a+kh)$, $k=1, 2, 3$, two systems of three equations with three unknown values $(y_1y_2y_3)$ and $(y_1'y_2'y_3')$:

$$(5)\quad\begin{aligned}18y_1-9y_2+2y_3&=11y_0+6hy_0',\\-3y_1+6y_2-y_3&=2y_0+6hy_1',\\-6y_1+3y_2+2y_3&=-y_0+6hy_2',\\18y_1'-9y_2'+2y_3'&=11y_0'+6hy_0'',\\-3y_1'+6y_2'-y_3'&=2y_0'+6hy_1'',\\-6y_1'+3y_2'+2y_3'&=-y_0'+6hy_2'',\end{aligned}$$

where $y_k=y(a+kh)$, $y_k'=y'(a+kh)$, $y_k''=\varphi(a+kh, y_k, y_k')$, $k=0, 1, 2, 3$.

For border problems and eigenvalue problems, take e.g., an equation of the second order with end conditions

$$y'(a)=y_0'=0,\quad y(b)=y_3=0;$$

taking in (5) $h=\frac{1}{3}(b-a)$ and $y_0=y_3=0$, one may solve system (5) for $y(a+kh)$ and $y'(a+kh)$, $k=0, 1, 2, 3$, where the roots will be y_0, y_1, y_2, y_1', y_2', y_3'. For eigenvalue problems one may take

$$y'' = \varphi(\lambda, x, y, y'), \tag{6}$$

depending of a certain parameter λ, where the second derivative y'' in a discrete point number k will have the form

$$y_k'' = \varphi(\lambda, a+kh, y_k, y_k') \quad (k = 0, 1, 2, 3).$$

If the eigenvalues λ are known, one may apply the system of equations (5) for computing y_k and y_k' for given end conditions, and one may add to the system (5) one or two of the equations for the point $x=a+3h$:

$$\begin{aligned} 9y_1-18y_2+11y_3 &= 2y_0+6hy_3', \\ 9y_1'-18y_2'+11y_3' &= 2y_0'+6hy_3''. \end{aligned} \tag{7}$$

When the eigenvalues are to be found, we may try to compute the roots λ, y_k, y_k' (often by means of functional iteration) if it is possible to get from the end conditions a number of equations equal to that of unknowns. If it is not possible, one may add one or two of the equations (7). The computation of the eigenvalues λ will be simplified for linear differential equations, because the equations (5) will then be linear in y_k and y_k'. This system must have roots different from zero, so that its determinant (containing the sought parameter λ) must be taken equal to zero. From this condition one obtains an equation for λ. The roots of this equation are approximate eigenvalues of the problem. *J. S. Naleszkiewicz* (Warsaw)

8662:
Sefton, P.; Vaillancourt, R. A simple technique for coding differential equations. Comm. ACM **3** (1960), 616–617.

8663:
Courant, Richard. Remarks about the Rayleigh-Ritz method. Boundary problems in differential equations, pp. 273–277. Univ. of Wisconsin Press, Madison, 1960.

The present note calls attention to the effectiveness of the Rayleigh-Ritz method for numerical computation particularly if the problem is suitably modified so as to improve convergence. The convergence properties depend upon the choice of the function to be minimized. Various choices of such a function are described. Moreover a method for relaxing the boundary conditions and obtaining solutions satisfying the boundary conditions in the mean, to a prescribed degree of accuracy, is described.

M. R. Hestenes (Los Angeles, Calif.)

8664:
Nitsche, Joachim; Nitsche, Johannes C. C. Error estimates for the numerical solution of elliptic differential equations. Arch. Rational Mech. Anal. **5**, 293–306 (1960).

The differential equation $Lu \equiv a(x, y)u_{xx}+2b(x, y)u_{xy}+c(x, y)u_{yy}=\rho(x, y)$ in the unit square with $u=0$ on the boundary is considered. It is assumed only that L is uniformly elliptic, that the coefficients are continuous, and that ρ is square integrable. The differential operator is replaced by a finite difference operator. If ρ is discontinuous, the corresponding mesh function is obtained by averaging ρ in a neighborhood of each mesh point.

A bound for the truncation error at any point, in terms of the ellipticity constants, the moduli of continuity of the coefficients, and the L_2 norm of f, is obtained. If the coefficients are Lipschitz continuous, this bound is $O(h^{2/5})$, where h is the mesh size.

H. F. Weinberger (Minneapolis, Minn.)

8665:
Nitsche, Johannes C. C.; Nitsche, Joachim. Fehlerabschätzung für die numerische Berechnung von Integralen, die Lösungen elliptischer Differentialgleichungen enthalten. Arch. Rational Mech. Anal. **5**, 307–314 (1960).

The truncation error in the preceding paper is obtained by applying a Sobolev inequality to a bound for an L_2 norm of the difference between the second derivatives of the solution and the corresponding second differences of the finite difference approximation.

In this paper it is pointed out that this L_2 bound can be used directly to bound the error made in approximating an integral of the form $\iint p(x, y)u_{xx}dxdy$, where p is a given square integrable function, by a sum involving the finite difference approximation to u. The error bound is again $O(h^{2/5})$. *H. F. Weinberger* (Minneapolis, Minn.)

8666:
Oleĭnik, O. A. Convergence of certain difference schemes. Dokl. Akad. Nauk SSSR **137** (1961), 523–526 (Russian); translated as Soviet Math. Dokl. **2**, 313–316.

Two difference methods for solving the nonlinear equation $\partial u/\partial t+\partial\varphi(u)/\partial x=0$ are proven to be convergent under certain conditions. In the first, due to Lax, the boundary condition is $u(0, x)=u_0(x)$, $-\infty<x<+\infty$, $|u_0(x)|\leq M$, and the difference equation is

$$u_n^{k+1} = (u_{n+1}^k-u_{n-1}^k)/2-[\varphi(u_{n+1}^k)-\varphi(u_{n-1}^k)]h/2l,$$

where $u_n^k=u_{nl}(kh, nl)$. In the second, due to Rothe, the boundary conditions are $u(0, x)=u_0(x)$, $u(t, 0)=u_1(t)$ in the region $t\geq 0$, $x\geq 0$, and the difference equation is $du^{n+1}/dt+[\varphi(u^{n+1})-\varphi(u^n)]/l=0$, where $u^n(t)=u_l(t, nl)$, $u^0(t)=u_1(t)$, and $u^n(0)=u_0(nl)$. *F. Goodspeed* (Quebec)

8667:
Keller, Herbert B. On some iterative methods for solving elliptic difference equations. Quart. Appl. Math. **16** (1958), 209–226.

The author considers various classes of iterative methods for solving elliptic difference equations. The classes include many of the currently used stationary linear iterative schemes as well as some generalizations of them. The eigenvalues of any method of the class are known functions of the eigenvalues of a single method of the class, which is known as the "reference method". It is thus possible, at least in principle, to determine the best method of the class, provided that the eigenvalues of the reference method are known.

The system of linear equations corresponding to a two-dimensional linear elliptic difference equation, based on the use of a five-point formula, is written in the form $M\phi=S$ where ϕ is a column vector with one component for each interior mesh point, S is a given column vector,

and M is a given matrix such that $M=I-L-R-B-T$. The non-zero elements of the matrices L, R, B, and T refer respectively to adjacent mesh points to the left, to the right, below and above the mesh point corresponding to the row in question. Much of the analysis of the iterative methods considered depends on the fundamental theorem that, for any non-zero scalars x and y, $|M|=|I-xL-x^{-1}R-yB-y^{-1}T|$. While the proof, based on geometric arguments, is given only for the rectangle, the extension to more general regions is indicated. It may be of interest to note that an algebraic proof for the more general case can be given using the method of D. Young for the case $x=y$ [Trans. Amer. Math. Soc. **76** (1954), 92–111; MR **15**, 562; Theorem 2.2]. All that is necessary is to introduce two vectors δ_i and ε_i which give, respectively, the row number and the column number of the ith mesh point.

Iterative methods are considered which are of the form $N\phi^{(\nu+1)}=P\phi^{(\nu)}+S$, where $N-P=M$, and where $N(\gamma)=\gamma_0 I-\gamma_1 L-\gamma_2 R-\gamma_3 B-\gamma_4 T$. The γ_i are so chosen that $|N(\gamma)|\neq 0$, and $\gamma_1\gamma_2\gamma_3\gamma_4=0$. The rate of convergence depends on the spectral radius of the matrix $N^{-1}P$. For the first class of methods the γ_i satisfy the conditions $\gamma_1\gamma_2=\gamma_3\gamma_4$, and $\gamma_1+\gamma_2=\gamma_3+\gamma_4$. The reference method for this class is referred to as Richardson's method (also known as the Jacobi method), and $\gamma_0=1$, $\gamma_1=\gamma_2=\gamma_3=\gamma_4=0$. Other methods in the class include the Gauss-Seidel method and the successive overrelaxation method with some, but not all, "consistent" orderings in the sense of Young [loc. cit.]. The known relation between the eigenvalues of the latter method and those of Richardson's method is obtained. In case all of the eigenvalues of Richardson's method are real, then the successive overrelaxation method is the best of this class of methods. However, it is conjectured that this need not be true in other cases.

The second class of methods, where $\gamma_0=\gamma_1=\gamma_2$, contains row iterative methods, including the simultaneous line displacement method (referred to as the line Richardson method) as well as the successive line overrelaxation method. The former method, where $\gamma_0=\gamma_1=\gamma_2=1$ and $\gamma_3=\gamma_4=0$, is selected as the reference method. It is shown that the same relation exists between the eigenvalues of the successive line overrelaxation method and those of the line Richardson method as between the corresponding point methods. This was also proved by Gates, Arms, and Zondek [J. Soc. Indust. Appl. Math. **4** (1956), 220–229; MR **22** #10167]. The third class of methods considered includes column iterative methods.

It is shown that if the matrices $(L+R)$ and $(B+T)$ commute and have real eigenvalues, and if the point Richardson method converges, then the line Richardson method converges faster. (This has recently been proved under more general conditions by R. Varga [*Boundary problems in differential equations*, pp. 121–142, Univ. of Wisconsin Press, Madison, 1960; MR **22** 12704].) A detailed comparison of line iterative methods and point iterative methods is given for Laplace's equation in a rectangle. It is shown that successive line overrelaxation converges faster than successive point overrelaxation by a factor of approximately $2^{1/2}$. It is not assumed that $\Delta x=\Delta y$. This result has previously been proved for the case $\Delta x=\Delta y$ by Gates, Arms, and Zondek [op. cit.].

Concerning the statement made in the introduction that iterative methods usually require on the order of n^2 operations, where n is the number of equations, it should be stated that in the solution of many systems derived from elliptic difference equations the use of the successive overrelaxation method requires on the order of $n^{3/2}$ operations. *D. M. Young, Jr.* (Austin, Tex.)

8668:

Zagustin, A. Numerical integration of the Poisson equation. Univ. Central Venezuela. Bol. Fac. Ing. No. 6 (1960), 3–19. (Spanish)

The author discusses several analogues of the Poisson equation using five and seven point formulae. He shows by numerical evidence that the seven point formula leads to greater accuracy using many fewer interior points than is obtained from the five point formula for a simple torsion problem. *J. Douglas, Jr.* (Houston, Tex.)

8669:

Волынский, Б. А. [Volynskiĭ, B. A.]; Бухман, В. Е. [Buhman, V. E.] ★Модели для решения краевых задач. [Models for solution of boundary problems]. Edited by L. A. Lyusternik. Fiziko-Matematičeskaya Biblioteka Inženera. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1960. 451 pp. 13.80 r.

In this book are thoroughly explored not only the practical questions arising in the use of models for approximate solution of boundary problems, but also the theory of models in general. The theoretical part is based on the study of mathematical analogies and, in the last chapter, some new and interesting possibilities in the use of models are pointed out. Many special networks and integrating stars are described in detail as, for instance, models for the solution of Neumann's problem in the three-dimensional case, biharmonic equation, etc. In most cases bounds for errors are studied and limits for obtainable accuracy are computed. *E. Kogbetliantz* (New York)

8670:

Esch, Robin E. A necessary and sufficient condition for stability of partial difference equation problems. J. Assoc. Comput. Mach. **7** (1960), 163–175.

For the solution of the difference equation (1) $v^{n+1}(\mathbf{x})=C(\Delta t)v^n(\mathbf{x})$, the Fourier series (2) $v^n(\mathbf{x})=\sum_{\mathbf{k}} A^n(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$ implies $A^{n+1}(\mathbf{k})=G(\Delta t,\mathbf{k})A^n(\mathbf{k})$, where $0\leq n\Delta t\leq T$ and G is called the amplification matrix. The author constructs a modified Jordan representation $G=RKR^{-1}$, where R and K vary continuously as $\Delta t\to 0$, in order to prove the main result: A necessary and sufficient condition for the stability of (1) is that in the limit as $n\to\infty$, $0\leq n\Delta t\leq T$, each eigenvalue $\lambda=\lambda(\mathbf{k},\Delta t)$ of G must have magnitude such that $n^{p-1}\lambda^n$ is bounded, where p is the size of the Jordan box in which λ appears. This must hold uniformly over all $\mathbf{k}$ required in the sum (2).

E. Isaacson (New York)

8671:

Rusanov, V. V. The solution of simultaneous difference equations. Dokl. Akad. Nauk SSSR **136** (1961), 33–35 (Russian); translated as Soviet Math. Dokl. **2**, 23–25.

An algorithm is described for the successive elimination of unknowns from systems of simultaneous equations of the very sparse type arising when partial differential

equations are transformed into such systems by substituting approximate difference expressions for their derivatives. *F. Goodspeed* (Quebec)

8672:

Mika, J.; Żelazny, R. Polynomial approximation in neutron transport theory. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **8** (1960), 59–62. (Russian summary, unbound insert)

Preliminary results are given on the approximate solution of the plane, monoenergetic, isotropic scattering transport equation by expansions which are in general orthogonal polynomials (i.e., Jacobi polynomials, $P_n^{\alpha\beta}(\mu)$). It is observed that the well-known phenomena in spherical harmonics expansions of better accuracy for odd orders, is a general feature only of Gegenbauer polynomials (i.e., $\alpha=\beta$). Test calculations to determine the "best" Gegenbauer polynomial approximation of first order for critical bare slabs favor the spherical harmonics method only for very thick slabs. *H. B. Keller* (New York)

8673:

Smith, M. G. An approximate solution of the one-dimensional transport equation. Quart. J. Mech. Appl. Math. **13** (1960), 119–128.

The one-dimensional, monoenergetic, anisotropic scattering transport equation

$$(*) \qquad \mu\frac{\partial i}{\partial z}+i=\tfrac{1}{2}\omega\sum_{r=0}^{1}a_rP_r(\mu)\int_{-1}^{1}i(z,\mu')P_r(\mu')\,d\mu'$$

is considered in a grey (i.e., $0\leq\omega\leq 1$) slab, $0\leq z\leq z_0$. After defining new variables the sequence of moment equations (with respect to μ) are terminated by assuming the first and second moments to be proportional, and the resulting second order differential equation is solved. Three arbitrary parameters which enter are then determined by requiring the approximate solutions to have exponential decay as $z\to\infty$ and to minimize a quadratic norm of the "error". A comparison with an accurate numerical solution of one special case of (*) is included and it is stated that the method can be extended to other geometries and more complicated anisotropics. *H. B. Keller* (New York)

8674:

Brik, V. A. Error accumulation in digital computations. Avtomat. i Telemeh. **21** (1960), 595–600 (Russian. English summary); translated as Automat. Remote Control **21**, 412–415.

Author's summary: "The paper describes a method, based on diffusion theory, of estimating the errors of computing devices used in automatic control systems, as well as special-purpose and general-purpose digital computers. An investigation is made of the errors of cyclical computational processes, in which the successively accumulated elementary errors can be considered as independent random variables with an approximately normal distribution density."

8675:

Robinson, S. M.; Struble, G. W. A short method for measuring error in a least-squares power series. Comm. ACM **3** (1960), 351.

8676:

Nemcova, L. G. Nomographing equations of third nomographic order. Vyčisl. Mat. **6** (1960), 100–113. (Russian)

The author gives the geometric sense of the parameters of the projective and nonprojective transformations of equations of the third nomographic order, and shows a new method of nomographing these equations which enables one to use the parameters introduced for improvement of the diagram. As an example, a nomogram is constructed for $z^2=x+y$. *D. Mazkewitsch* (Cincinnati, Ohio)

8677:

Носова, Л. Н. [Nosova, L. N.]. ★Таблицы функций Томсона и их первых производных. [Tables of Kelvin functions and their first derivatives]. Akademiya Nauk SSSR, Vyčislitel'nyĭ Centr. Matematičeskie Tablicy. Izdat. Akad. Nauk SSSR, Moscow, 1960. 423 pp. 4.90 r.

The Thomson (or Kelvin) functions are defined by $\operatorname{ber} x+i\operatorname{bei} x=J_0(xi\sqrt{i})$, $\ker x+i\operatorname{kei} x=K_0(x\sqrt{i})$. The tables give ber, bei, ker, kei and their first derivatives to 7S. For $x=0(.01)10$, these are given directly, and for $x=10(.01)100$, in terms of auxiliary functions. The auxiliary functions are the products by $\exp(\pm x/\sqrt{2})$, the sign being chosen appropriately. Mean second differences are tabulated throughout. There is also a table of $\exp(x/\sqrt{2})$ to 7S for $x=10(.01)100$ and for $x=0(.00001).01$.

The tables were computed on the automatic computer "СТРЕЛА" using series and asymptotic expansions and were checked using differences.

There is an introduction which lists expansions, gives graphs of the functions, discusses and illustrates interpolation in the tables. The bibliography lists earlier tables of the functions, all of which are completely superseded in extent and precision by the present volume.

John Todd (Pasadena, Calif.)

8678:

Bornitz, Ulrich; Schubert, Andreas. ★Siebenstellige Logarithmen der Fakultäten der Zahlen 0 bis 12000. Herausgegeben im Auftrage des Instituts für Maschinelle Rechentechnik an der Technischen Hochschule Dresden. B. G. Teubner Verlagsgesellschaft, Leipzig, 1960. 51 pp. DM 10.30.

Cette table donne les logarithmes décimaux de $n!$ pour: $n=0(1)$ 12 000 avec 7 décimales, le nombre de chiffres de la partie entière variant de 0 à 5.

Les calculs furent effectués sur la calculatrice digitale D_1 de la Technische Hochschule de Dresde avec 14 décimales, pour éviter les cas douteux lors de l'arrondi des résultats.

La clarté typographique est bonne, mais la disposition adoptée peut surprendre. Il convient, en particulier, de faire attention à la reconstitution de la partie entière du résultat, dont seuls les deux derniers chiffres sont donnés, les autres n'étant fournis qu'en haut de colonne, sauf changement de valeur en cours de colonne.

G. Brillouet (Nantes)

8679:

Haselgrove, C. B.; Miller, J. C. P. ★Tables of the Riemann zeta function. Royal Society Mathematical Tables, Vol. 6. Cambridge University Press, New York, 1960. xxiii+80 pp. $9.50.

This volume gives the first tabulation of the Riemann function $\zeta(s)=\sum_{n=1}^{\infty} n^{-s}$ in the complex plane, although only for $\mathrm{Re}(s)=\frac{1}{2}$ and 1. There are five separate tables.

Table I gives 6D values of the real and imaginary parts of $\zeta(\frac{1}{2}+it)$ and $\zeta(1+it)$ together with $Z(t)$ and $\theta(t)$ where $Z(t)e^{-i\pi\theta(t)}=\zeta(\frac{1}{2}+it)$ and $\pi\theta(t)=\mathrm{ph}\{\Gamma(\frac{1}{4}+\frac{1}{2}it)/\pi^{it/2}\}$ for $t=0(.1)100$.

Table II gives only the "signed modulus" $Z(t)$, for $t=100(.1)1000$, to 6D. This is the function used for computing the zeros of $\zeta(\frac{1}{2}+it)$.

Table III is concerned with the zeros γ_n of $Z(t)$. The first 1600 zeros are given to 6D together with the absolute values of $\zeta'(\frac{1}{2}+\gamma_n)$ to 5D. For the first 650 zeros the quantities g_{n-1} and ϕ_n where $\theta(g_n)=n\pi$, $\pi\phi_n=\mathrm{ph}\zeta'(\frac{1}{2}+i\gamma_n)$ are given to 6D. The 1600th zero is given as 2090.036207.

Table IV deals with special ranges of t on the critical line, namely, $t=7000+u$, $17120+u$, $100{,}000+u$ and $250{,}000+u$ for $u=0(.1)25$ where the function $Z(t)$ is given to 6D. The zeros γ of $Z(t)$ in these ranges and $|Z'(\gamma)|$ are also given. The first two ranges were chosen because of the close pairs of zeros:

7005.062866	17143.786536
7005.100565	17143.821844.

The largest value of $|\zeta(\frac{1}{2}+it)|$ so far computed occurs near $t=17123$ and is 19.0.

Table V is an auxiliary table of $\pi^{-1}\mathrm{ph}\Gamma(\frac{1}{2}+it)$ for $t=0(.1)50(1)600(2)1000$. Pages xxi and xxii of the introduction give 10 to 20 D values of the coefficients in the power series expansions of certain functions ϕ_r used in the reviewer's form of the Riemann-Siegel formula for $\zeta(\frac{1}{2}+it)$ and of the corresponding functions ψ_r for dealing with $\zeta(1+it)$.

This is the first table of the Royal Society to be produced automatically by electronic computers. Any other method would not have been feasible. A great many terms of the Euler-Maclaurin or Riemann-Siegel series were used to calculate each entry. For some cases 15 figure logarithms were needed as input data to obtain barely 6 decimal accuracy. The large zeros of Table IV required inverse interpolation involving 14th differences.

The tables will be very helpful in future exploration of inequalities involving certain numerical functions. Haselgrove had already used the first 600 zeros of $\zeta(s)$ to disprove Pólya's conjecture that there are always at least as many numbers $\leq x$ having an odd number of prime factors as those having an even number of prime factors [Mathematika **5** (1958), 141–145; MR **21** #3391.]

D. H. Lehmer (Berkeley, Calif.)

COMPUTING MACHINES

See also A7977, 8606, 8662, 8674, 9383, 9415.

8680:

Ralston, Anthony; Wilf, Herbert S. (Editors). ★Mathematical methods for digital computers. John Wiley & Sons, Inc., New York-London, 1960. xi+293 pp. $9.00.

This book consists of 26 papers individually reviewed below. They are divided into 6 groups, as follows. (1) Generation of elementary functions [#8681]. (2) Matrices and linear equations [#8682–8687]. (3) Ordinary differential equations [#8688–8691]. (4) Partial differential equations [#8692–8696]. (5) Statistics [#8697–8700]. (6) Miscellaneous methods [#8701–8706]. Each paper presents the theory of the mathematical method, the calculation procedure, the digital computer flow chart and data about memory requirements and running time.

8681:

Kogbetliantz, E. G. Generation of elementary functions. Mathematical methods for digital computers, pp. 7–35. Wiley, New York, 1960.

This is an interesting account of methods available for approximating elementary functions and their application to automatic computers. The subject, as the author says, is developing fast, so this is an interim account. Part A describes the mathematical tools, mainly power series and rational approximations of various kinds. Part B gives a considerable number of typical applications in detail. The chapter is a difficult one to read, being very individual and condensed in style and in printing. (The individuality is typified by the author's use of Argtanhyp x for the function the reviewer prefers to call $\mathrm{th}^{-1}x$.) Nevertheless it should be studied carefully.

J. C. P. Miller (Cambridge, England)

8682:

Orden, Alex. Matrix inversion and related topics by direct methods. Mathematical methods for digital computers, pp. 39–55. Wiley, New York, 1960.

This and the following five articles constitute part II, "Linear equations and matrices", of the collection edited by Ralston and Wilf. Judged by these six articles alone, the Ralston-Wilf volume is intended to be a handbook for programmers, in which known algorithms are brought together in a form that should make it easy to code them for a digital computer. Each article follows a rigid outline, with minor deviations: (1) Function (purpose of the computer programs); (2) Mathematical discussion (usually a review of one or two papers); (3) Summary of the calculation procedure; (4) Flow chart; (5) Description of the flow chart; (6) Subroutines needed; (7) Sample problem (of order 2 or 3); (8) Memory requirements; (9) Estimation of the running time; (10) References.

One misses two vital topics of linear algebra, which may have been omitted because programs had not yet developed to the point where they could enter a handbook. One is the calculation of eigenvalues of a nonsymmetric matrix; the other is the efficient use of magnetic tapes for large matrices.

Let A be an $m\times m$ matrix, B an $m\times r$ matrix, both real and both presumed to be sufficiently dense that all elements will be stored in a computer. The programs considered in the present article are the most comprehensive in part II, and will do any subset of the following tasks: (1) compute A^{-1}, if it exists; (2) solve $AX=B$ for an $m\times r$ matrix X, provided the system is consistent; (3) find the determinant $\delta(A)$; and/or (4) find $\rho(A)$, the rank of A, and locate a nonsingular minor of order $\rho(A)$.

The method is the Gauss-Jordan complete elimination by columns. The mathematical discussion considers round-off, storage, fixed versus floating point, the importance of searching for the largest pivot in each column, etc. In all these matters the author describes and advocates modern computing practices, but there are neither references nor quantitative justifications for them. The

flow charts are detailed, and bring out the dominance of logical over arithmetic processes in these problems. The author claims that his package needs no more than 400 words for the program and temporary storage; additional storage is needed for any subroutines (e.g., double precision arithmetic) and the matrices. {There is no hint of the type of computer the author has in mind here.} To compute A^{-1} is said to require αm^2 multiplication times, where α is usually between 2 and 3.

G. E. Forsythe (Stanford, Calif.)

8683:

Van Norton. The solution of linear equations by the Gauss-Seidel method. Mathematical methods for digital computers, pp. 56–61. Wiley, New York, 1960.

The present article deals with the solution of a nonsingular system of N linear algebraic equations in N unknowns by the classical iteration called the Gauss-Seidel method (or the Seidel method, or the method of successive displacements). The author reviews the theory of this linear iteration, and gives two standard sufficient conditions for its convergence. There is no discussion of factors governing the speed of convergence. The flow chart is trivially simple. {Reviewer's note: Use of the method on substantial problems would often require some acceleration techniques, and these would surely require a more elaborate flow chart.}

G. E. Forsythe (Stanford, Calif.)

8684:

Beckman, F. S. The solution of linear equations by the conjugate gradient method. Mathematical methods for digital computers, pp. 62–72. Wiley, New York, 1960.

The conjugate-gradient method was described by Hestenes and Stiefel [J. Res. Nat. Bur. Standards **49** (1952), 409–436 (1953); MR **15**, 651], a paper that the author fails to cite. It is an N-step iterative process for solving a linear system $Ax=k$ of N equations in N unknowns, where A is a symmetric and positive definite matrix. The present paper is a careful exposition of the method, with an indication of some of its variations. The author brings out the points where high accuracy is needed, and discusses time and accuracy. He states that the conjugate gradient method can be used with great economy for large sparse matrices A, but that it takes about six times as long as elimination for a dense matrix. The flow chart shows the simplicity of the method, but is not so detailed as many in this handbook.

G. E. Forsythe (Stanford, Calif.)

8685:

Wilf, Herbert S. Matrix inversion by the method of rank annihilation. Mathematical methods for digital computers, pp. 73–77. Wiley, New York, 1960.

The present article consists mainly of the addition of a flow chart and a discussion thereof to an article by the same author [J. Soc. Indust. Appl. Math. **7** (1959), 149–151; MR **21** #6689]. The algorithm obtains the inverse of a matrix of order N by successive use of a formula of Sherman and Morrison [Ann. Math. Statist. **21** (1950), 124–127; MR **11**, 693] for the change of the inverse of a matrix corresponding to the change of one element. The number of multiplications and divisions is asymptotically $(7/3)N^3$, as $N\to\infty$. *G. E. Forsythe* (Stanford, Calif.).

8686:

Oswald, Florence Jeanne. Matrix inversion by Monte Carlo methods. Mathematical methods for digital computers, pp. 78–83. Wiley, New York, 1960.

This is an exposition of a Monte Carlo method due to von Neumann and Ulam and expounded by the reviewer and Leibler [Math. Tables Aids Comput. **4** (1950), 127–129; **5** (1951), 55; MR **12**, 361] for inverting certain types of matrices. Detailed flow charts, commentary, and a sample problem of order 3 are included.

G. E. Forsythe (Stanford, Calif.)

8687:

Greenstadt, John. The determination of the characteristic roots of a matrix by the Jacobi method. Mathematical methods for digital computers, pp. 84–91. Wiley, New York, 1960.

Let A be a real symmetric matrix. The author discusses the Jacobi method of diagonalizing A to machine accuracy by successive rotations of coordinate 2-planes. The recommended variant is the "threshold" method of Pope and Tompkins [J. Assoc. Comput. Mach. **4** (1957), 459–466; MR **20** #2839]. The flow chart is not as detailed as some in this handbook, in that it refers to formulas in the text for the arithmetic.

G. E. Forsythe (Stanford, Calif.)

8688:

Ralston, Anthony. Numerical integration methods for the solution of ordinary differential equations. Mathematical methods for digital computers, pp. 95–109. Wiley, New York, 1960.

Informal discussion, with valuable bits of practical advice, of multi-step methods for solving the initial value problem $y'=f(x, y)$, $y(x_0)=y_0$. Emphasized are the device of estimating local discretisation error by comparing predicted and corrected values, starting methods, criteria and methods for changing the interval, and, in a heuristic way, "stability". A model procedure is described analytically and by means of a flow diagram.

P. Henrici (Los Angeles, Calif.)

8689:

Romanelli, Michael J. Runge-Kutta methods for the solution of ordinary differential equations. Mathematical methods for digital computers, pp. 110–120. Wiley, New York, 1960.

Discussion of the Runge-Kutta method for the numerical integration of a system of ordinary differential equations of order one, both in its classical form and in the form due to S. Gill [Proc. Cambridge Philos. Soc. **47** (1951), 96–108; MR **12**, 538], including a detailed derivation of the formulas for the case of a single equation. There are some remarks on the choice of the stepsize and on extrapolation to $h=0$. Also given are a flow diagram and a numerical example. *P. Henrici* (Los Angeles, Calif.)

8690:

Wachspress, Eugene L. The numerical solution of boundary value problems. Mathematical methods for digital computers, pp. 121–127. Wiley, New York, 1960.

A finite difference method for solving the Sturm-Liouville boundary value problem

$$-\frac{d}{dx}[p(x)Y']+q(x)Y = f(x), \quad \alpha < x < \beta,$$

$$\alpha_1 Y(\alpha)-\alpha_2[p(\alpha)Y'(\alpha)] = \alpha_3,$$

$$\beta_1 Y(\beta)-\beta_2[p(\beta)Y'(\beta)] = \beta_3,$$

is described (p, q, f piecewise continuous; $p>0$, $q\geq 0$). The meshpoints are spaced arbitrarily. In each subinterval the functions p, q, f are replaced by their mean values, and the resulting differential equation with constant coefficients is solved explicitly. The tridiagonal system of linear equations for the approximate values of the solution at the meshpoints thus obtained is solved by Gaussian elimination. There are some remarks on the propagation of round-off error in the elimination process, but no formal discussion of convergence or discretisation error is given. The computational procedure is illustrated by a flow diagram. {Some additional condition on α_1, α_2, β_1, β_2 such as positivity is needed in order to ensure the existence of a solution.} *P. Henrici* (Los Angeles, Calif.)

8691:

Certaine, J. The solution of ordinary differential equations with large time constants. Mathematical methods for digital computers, pp. 128–132. Wiley, New York, 1960.

This article is concerned with the numerical integration of systems of differential equations of the form $z' = -Dz+s(z,\tau)$, where D is a diagonal matrix with large non-zero elements and where the elements of the functional matrix of s with respect to the components of z are of moderate size. It is known that such systems are difficult to integrate numerically by conventional methods unless the time step is chosen extremely small. An alternate procedure, which amounts to writing the differential equation as a Volterra integral equation for the function $y=e^{(\tau-\tau_0)D}z$ and using numerical quadrature, is recommended. *P. Henrici* (Los Angeles, Calif.)

8692:

Keller, Herbert B. The numerical solution of parabolic partial differential equations. Mathematical methods for digital computers, pp. 135–143. Wiley, New York, 1960.

The author discusses the numerical solution of

$$u_t-a(x,t)u_{xx}-2b(x,t)u_x+c(x,t)u = d(x,t)$$

by replacing u and its space derivatives by the usual two level difference equation involving linear combinations of appropriate differences at adjacent time levels. He derives a maximum principle and uses it to establish solvability of the linear equations and convergence of the solution of the difference equation to that of the differential equation. A brief description of the actual computing problem is included. The entire discussion is at an elementary level and is quite clear. Nothing new is presented in this survey paper. *J. Douglas, Jr.* (Houston, Tex.)

8693:

Sheldon, J. W. Iterative methods for the solution of elliptic partial differential equations. Mathematical methods for digital computers, pp. 144–156. Wiley, New York, 1960.

The author gives an expository discussion of some numerical methods for solving elliptic partial differential equations by finite difference methods. The discussion is devoted to iterative methods for solving the systems of linear equations derived from difference equations rather than to the accuracy of the solutions of the difference equation as a solution of the differential equation. First the difference equation corresponding to a self-adjoint elliptic equation is derived. If the values of the function on the boundary of the region are prescribed and if the boundary is assumed to consist of mesh lines, then the matrix of the corresponding linear system is symmetric. Among the iterative methods considered for solving the linear system are linear iterative processes of first degree in the sense of Forsythe [Bull. Amer. Math. Soc. **59** (1953), 299–329; MR **15**, 65] and related acceleration procedures. The effectiveness of a given method is measured with the aid of the spectral norm and the spectral radius of an associated matrix.

Acceleration processes based on the use of Chebyshev polynomials are discussed. Second degree processes, while requiring extra storage, provide greater computational stability and have the property that the intermediate results as well as the final result is, in a sense, optimal. These considerations are applied to the Jacobi method and to the Gauss-Seidel method as well as to the corresponding acceleration procedures. By a modification of the acceleration procedure for the Gauss-Seidel method, somewhat faster convergence is obtained.

The successive overrelaxation method is discussed and the known results are derived for matrices which are cyclic of index 2 [V. Romanovsky, Acta Math. **66** (1936), 147–251] which is equivalent to "Property (A)" in the sense of Young [Trans. Amer. Math. Soc. **76** (1954), 92–111; MR **15**, 562]. The rapidity of convergence of the method is shown to be approximately the same as the "Chebyshev accelerated" Gauss-Seidel method. Additional topics covered include the application to the Dirichlet problem, a brief discussion of block iteration and alternating direction methods and a flow diagram for the solution by a computer of a simple illustrative problem.

While the exposition is reasonably good as far as it goes, it is somewhat out of date and does not cover some important topics in sufficient detail. For example, it is stated that the degree of improvement from line relaxation is often "disappointingly small". Actually, Cuthill and Varga [#8651] have shown that in many cases with line relaxation one can reduce the number of iterations by a factor of $2^{-1/2}$ without increasing the work required per iteration. More recently Varga has obtained larger savings by the use of two-line relaxation. Another method which should have been discussed in more detail is the alternating direction method of Peaceman and Rachford. In spite of the fact that the theoretical analysis for method in the general case has not been completely worked out, the method has been used with considerable success for many problems. It certainly should have been presented if the chapter was intended to cover the more widely used iterative methods. {One misprint should be noted: on page 152, the word "recreational" on line 21 of the first column should be replaced.}

D. M. Young, Jr. (Austin, Tex.)

8694:

Klahr, Carl N. A Monte Carlo method for the solution of elliptic partial differential equations. Mathematical methods for digital computers, pp. 157–164. Wiley, New York, 1960.

An expository article which shows how to translate into computer practice certain algorithms given by the reviewer [Proceedings, Seminar on Scientific Computation, Nov., 1949, pp. 87–109, IBM Corp., New York, 1950; MR **13**, 286]. *J. H. Curtiss* (Miami, Fla.)

8695:

Lister, Mary. The numerical solution of hyperbolic partial differential equations by the method of characteristics. Mathematical methods for digital computers, pp. 165–179. Wiley, New York, 1960.

This article gives an exposition of the use of characteristics for solving hyperbolic partial differential equations in two dependent and two independent variables, with illustration from one-dimensional isentropic flow. Equations for computation are given in detail for solution at points on the characteristic grid, and also for solution on a rectangular grid, which effectively involves one-dimensional interpolation at each step. The second method, with the possibility of "extrapolation to zero grid size", is generally preferred, and details are given of a flow-diagram, storage required, and estimation of running time when the equations are solved on a digital computer, together with a numerical example. *L. Fox* (Oxford)

8696:

Fox, P. The solution of hyperbolic partial differential equations by difference methods. Mathematical methods for digital computers, pp. 180–188. Wiley, New York, 1960.

The author considers primarily the solution of the wave equation $\partial^2 y/\partial t^2 = c^2(\partial^2 y/\partial x^2)$ with boundary conditions $y(x, 0) = f_1(x)$ and $\partial y(x, 0)/\partial t = f_2(x)$. The solution of an equivalent first order system by explicit, implicit, and staggered finite difference schemes is discussed from the point of view of convergence and stability. Particular references are made to the paper of P. Lax and R. Richtmyer [Comm. Pure Appl. Math. **9** (1956), 267–293; MR **18**, 48] and to the book by R. Richtmyer, *Difference methods for initial-value problems* [Interscience, New York, 1957; MR **20** #438]. A description of the procedure for solving a typical problem on a high-speed computer is given. A flow chart and numerical results for a sample case are included. *D. M. Young, Jr.* (Austin, Tex.)

8697:

Efroymson, M. A. Multiple regression analysis. Mathematical methods for digital computers, pp. 191–203. Wiley, New York, 1960.

A step-by-step description, with a flow chart, of an algorithm, suitable for digital computers, for obtaining the solution to the following problem: Given a set of values of a number of variables, find a least-squares regression equation for determining the dependence of a selected variable upon a subset of the others, including in the subset only those variables whose individual contributions to the variance of the dependent variable exceed a certain threshold. As the contribution of each variable is dependent upon which variables are already included, this problem does not necessarily have a unique solution. The algorithm described provides that solution which is obtained by a procedure of adding or deleting one variable at a time. *B. A. Chartres* (Sydney)

8698:

Harman, Harry H. Factor analysis. Mathematical methods for digital computers, pp. 204–212. Wiley, New York, 1960.

This is a brief account of factor analysis (in which an attempt is made to describe a set of variables in terms of a smaller number of variables or factors) and the description of a recommended calculational method. It contains an introduction to the subject, mathematical discussion, and summary of a calculational procedure together with remarks on subroutines, memory requirements, and running time. The mathematical discussion includes a derivation of the principal factor method as applied to a reduced correlation matrix. The recommended procedure is essentially a modified Jacobi method.

P. S. Dwyer (Ann Arbor, Mich.)

8699:

Southworth, Raymond W. Autocorrelation and spectral analysis. Mathematical methods for digital computers, pp. 213–220. Wiley, New York, 1960.

This paper describes a program to compute the autocorrelation coefficients, the raw spectral estimates and the smoothed spectral estimates for a stationary time-series from data sampled at equal time intervals. A detailed flow-chart is provided with sufficient explanatory material to make the construction of a FORTRAN code, for example, a simple matter.

W. Freiberger (Providence, R.I.)

8700:

Hartley, H. O. Analysis of variance. Mathematical methods for digital computers, pp. 221–230. Wiley, New York, 1960.

This paper is an elaboration of the author's article in Biometrics **12** (1956), 110–122 [MR **18**, 74] for organizing analysis of variance computations on a computer.

M. Zelen (Washington, D.C.)

8701:

Wilf, Herbert S. The numerical solution of polynomial equations. Mathematical methods for digital computers, pp. 233–241. Wiley, New York, 1960.

This is an expository paper, which begins with a brief summary of results on the localization of roots, including Sturm's theorem, then describes, and provides a flow chart for, a practical algorithm. The method is to obtain a first approximation by use of a modified form of Bernoulli's method, and thereafter to improve it by use of Bairstow's method.

A. S. Householder (Oak Ridge, Tenn.)

8702:

Ralston, Anthony. Methods for numerical quadrature. Mathematical methods for digital computers, pp. 242–248. Wiley, New York, 1960.

The author abstracts a small number of the known

formulas for numerical quadrature. In this area, at least as reflected here, the existence of digital computers seems to have had minimal effects on methods. A formulas or methods book in which summaries are given of standard procedures is useful. In this case, there are several rather too hasty conclusions given for the reviewer's taste. For example (p. 244): "Newton-Cotes closed type formulas are significantly more accurate than open-type formulas" (for a given number of points). This, it is claimed, is a reason for not using open-type formulas. If one sets $n=0$ and uses the 1-point "open" formula it has superiority (on the author's scales) over the trapezoidal (2-point) [cf. Hammer, Math. Mag. **31** (1957/58), 193–195; MR **20** #6191]. As a matter of fact, the choice of points for the ordinary open formulas is not well taken, since they should begin at a half-interval step from the endpoint. The open formulas are also to be preferred for instances in which the integrand may not be defined on the endpoint.

Also the author states (p. 246): "From (15) we see that Gaussian quadrature formulas of high order suffer even more than Newton-Cotes formulas from having high-order derivatives in the error terms. For this reason we shall be chiefly interested in low order Gaussian quadrature formulas." Here what was claimed as an advantage for closed-type formulas above becomes a disadvantage. The fact is that any high-order formula is usually useful only if the integrand function is very well approximated by polynomials.

For machine computation the validity of numerical convergence tests (e.g., comparison of the estimate and the estimate for halved intervals) is perhaps more to the point than error terms using derivatives, although in calculating integrals of simple functions the latter have a role.

Numerical integration in higher dimensions is ignored in the book except for the next paper [#8703]. Now the reader will find that if he has other problems of integration such as iterated integration, integration over a square, a circle, a cube or a sphere, no specific help is given. A handbook such as this might be expected at least to indicate a few of the recent developments and to give references to a much broader set of papers and tables. The six references in this paper are not adequate for tables or formulas. *P. C. Hammer* (Madison, Wis.)

8703:

Kahn, Herman. Multiple quadrature by Monte Carlo methods. Mathematical methods for digital computers, pp. 249–257. Wiley, New York, 1960.

An expository article written in an informal style which (a) briefly sketches the probability theory underlying the estimation of (deterministic) multiple integrals by probability sampling methods, (b) names some of the variance-reducing techniques, describing them only vaguely but giving references, (c) indicates some of the known methods of generating probability distributions numerically, (d) gives an example of the evaluation on a digital computer of a nine-fold integral arising in electron physics. The example does not illustrate (b).

J. H. Curtiss (Miami, Fla.)

8704:

Goertzel, G. Fourier analysis. Mathematical methods for digital computers, pp. 258–262. Wiley, New York, 1960.

The Fourier coefficients of the expansion of a function defined at discrete, equidistant points are evaluated by means of a simple iterative procedure based upon the Euler-Maclaurin expansion. Only two trigonometric functions have to be looked up or calculated, and the method is also shown to apply to the evaluation of Chebyshev polynomials. A computer flow diagram is provided and memory requirements are estimated. An efficient and useful method. *A. D. Booth* (London)

8705:

Arden, Dean N. The solution of linear programming problems. Mathematical methods for digital computers, pp. 263–279. Wiley, New York, 1960.

"Discusses briefly the major properties of linear programming problems and the algorithms in most common use for the solution of these problems, the simplex method and the dual simplex method. Proofs and descriptions are given with attention to the details which arise in computational work. In particular, the question of keeping track of permutations of variables is handled explicitly with the aid of matrices." (From the introduction)

After proving the main theorems, the author proceeds to extensive discussion of computational practice with digital computers. *C. Davis* (Providence, R.I.)

8706:

Bashkow, T. R. Network analysis. Mathematical methods for digital computers, pp. 280–290. Wiley, New York, 1960.

Computer technique is given for the steady-state analysis of a special class of ladder networks.

G. Kron (Schenectady, N.Y.)

8707:

★The Computing and Data Processing Society of Canada. 2nd Conference: June 6, 7, 1960. Proceedings. University of Toronto Press, Toronto, 1960. vi+365 pp. $5.00.

A set of 29 papers of which those with mathematical interest will be reviewed separately.

8708:

Güntsch, Fritz Rudolf. ★Einführung in die Programmierung digitaler Rechenautomaten, mit besonderer Berücksichtigung der Z 22. Walter de Gruyter & Co., Berlin, 1960. 142 pp. (2 inserts) DM 24.00.

This book is intended to introduce the reader into programming for the Z22 (a medium size electronic computer which is in use at many computer installations in Germany) as well as to give him additional facts valid also for other computers. A few technical details of the Z22 are given in the introduction; the other chapters are dedicated entirely to programming. In chapters 3–5, allocation of addresses and loop control are explained very carefully. Chapter 6 then treats the so-called "changes of the second kind", i.e., operation needed to call in a subroutine correctly. The author proposes mainly the Bauer-Samelson technique which stores parameters in consecutive storage places immediately after the jump to the subroutine, from where they may be transferred to the subroutine. For vectors and matrices this may be too

clumsy; therefore only block addresses and dimensions are given as parameters in such cases. Chapter 7 gives some information concerning "changes of the first kind", namely modifications (performed at input time) needed to make a subroutine operative if it is not stored in the places for which it was originally designed. Chapter 8 shows how the techniques of chapters 6 and 7 may be simplified with the aid of index-registers: instead of actually changing the subroutine, its initial address and the initial address of the parameter block are stored in separate index registers and only the contents of these are changed with every jump to and from a subroutine. In addition, index-registers are of course used to simplify the computation with quantities the addresses of which depend on other quantities. In chapter 9 compiling and interpretive technique are mentioned. Chapters 10 and 11 give information on the internal code of the Z22 computer (in the other chapters only the much easier external code is used) as well as details concerning certain organisational subroutines such as printing.

The book is easy to read and certainly will be welcomed by the prospective user of the Z22 as well as by users of other machines. *H. Rutishauser* (Zürich)

8709:

Leveson, J. H. (Editor). ★Electronic business machines. Philosophical Library Inc., New York, 1960. 272 pp. $15.00.

This book, which is divided into three sections (Programming for business purposes, Business management and electronic data processing, Computer equipment and applications), results from a series of lectures delivered at Dundee Technical College in 1958. The individual sections are competently treated, but the work suffers from the usual redundancies present when nineteen chapters are written by nine authors. The price of the book is excessive, especially in the light of the many alternatives available.

[See Alt, *Electronic digital computers: their use in science and engineering*, Academic Press, New York, 1958; MR **21** #4564; Gotlieb and Hume, *High-speed data processing*, McGraw-Hill, New York, 1958; MR **20** #2864; Jeenel, *Programming for digital computers*, McGraw-Hill, New York, 1959; MR **21** #7607; Mandel, *Fundamentals of digital computers*, Prentice-Hall, Englewood Cliffs, N.J., 1958; MR **21** #443; McCracken, Weiss, and Lee, *Programming business computers*, Wiley, New York, 1959; MR **21** #6110; Proc. 5th Annual Comput. Appl. Sympos. (October 29–30, 1958), Technology Center, Chicago, Ill., 1959; MR **20** #6195.] *C. C. Gotlieb* (Toronto)

8710:

Goode, Omar S. ★Analysis of items in large scales: Generalized programs for the IBM 650 computer. Introduction by Ralph M. Stogdill. Special Bulletin No. X-59. Bureau of Business Research, The Ohio State University, Columbus, Ohio, 1960. v + 205 pp. $3.00.

This monograph provides complete explanation of the use of the IBM 650 for large scale item analysis. Detailed instructions cover card format, machine procedures, specifications of machine operations, and the relevant subroutines. The complete program consists of a set of routines for finding the means, standard deviations of a test score or subtest score, item-subtest correlations, item intercorrelations, test or subtest reliability coefficients, and a Wherry-Winer factor analysis of items. The subroutines include matrix multiplication and matrix inversion. The monograph is particularly suited to social scientists with little experience in the utilization of electronic computers. A short example would have increased the usefulness of the monograph. Complete details are provided and sets of cards for the various routines may be obtained.

L. A. Aroian (Los Angeles, Calif.)

8711:

Czapla, V. Digitale Rechenanlagen in der Sowjetunion. Math.-Tech.-Wirtschaft **8** (1961), 16–18.

Author's summary: "Eine Übersicht über den Bau und den Einsatz digitaler Rechenanlagen in der Sowjetunion wird gegeben. Am Ende des Beitrages folgt eine Tabelle über die Daten verschiedener sowjetischer Rechenanlagen."

8712:

Good, I. J. A comparison of some methods of calculating covariance functions on an electronic computer. Comput. J. **3** (1960/61), 262–265.

Three methods are compared for calculating the lagged products $c_s = \sum_{r=1}^{N-s} a_r b_{r+s}$ $(s = -M, -M+1, \cdots, M)$ in each of which several numbers are packed into a single word of the computer memory. In method I, both a's and b's are packed into single words, taking full advantage of parallel multipliers, if the computer has them; in method II, only a's are packed; and method III, using logical instructions, is appropriate mainly if each a and b consists of one binary digit only.

W. Freiberger (Providence, R.I.)

8713:

Neubüser, J. Untersuchungen des Untergruppenverbandes endlicher Gruppen auf einer programmgesteuerten elektronischen Dualmaschine. Numer. Math. **2** (1960), 280–292.

A description is given of a program designed to analyze the structure of finite groups with the aid of a binary digital computer instead of theorems. The group elements are represented by permutations on n marks. One or more words are used to store one permutation, using n subwords, each sufficiently large to store a number from 1 to n. Thus, for $n = 19$, 5 bits are required for each mark and so 7 marks are packed into each of 2 words and the remaining 5 marks are packed into a third word. (A 38 bit word is used.) The program generates the group from the given generators and proceeds to analyze the group by means of cyclic subgroups. It reports out (a) the order of the group, (b) the number of cyclic subgroups of prime power order, (c) the total number of subgroups, and (d) the number of classes of conjugate subgroups. Space and time restrictions of the machine, a Zuse 22, limit the program to groups of order about 300, without outside human assistance. *D. H. Lehmer* (Berkeley, Calif.)

8714:

Perry, C. Conversion between floating point representations. Comm. ACM **3** (1960), 352.

8715:

Smith, Philip A. Some problems and approaches to automation of medical diagnosis. Behavioral Sci. **6** (1961), 88–91.

8716:

Maslov, A. A.; Purlov, Yu. G. A universal function generator based on the principle of quadratic approximations. Avtomat. i Telemeh. **21** (1960), 237–244 (Russian. English summary); translated as Automat. Remote Control **21**, 164–174.

Authors' summary: "Methods are presented for quadratic approximations for functions given either analytically or graphically. Simple formulas and relationships are derived for finding the rule for argument partitioning. For obtaining a quadratic relationship, a variant of a diode element is considered.

"The paper describes the circuit of a universal function generator using quadratic approximations and capable of being implemented in the form of an attachment to a type EMU-8 analog computer."

8717:

Hechtel, J. R.; Seeger, J. A. Accuracy and limitations of the resistor network used for solving Laplace's and Poisson's equations. Proc. IRE **49** (1961), 933–940.

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See also 8781, 8782, 9099.

8718:

★Handbuch der Physik. Bd. III/1. Prinzipien der klassischen Mechanik und Feldtheorie. Herausgegeben von S. Flügge. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. vii + 902 pp. DM 198.00.

This volume contains two articles: "Classical dynamics" by J. L. Synge and "The classical field theories" by C. Truesdell and R. Toupin, both of which are reviewed in this issue [#8719, 8778].

8719:

Synge, J. L. Classical Dynamics. Handbuch der Physik, Bd. III/1, pp. 1–225. Springer, Berlin, 1960.

The brief introductory chapter A (10 pp.) is primarily concerned with the relation between Newtonian and relativistic dynamics of a particle. Chapter B on kinematics (32 pp.) deals with displacements of a rigid body (orthogonal matrices, Eulerian parameters, quaternions, Pauli matrices), kinematics (velocity, acceleration, and hodograph of a particle, angular velocity of a rigid body, moving axes), mass distributions and force systems (mass center, moments and products of inertia, principal axes of inertia, linear and angular momentum, kinetic energy, force systems), and generalized coordinates (holonomic and non-holonomic systems, generalized forces, work, potential). Dynamics of a particle is the subject of chapter C (13 pp.) which has sections on the equations of motion (basic equations, energy and angular momentum, moving frames of reference), one-dimensional motions (simple harmonic oscillator, damping, circular and cycloidal pendulum). Chapter D on dynamics of systems of particles and of rigid bodies (42 pp.) contains sections on the equations of motion (principles of linear and angular momentum, d'Alembert's principle, energy, Lagrange's equations, ignorable coordinates, Hamilton's and Appell's equations, equations of motion of a rigid body, moving frames of reference), systems without constraints (two-body problem, capture and scattering, n-body problem, periodic structures), rigid body with a fixed point (rigid body under no forces, spinning top, gyroscopic stiffness, gyrocompass) and impulsive motion (this section does not appear in the table of contents; its subheadings are impulsive forces and moments, impulsive work, Lagrange's equations collisions, coefficient of restitution, minimal theorems in impulsive motion). Chapter E (101 pp.) presents the general dynamic theory; its sections are entitled geometrical representations of dynamics (the role of general dynamic theory, representative spaces, topological remarks), the space of events (the homogeneous and the ordinary Lagrangian, first form of Hamilton's principle, Lagrange's equations of motion, the energy equation and the Hamiltonian, second form of Hamilton's principle, canonical equations, equivalence of Lagrangian and Hamiltonian dynamics, examples of Lagrange-Hamilton correspondences, reciprocity theorem, Hamilton's two-point characteristic or principal function, the Hamilton-Jacobi equation, dynamics based on a chosen two-point characteristic function, coherent systems of rays or trajectories, the one-point characteristic function, waves of constant action, Huygens' construction, determination of waves from initial data, method of characteristic curves, Jacobi's complete integral of the Hamilton-Jacobi equation, the practical use of Jacobi's theorem, separation of variables), momentum-energy space (momentum-energy characteristic function encounters), configuration space (reinterpretation of dynamics in configuration space, rays and waves in a coherent system, isoenergetic dynamics and its relation to general dynamics in the space of events, Maupertuis action, the two-point characteristic function for isoenergetic systems, the homogeneous Lagrangian and Jacobi's principle of least action, the kinematical line element, least curvature), the space of states and energy (the energy surface and the energy function, canonical transformations, the bilinear invariant, generating functions, Poisson and Lagrange brackets, canonical transformations generated by canonical equations, the basic relative integral invariant, transformation of the natural congruence to straight lines by solving the Hamilton-Jacobi equation, the order of the canonical equations and its reduction by means of a first integral), the space of states (circulation theorem, transformation of coordinates, canonical transformations), phase space (basic theory for conservative systems, non-conservative systems, canonical transformations, Poisson and Lagrange brackets, absolute integral invariants, Liouville's theorem, action-angle variables, the periodic property of angle variables), and small oscillations (reduction of energies to normal form, normal modes and frequencies, degeneracy, the effect of constraints, dissipative systems, gyroscopic stability, forced oscillations, resonance, operational methods, oscillations about steady motion or about singular point in phase space, transformation of Hamiltonian normal form, perturbations). Chapter F (25 pp.) on relativistic dynamics contains sections on Minkowskian space-time and the

laws of dynamics (Lorentz transformations, kinematics in space-time, 4-momentum, equations of motion of a particle, Lagrangian and Hamilonian dynamics, the free particle, the two-event characteristic function and the Hamilton-Jacobi equation), some special dynamical problems (hyperbolic motion, particle in a potential field, charged particle in electromagnetic field, relativistic Kepler problem), de Broglie waves (coherent systems of trajectories in space-time and associated waves, de Broglie wavelength and frequency), and relativistic catastrophes (conservation of 4-momentum, inelastic and elastic collisions, Compton effect, angular momentum and mass center).

Throughout the work, the exposition has the clarity and freshness that are the author's hall-mark.

W. Prager (Providence, R.I.)

8720:

Downey, Glenn L.; Smith, Gerald M. ★Advanced dynamics for engineers. International Textbook Co., Scranton, Pa., 1960. viii + 354 pp. $12.00.

This textbook is suitable for an advanced undergraduate level course in engineering mechanics, and its principal attributes are its extensive collection of highly practical examples and problems. It is not, nor is it intended to be, suitable for students of physics or applied mathematics, as the basic concepts and postulates of classical mechanics are dealt with in a perfunctory and somewhat sloppy manner, and some of the mathematical exposition is also of that character. In fact, this reviewer feels that in a modern engineering curriculum, too, it is a questionable technique to lead the student through a large compendium of various types of problems, while neglecting the effort to give him a unified theoretical picture of the field.

Vector notation is used throughout, but matrices are not introduced although they could be used to advantage in several topics. In spite of their great utility as well as elegance, Lagrange's equations are introduced only near the end and then only in connection with vibration problems. A few examples of electromechanical analogies are interspersed in the text, and the book closes with a rather useful introduction to the use of analog computers.

E. T. Kornhauser (Providence, R.I.)

8721:

Dubošin, G. N. The mutual potential of a sphere and a solid of revolution. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 4, 55–60. (Russian)

The mutual potential of a sphere with arbitrary spherical density distribution and a solid of revolution, whose density is a given integrable function of the distances from the axis of revolution and from a plane which is perpendicular to the axis of revolution, is determined in the form of the following series:

$$(1) \qquad U(r, \nu) = \frac{fm_0 m}{r} \sum_{k=0}^{\infty} A_k X_k(\nu) \frac{c^k}{r^k}.$$

Here $X_k(\cos\theta) = (-1)^k X_k(\nu)$, a Legendre polynomial of order k where r is the distance of a point considered from the pole situated at the axis of revolution, f is the constant of gravitation, m_0 is the mass of the sphere, m is the mass of the solid, c is the greatest among the three characteristic lengths of the solid of revolution (the two are the distances of the pole to the bounding planes of the solid which are perpendicular to the axis of revolution and the third is the largest cross-section radius). Finally, the coefficients A_k are all bounded and can be obtained by evaluating the double definite integrals.

The author states that the form (1) is especially suitable for certain applications in celestial mechanics as it is for instance in the problem of the determination of helicoidal motion of an artificial satellite of the earth or of a cosmic rocket.

T. P. Anđelić (Belgrade)

8722:

Manna, F. Zur Ermittlung der Wirkungslinie des Massenwiderstandes eben bewegter Scheiben und Getriebeglieder. Ing.-Arch. **30** (1961), 148–151.

8723:

Rocard, Y. ★General dynamics of vibrations. Frederick Ungar Publishing Co., New York, 1960. xii + 522 pp. $17.50.

This book differs from the usual engineering treatments of the theory of vibrations in the range of the physical applications considered. These are drawn from the acoustics and electronics of radio engineering, from applied elasticity, from gyroscopes and from pure physics. The book's value to applied mathematicians will be twofold, as a provider of illustrations for a theoretical course in vibrations and as a source of the standard engineering treatments of practically important oscillation phenomena. Topics covered include the general concept of impedance, stability, non-linear oscillations and the theory of sound with special reference to radiation and diffraction.

D. R. Bland (Manchester)

8724:

Bottema, O. On the stabilization of an equilibrium by rotation. Bul. Inst. Politehn. Iaşi (N.S.) **5** (**9**) (1959), no. 3–4, 75–78. (Russian and Romanian summaries)

A conservative system with two degrees of freedom under or not under the action of linear damping force is referred to the principal coordinate axes. The conditions for stability of the equilibrium-configuration of the system are discussed in the case where the uniform rotating motion of the coordinate plane about its origin is superposed on the small vibratory motion of the system about equilibrium at the origin.

K. Yamada (Nagaoka)

8725:

Nohel, John A. Stability of perturbed periodic motions. J. Reine Angew. Math. **203** (1960), 64–79.

The author gives sufficient conditions for the existence of periodic solutions of the differential equation $x' = Ax + \mu f(x, t, \mu)$, where x and f are n-vectors, A is an $n \times n$ matrix and μ is a small parameter. The paper is closely related to a paper of Coddington and Levinson [*Contributions to the theory of non-linear oscillations, Vol. II*, pp. 19–35, Princeton Univ. Press, Princeton, N.J., 1952; MR **14**, 981] dealing with the same equation in the case that A is a constant matrix written in a certain canonical form. The author considers the case that A, besides satisfying other conditions, has at least one pair of eigenvalues of the form $\pm iN$, N being an integer which may be zero. The author also extends the theory of Coddington and Levinson to the case that A is a periodic matrix. The results are too involved to be stated here.

M. M. Peixoto (Rio de Janeiro)

8726:

Šimanov, S. N. On the theory of oscillations of quasi-linear systems with constant lags. Avtomat. i Telemeh. **21** (1960), 706–709 (Russian. English summary); translated as Automat. Remote Control **21**, 490–492.

Author's summary: "The conditions are given for the existence and stability of almost periodic oscillations in quasi-linear systems with time delays."

8727:

Roĭtenberg, Ya. N. On the placing of gyroscopic compass into a meridian during the acceleration of the rotors of the gyroscopes. Prikl. Mat. Meh. **24** (1960), 88–92 (Russian); translated as J. Appl. Math. Mech. **24**, 113–119.

In una sua nota anteriore [Prikl. Mat. Meh. **23** (1959), 656–665; MR **22** #1762] l'A. riduce all'integrazione di un sistema di equazioni integrali il problema di determinazione delle sollicitazioni di comando $q_j(t)$ atti di far portare, sin dal momento $t=t_1>t_0$ in poi, in un moto prefissato $r_{p_i}(t)$ $(i=1, 2, \cdots, n)$, la traiettoria $y_k(t)$ corrispondente al sistema non stazionario

$$\text{(a)} \qquad \sum_{k=1}^{m} f_{jk}(\mathscr{D})y_k = x_j(t)+q_j(t) \quad (j = 1, 2, \cdots, m),$$

oppure la traiettoria y_k del sistema impulsivo

$$\text{(b)} \qquad \sum_{k=1}^{m} f_{jk}(T)y_k = x_j(t)+q_j(t) \quad (j = 1, 2, \cdots, m),$$

ove $x_j(t)$ sono le forze esteriori date, $f_{jk}(\mathscr{D})$ sono i polinomi in $\mathscr{D}=d/dt$ e $f_{jk}(T)$—i polinomi in T, definito tramite l'eguaglianza $T^s y_k(t)=y_k(t+s\tau)$, $\tau=\text{const}$.

In questa sua nota che ne constituisce la continuazione di un altro lavoro [ibid. **23** (1959), 961–963; MR **22** #5157], l'A., dopo aver ridotto mediante soppressione dei termini d'inerzia il sistema di equazioni del moto di una bussola giroscopica i cui giroscopi hanno i momenti proprii variabili, perviene, nella trascrizione matriciale, alle espressioni determinanti il moto della bussola giroscopica considerata, in corrispondenza alle condizioni iniziali arbitrari $z_k(0)$. Si procede poi, seguendo il primo lavoro citato, alla selezione in funzione del tempo della legge di variazione della forze esteriore supplementare, imponendovi la condizione affinchè nell'istante $t=t_1$, mentre si suppone cessata l'accelerazione dei rotori dei giroscopi la bussola giroscopica si situa in un meridiano. L'articolo finisce con un esempio numerico. *D. Mangeron* (Iași)

8728:

St. Kalitzin, N. Näherungstheorie des rotations-symmetrischen Kreisels mit veränderlicher Masse. Nuovo Cimento (10) **15** (1960), supplemento, 282–289.

Verfasser geht von der Grundgleichung

$$\text{(i)} \qquad \mathbf{I}(\boldsymbol{\omega}_2\times\boldsymbol{\omega}_1) = \mathbf{M}$$

der Näherungstheorie der Kreiselerscheinungen aus im Falle eines rotationssymmetrischen Kreisels mit konstanter Masse. Dabei bezeichnet **I** das Trägheitsmoment des Kreisels in Bezug auf die Drehachse $\overline{OO}_1$; $\boldsymbol{\omega}_1$ ist der Vektor der Winkelgeschwindigkeit des Kreisels um $\overline{OO}_1$; $\boldsymbol{\omega}_2$ ist der Vektor der Winkelgeschwindigkeit der Achse $\overline{OO}_1$. **M** ist das Moment der äußeren Kräfte in Bezug auf einen unbeweglichen Punkt auf der Drehachse. $\boldsymbol{\omega}_1$ wird konstant und $\boldsymbol{\omega}_2 \ll \boldsymbol{\omega}_1$ vorausgesetzt. In der entwickelten Näherungstheorie nimmt Verfasser an: (1) der Drehimpuls **K** behält für $\boldsymbol{\omega}_2 \ll \boldsymbol{\omega}_1$ die Richtung der Kreiselachse $\overline{OO}_1$; (2) bei Berechnung von **K** kann $\boldsymbol{\omega}_2$ vernachlässigt werden; (3) die Massenabnahme des Punktes P_i soll den Kreisel sofort verlassen. Die gesuchte Verallgemeinerung der Grundgleichung (i) ist

$$\text{(ii)} \qquad \mathbf{I}(\boldsymbol{\omega}_2\times\boldsymbol{\omega}_1) = \mathbf{e}_1\times(\mathbf{N}\times\mathbf{e}_1).$$

Dabei ist $\mathbf{e}_1$ der Einheitsvektor auf $\boldsymbol{\omega}_1$, **N** bezeichnet den Vektor $\mathbf{M}+\sum \mathbf{r}_i\times\dot{m}_i\mathbf{u}_i$, und m_i sind die zeitlich veränderlichen Massen der Punkte P_i $(i=1, 2, \cdots, n)$ von den Radienvektoren $\mathbf{r}_i$ ($\mathbf{u}_i$ Geschwindigkeit von dm_i). Die Gleichung (ii) läßt sich auch noch im Falle relativistischer Geschwindigkeiten der Massenabflüsse verwenden, wenn die übrigen Voraussetzungen in Kraft bleiben. Als praktische Anwendungen bieten sich Berechnungen rotierender Sterne veränderlicher Masse (Supernovae) sowie solche von Raketen, welche durch Drehung um ihre Längsachse stabilisiert werden. *M. Pinl* (Cologne)

8729:

Thorne, C. J. Some gyroscopic oscillations. J. Appl. Mech. **28** (1961), 57–66.

Approximate solutions of the differential equations of a gyroscope computed on both an analog and a digital machine. Oscillations are observed when the approximations contain nonlinear terms and the gyroscope is subject to constant acceleration and a "correcting" torque. One of the approximations includes viscous friction terms. The approximate equations are obtained under the assumption that certain angles and their derivatives are small. More details and more curves are said to be given in NAVORD Report 6542, Parts 1 and 2.

J. P. LaSalle (Baltimore, Md.)

8730:

Thomson, W. T. Shock spectra of a nonlinear system. J. Appl. Mech. **27** (1960), 528–534.

Author's summary: "The peak response under pulse excitation, of a single-degree-of-freedom system with nonlinear spring approximated by two straight lines, is analytically determined. Shock spectra for the step and delta function are developed with damping as parameter. For the rectangular pulse excitation, the shock spectra for zero damping are presented with pulse duration as a parameter. For the arbitrary pulse shape, a general procedure is discussed and illustrated for the sine pulse of differing pulse duration. Generalized phase-plane plots associated with the problem are included. The shock spectra presented are applicable only when the first peak corresponds to the maximum response, thereby avoiding the questions of unloading."

STATISTICAL THERMODYNAMICS AND MECHANICS

See also 8672, 8673, 8783, 9022, 9213, 9264, 9265, 9266.

8731:

Uhlenbeck, G. E. Some fundamental problems in statistical physics. Proc. Internat. Congress Math. 1958, pp. 256–267. Cambridge Univ. Press, New York, 1960.

A survey of the fundamental problems confronting statistical mechanics. These include (for equilibrium problems) more powerful methods of evaluating the partition function, in non-equilibrium problems the discussion of the mathematical properties of the Boltzmann equation (uniqueness of solutions, linearized Boltzmann equation, characteristics, more particularly propagation and dispersion), the derivation of the macroscopic laws from the Boltzmann equation (Chapman-Enskog theory), and the derivation of the Boltzmann equation itself from Liouville's equation (Bogolyubov's approach).

P. G. Bergmann (Syracuse, N.Y.)

8732:

Prigogine, I. (Editor). ★Advances in Chemical Physics, Vol. III. Interscience Publishers, New York-London, 1961. ix + 372 pp. $11.50.

A set of eight articles. Those of mathematical interest will be reviewed separately in these pages.

8733:

Naze, Jacqueline. Propriétés des opérateurs de collision linéaires. Étude du cas général. C. R. Acad. Sci. Paris **251** (1960), 651–653.

L'auteur envisage les opérateurs de collision linéarisés dans le cas d'un gaz simple (opérateurs J) et dans le cas d'un mélange binaire dont l'un des constituants possède une distribution maxwellienne des vitesses (opérateurs K). Les opérateurs K commutent avec les rotations et leur action sur le produit d'une fonction sphèrique par une fonction isotrope de le vitesse conduit ou produit de le même fonction sphèrique par une nouvelle fonction isotrope de la vitesse. Ces proprietès s'étendent aux opérateurs J.

H. Cabannes (Paris)

8734:

Naze, Jacqueline. Propriétés des opérateurs de collision linéaires. Étude d'un cas particulier. C. R. Acad. Sci. Paris **251** (1960), 854–856.

Cette note fait suite à la précedente; l'auteur y montre que pour une loi d'interaction en r^{-5}, les fonctions propres de l'opérateur K sont des produits de fonctions sphèriques et de polynomes de Sonine, étendant ainsi au cas des mèlanges binaires les résultats établis par Wang-Chang et Uhlenbeck [Handbuch der Physik, Bd. 12, Springer, Berlin, 1958; p. 370] dans le cas d'un gaz simple. Le rapport des masses des particules des deux constituants est arbitraire.

H. Cabannes (Paris)

8735:

Gross, E. P.; Ziering, S. Heat flow between parallel plates. Phys. Fluids **2** (1959), 701–712.

Authors' summary: "A study is made of the heat flow between parallel plates of slightly different temperatures. The problem is described by the linearized Boltzmann equation subject to microscopic boundary conditions. The distribution function is approximated by half-range polynomials in velocity space and the space-dependent coefficients are determined by forming half-range moment equations. An approximation with four pairs of space functions gives an accurate treatment of the heat flow and of the density and temperature profiles for the entire range of conditions from free molecule to hydrodynamic. Numerical results are obtained for the temperature slip and molecular boundary structure for hard-sphere molecules. Cruder methods of fixing the coefficients are compared." *M. D. Friedman* (Needham Heights, Mass.)

8736:

Ziering, S. Shear and heat flow for Maxwellian molecules. Phys. Fluids **3** (1960), 503–509.

Cet article applique une méthode de développement de la fonction de distribution des vitesses adaptée à l'étude des discontinuités de la fonction ou de ses dérivées [E. P. Gross et S. Ziering, Phys. Fluids **1** (1958), 215–224; **2** (1959), 701–712; MR **22** #7319, 8735]. On trouve ici les résultats de calculs numériques effectués dans le cas de molécules Maxwelliennes (loi en $1/r^5$) pour les écoulements entre des plaques planes parallèles, et on les compare à ceux obtenus dans les articles précédents pour les sphères dures.

Une discussion détaillée compare l'ensemble de ces résultats à ceux de Mott-Smith, Wang-Chang et Uhlenbeck en particulier. Il apparait que le développement ici proposé converge plus rapidement que ceux qu'ont utilisés ces auteurs; les propriétés de l'écoulement en régime continu ($d/\lambda \gg 1$) sont indépendantes de la loi d'intraction moléculaire (cf. la théorie d'Enskog-Chapman).

J. Naze (Marseille)

8737:

Konowalow, Daniel D.; Taylor, Marion H.; Hirschfelder, J. O. Second virial coefficient for the Morse potential. Phys. Fluids **4** (1961), 622–628.

Authors' summary: "Tables and algorithms are given for the classical second virial coefficient and its first two temperature derivatives for gases obeying a Morse potential $\varphi(r) = \varepsilon(x^2 - 2x)$, where $x = \exp[-(c/\sigma)(r - r_m)]$. The calculations cover the temperature range $0.3 \leq kT/\varepsilon \leq 400$."

8738:

Konowalow, Daniel D.; Hirschfelder, Joseph O. Morse potential parameters for O-O, N-N, and N-O interactions. Phys. Fluids **4** (1961), 637–642.

8739:

Konowalow, Daniel D.; Hirschfelder, J. O. Intermolecular potential functions for nonpolar molecules. Phys. Fluids **4** (1961), 629–636.

Authors' summary: "The Morse potential function is used to represent the intermolecular potential for several nonpolar substances."

8740:

Siegel, Armand. Differential-operator approximations to the linear Boltzmann equation. J. Mathematical Phys. **1** (1960), 378–390.

Let

$$B_{rs}(\lambda) \equiv c(\lambda) \sum_{m=0}^{\infty} \lambda^m b_{rs}^{(m)}$$

be a negative semi-definite Hermitian matrix depending on a small parameter λ. This paper gives a method of

modifying the $(m+1)$th partial sum of this series (by "completing the square") to give an approximation to $B_{rs}(\lambda)$ which is correct to order λ^m and is itself negative semi-definite.

The method is applied to the linearized Boltzmann equation

$$\partial P(V,t)/\partial t = \int B(V,V',\lambda)P(V',t)\,dV',$$

using a Hermite-polynomial representation in which the kernel is represented by a symmetric real matrix of the form $B_{rs}(\lambda)$. For the particular case of Brownian motion of heavy particles of mass M in a gas of light particles of mass $M\lambda/(1-\lambda)$, the function $c(\lambda)$ and the matrices $b_{rs}^{(m)}$ are worked out using a technique involving "creation" and "destruction" differential operators. The lowest-order approximation to $B_{rs}(\lambda)$ is equivalent to the Fokker-Planck equation. The next approximation, a fourth-order differential equation, is also obtained.

O. Penrose (London)

8741:

Johnston, T. Wyatt. Cartesian tensor scalar product and spherical harmonic expansions in Boltzmann's equation. Phys. Rev. (2) **120** (1960), 1103–1111.

8742:

Nettleton, R. E. Density fluctuations and heat conduction in a pure liquid. Phys. Fluids **4** (1961), 74–84.

The author of this paper takes the unusual view that the mechanism of thermal conductivity in liquids combines the processes effective in gases and crystals. As what the author calls the "self-diffusion" component, characteristic of gases, is small in liquids, this means that thermal conductivity is associated mainly with elastic waves of high frequency (phonons). Without discussing the effect of this hypothesis at the molecular level, Nettleton develops a macroscopic theory which differs from his earlier work in taking account of terms in the thermal flux proportional to the density gradient. The result is a set of three inequalities derived from the consideration that the entropy generated in thermal conduction must be positive.

H. S. Green (Adelaide)

8743:

Fišer, I. Z; Kopeliovič, B. L. On a refinement of the superposition approximation in the theory of fluids. Dokl. Akad. Nauk SSSR **133** (1960), 81–83 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 761–763.

Techniques of statistical mechanics are difficult to apply to dense gases and fluids owing to the large number of degrees of freedom available to the particles. One approach is through the study of correlations among the motions of successively larger groups of particles. This leads by formal analysis to an infinite system of integro-differential equations for the correlation functions. Various schemes have been proposed for solving these equations by systematic approximation methods. The authors discuss a variant of the method used by J. G. Kirkwood, V. A. Lewinson, and B. J. Alder [J. Chem. Phys. **20** (1952), 929–938]. Their results are not described in detail, but it is indicated that improved results are obtained for the virial coefficients for dense gases, up to the fourth order. It is proposed to apply the theory to fluids by use of numerical computers.

E. L. Hill (Minneapolis, Minn.)

8744:

Stillinger, Frank H., Jr.; Kirkwood, John G. Theory of the diffuse double layer. J. Chem. Phys. **33** (1960), 1282–1290.

Authors' summary: "The charge moment expansion for potentials of mean force acting between the ions of an electrolyte is reviewed in a form applicable to surface phases. An integral equation is in this manner derived for approximate determination of the average charge distribution near a planar electrode. The solution of the linearized equation is constructed for an electrolyte consisting of charged hard spheres suspended in a dielectric continuum. For very dilute solutions, the predictions of the linearized Poisson-Boltzmann equation are verified; at higher concentrations, the average space charge in the neighborhood of the electrode tends to alternate in sign as a result of local latticelike ion arrangement imposed effectively by short range ion repulsions. Predicted values of the ζ potential relative to those of the linear Poisson-Boltzmann theory are reported." *H. L. Frisch* (Murray Hill, N.J.)

8745:

Gerjuoy, E.; Stuart, G. W. Ionization growth in a gas with a constant electric field. Phys. Fluids **3** (1960), 1008–1015.

Using Laplace transform techniques the authors calculate under certain simplifying assumptions the time rate of growth of ionization and the energy distribution for long times.

A. A. Blank (New York)

8746:

Glassgold, A. E.; Kaufman, A. N.; Watson, K. M. Statistical mechanics for the nonideal Bose gas. Phys. Rev. (2) **120** (1960), 660–674.

Bogolyubov's approximation for the ground state of a system of weakly interacting bosons is extended to very low but finite temperatures. Statistical mechanics for a moving system with constant velocity is discussed, and, especially, the two-fluid model is discussed in the framework of Bogolyubov's approximation along the line developed by Landau. Further, transport equations are derived from first principles for the system in quasi-equilibrium in which super and normal components are in their respective local equilibrium states. Under Bogolyubov's approximation, these transport equations are shown to coincide with those in the two-fluid hydrodynamics. This work is based on the microscopic point of view and may be considered as a justification of the phenomenological two-fluid model. The use of the Bogolyubov approximation restricts the application of the present work to systems with weak coupling and low density and at low temperature.

R. Kubo (Tokyo)

8747:

Teramoto, Ei. Lattice vibration and random walk problems. Progr. Theoret. Phys. **24** (1960), 1296–1306.

"Mathematical relations between the lattice vibration and the random walk problems are presented for the case of a system of one-dimensional harmonically coupled oscillators. Namely, it is shown that the Laplace and Mellin transforms of the frequency spectrum of the lattice vibration can be expressed in terms of the transition

probabilities of the random walk problems with the continuous and discrete time parameters respectively." (Author's abstract) *S. Sherman* (Detroit, Mich.)

8748:

Ehrman, J. R.; Fosdick, L. D.; Handscomb, D. C. Computation of order parameters in an Ising lattice by the Monte Carlo method. J. Mathematical Phys. **1** (1960), 547–558.

The long-range and short-range order parameters are computed for the Ising lattice (square, simple cubic, and body-centered cubic) using a Monte Carlo sampling scheme. The effect of size of lattice is illustrated and the results are compared with exact and approximate solutions where available. Apparently the least dependable result is the estimate of the Curie temperature and the long-range order near the Curie temperature.

S. Sherman (Detroit, Mich.)

8749:

Sykes, M. F. Some counting theorems in the theory of the Ising model and the excluded volume problem. J. Mathematical Phys. **2** (1961), 52–62.

"The problem of the exact enumeration of self-avoiding random walks on a lattice is studied and a theorem derived that enables the number of such walks to be calculated recursively from the number of a restricted class of closed graphs more easily enumerated than the walks themselves. The method of Oguchi for deriving a high-temperature expansion for the zero-field susceptibility of the Ising model is developed and a corresponding theorem enabling the successive coefficients to be calculated recursively from a restricted class of closed graphs deduced. The theorem relates the susceptibility to the configurational energy and enables the behavior of the antiferromagnetic susceptibility at the transition point to be inferred." (Author's abstract)

S. Sherman (Detroit, Mich.)

8750:

Domb, C.; Sykes, M. F. Use of series expansions for the Ising model susceptibility and excluded volume problem. J. Mathematical Phys. **2** (1961), 63–67.

"The present paper discusses the problem of making the most effective use of the coefficients of series expansions for the Ising model and excluded volume problem in estimating critical behavior. It is shown that after initial irregularities the coefficients appear to settle down to a smooth asymptotic behavior. Alternative methods of analysis are considered for the provision of a steady series of approximations to the critical point. Numerical conclusions are drawn for particular lattices for which additional terms have recently become available." (Authors' abstract) *S. Sherman* (Detroit, Mich.)

8751:

Syozi, Itiro; Naya, Shigeo. Symmetrical properties of two-dimensional Ising lattices. Progr. Theoret. Phys. **24** (1960), 829–839.

The consistency of expressions suggested for spontaneous magnetization and susceptibility is discussed for the cases of square, triangular, honeycomb, and generalized square lattices. *S. Sherman* (Detroit, Mich.)

8752:

Whitfield, G. D. Theory of electron-phonon interactions. Phys. Rev. (2) **121** (1961), 720–734.

The theory of electron-phonon interaction is developed in terms of modified Bloch functions, which allow the major part of the interaction to be calculated in terms of the strain tensor. The theory leads to a generalization of the deformation potential theorem. *S. Simons* (London)

8753:

von Roos, Oldwig. Plasma theory of electron-phonon interaction in metals. Phys. Rev. (2) **120** (1960), 1641–1647.

The transport theory recently developed by the author for a "free" electron gas, taking coulomb forces into account, is now extended to include the interaction with the positive ions. The ions induce a polarization of the electron cloud, which in turn affects the ionic motion. The effect of this is calculated for non-zero temperatures.

S. Simons (London)

8754:

Hone, Daniel. Exchange and correlation corrections to the electron-phonon interaction. Phys. Rev. (2) **120** (1960), 1600–1606.

Exchange and correlation corrections in electron-phonon interactions are taken into account by an extension of the method of Bandeen and Pines. Since the interaction matrix element depends on the initial electron wave vector, an average is taken over this wave vector. An alternative approach employing deformation potential techniques is also given. The results are compared with those previously obtained by Bailyn [see papers reviewed below].

S. Simons (London)

8755:

Bailyn, M. Transport in metals. I. Effect of the non-equilibrium phonons. Phys. Rev. (2) **112** (1958), 1587–1598.

Author's summary: "The coupled Boltzmann equations for phonons and electrons in a metal are solved simultaneously using first-order perturbation theory. Use is made of the Kohler variational principle to put the transport coefficients in terms of determinants. The final results are in the form of integrals involving the phonon-phonon relaxation time $\tau(\sigma)$ and the electron-electron relaxation time. The details of how a finite electrical conductivity exists by means of umklapp processes even when $\tau(\sigma)\to\infty$ (the Peierls equilibrium problem) is revealed in the electrical conductivity expression. In general, the effects may be described as 'phonon relaxation' effects, altering all the transport coefficients somewhat when $\tau(\sigma)$ gets large, and 'phonon-drag' or drift effects, altering appreciably only the thermo-electric power."

E. H. Sondheimer (London)

8756:

Bailyn, M. Transport in metals. II. Effect of the phonon spectrum and umklapp processes at high and low temperatures. Phys. Rev. (2) **120** (1960), 381–404.

An attempt is made to treat quantitatively the effect of the phonon spectrum on both the normal and umklapp contributions to the nonmagnetic transport coefficients of the alkali metals. The phonon equations of motion are

solved numerically, and spectrum averages are evaluated in terms of elastic constants. The evaluation of the shielding and ion parts of the electronic matrix element is discussed. Umklapp processes are found to be dominant at all temperatures in the ideal components of the electrical and thermal resistivities. It is suggested that this may account for the observed T^6 law in the electrical resistivity of sodium at low temperatures, but the computed magnitudes of the low-temperature resistivities are much too large. The dominant contributions come from the transverse phonons, particularly at low temperatures. The variational principle is used to calculate general expressions, valid for any Fermi surface shape, for the transport coefficients, including the phonon-drag term in the thermoelectric power. *E. H. Sondheimer* (London)

8757:

Englman, R. The discontinuities in the solutions of the transport equation and the variational principle. Proc. Phys. Soc. **76** (1960), 909–913.

The author points out that when magnetic fields and boundaries are both present simultaneously, the solution of the Boltzmann transport equation suffers a discontinuity. He considers the effect that this has on the proof of the variational theorem under these conditions.

S. Simons (London)

8758:

Helfand, Eugene. Transport coefficients from dissipation in a canonical ensemble. Phys. Rev. (2) **119** (1960), 1–9.

The author derives expressions for the transport coefficients such as diffusion constant, shear viscosity, bulk viscosity and thermal conductivity. He writes down first the formulae for second moments of appropriate physical quantities associated with each particular process using the phenomenological equations and then the microscopic expressions for the same quantities. By comparing these, the transport coefficients are obtained as integrals of correlation functions of currents of certain kinds. The results are the same as those derived previously by other authors. *R. Kubo* (Tokyo)

8759:

Kirkwood, John G.; Fitts, Donald D. Statistical mechanics of transport processes. XIV. Linear relations in multicomponent systems. J. Chem. Phys. **33** (1960), 1317–1324.

8760:

★Proceedings of Symposia in Applied Mathematics. Vol. XI: Nuclear reactor theory. American Mathematical Society, Providence, R.I., 1961. v+339 pp. $8.70.

The 19 papers in this volume will be reviewed separately.

8761:

Katz, Stanley. A stochastic model for emulsion polymerization. J. Soc. Indust. Appl. Math. **8** (1960), 368–375.

8762:

Montroll, Elliott W.; Ward, John C. Quantum statistics of interacting particles; general theory and some remarks on properties of an electron gas. Phys. Fluids **1** (1958), 55–72.

The quantum analog of the classical Mayer cluster expansion is discussed to treat correctly the quantum statistics, e.g., of interacting particles with short-range repulsive forces. Both the partition function $Z=\text{trace}\exp(-\beta H)$ and the grand partition function

$$Z_G = \text{trace}\sum z^N \exp(-\beta H_N)$$

are obtained from perturbative solutions of the Bloch equation $\partial\psi/\partial\beta=-H\psi$, which the authors study by means of standard propagator techniques and Feynman diagrams in (β, r) space. To study bosons or fermions, the necessary symmetrization or antisymmetrization must be made. Expansion of the requisite permanent or determinant, as in the Fredholm theory, leads to a series of closed iterated kernels that replace the single kernel suitable for Boltzmann statistics. The nth iterated kernel is referred to as an n-toron and is represented diagrammatically by a closed 1-path on a 2-torus that winds around n times in the β direction and once in the r direction. Interactions between particles are represented by the inter-toron emission and absorption of quanta. Toron self-energy effects involve intra-toron emission and absorption. Rules are presented that enable the contribution from an arbitrary diagram to be found. As an application the contribution of ring integrals to the grand partition function is determined for bosons and fermions. For the interacting electron gas the Debye-Hückel theory follows in the classical limit ($\hbar\to 0$), while at low temperatures ($\beta\to\infty$) the authors obtain the Gell-Mann–Brueckner equation for the correlation energy.

J. R. Klauder (Murray Hill, N.J.)

8763:

Montroll, Elliott W.; Ward, John C. Quantum statistics of interacting particles. II. Cluster integral development of transport coefficients. Physica **25** (1959), 423–443.

The analysis of the preceding paper [#8762] is extended to transport properties of systems of interacting particles. The transport theory of R. Kubo [J. Phys. Soc. Japan **12** (1957), 570–586; MR **20** #4940a] is presented in a propagator formalism, the perturbation series of which, due to the non-equilibrium processes under study, is representable by diagrams in the space $(r, \beta+it\hbar^{-1})$. The general analysis again involves various torons, but only Boltzmann statistics are discussed in this paper for which simplified diagrams apply. A two-component system is discussed in some detail and in some generality. As an application, the Drude conductivity formula for two species of noninteracting particles is derived. The paper concludes with a derivation of the Boltzmann equation in the Born approximation applicable in the classical limit ($\hbar\to 0$). The extension to the full scattering cross section by standard diagrammatic iteration techniques is discussed for the Coulomb interaction.

J. R. Klauder (Murray Hill, N.J.)

8764:

Peretti, Jean. Fonction de partition d'un gaz de Bose-Einstein imparfait (problème de l'hélium liquide). C. R. Acad. Sci. Paris **247** (1958), 2099–2101.

The author sketches a calculation of the grand partition function and of the corresponding thermodynamical potential q of an imperfect Bose-Einstein gas of hard spheres, up to first order in the ratio a/λ (where a is the radius of the spheres and λ is the thermal de Broglie wavelength). The method used is essentially based upon Montroll and Ward's theory [#8762 above].

The author does not mention the work of K. Huang, C. N. Yang, and J. M. Luttinger [Phys. Rev. (2) **105** (1957), 776–784; MR **18**, 836] who obtained the same result and moreover also the second order term in a/λ.

R. Balescu (Brussels)

8765:

Garcia-Colin, Leopoldo S.; Peretti, Jean. On the pair distribution function of a hard sphere Bose system. J. Mathematical Phys. **1** (1960), 97–106.

The binary collision expansion is used to obtain the pair distribution function for a hard-sphere boson system. Corrections to the ideal Bose gas results due to the hard core are obtained for large values of the distance between the two particles. *D. J. Thouless* (Birmingham)

8766:

Garcia-Colin, Leopoldo S. Pair distribution function of a hard sphere Bose system calculated by the pseudo-potential method. J. Mathematical Phys. **1** (1960), 87–96.

The pair distribution function of a dilute system of interacting bosons is calculated by the pseudo-potential method, using Bogolyubov's quasiparticle transformation. Results are obtained both for the gaseous phase and for the phase with particles condensed in momentum space, for all values of the separation of the two particles. The results for the gaseous phase disagree with those obtained earlier from the binary collision expansion [#8765], and it is shown why the earlier result is more accurate, where it is applicable. *D. J. Thouless* (Birmingham)

8767:

Landau, L. D. The properties of the Green function for particles in statistics. Ž. Èksper. Teoret. Fiz. **34** (1958), 262–264 (Russian); translated as Soviet Physics. JETP **7**, 182–184.

Author's introduction: "In an attempt to apply the methods which have recently been developed in quantum electrodynamics to statistics, one is faced with the problem of the properties that will be exhibited in that case by the fundamental quantities entering into these methods. We shall show here that some very general relations can be derived for the particle Green function."

8768:

Bonch-Bruevich, V. L.; Kogan, Sh. M. On the theory of the temperature Green's functions. Ann. Physics **9** (1960), 125–138.

The temperature Green's functions, unlike the ordinary ones, are defined as averages of the appropriate T products over the Gibbs ensemble rather than over the ground states. In Landau's work [see preceding review] on temperature Green's functions the spectral representations of the Lehman type are given but the relation to excitation spectrum has not been obtained. Here a system of Fermi particles interacting via the electromagnetic field is considered at an equilibrium state characterised by absolute temperature T and reduced chemical potential μ. Defining the temperature Green's function for fermions as

$$G(x, y) = i \operatorname{Trace}[e^{\beta\Omega+\mu N-\beta H} T\{\psi(x)\bar{\psi}(y)\}]$$

and using the equations of motion, the author obtains the following expressions:

$$\{F(x)-e\gamma_\mu \bar{A}_\mu(x)\}G(x, y)+\int M(x, z)G(z, y)\,dz = \delta(x-y),$$

$$\Box\mathfrak{D}_{\mu\nu}(x, y)+\int P_{\mu\lambda}(x, z)\mathfrak{D}_{\lambda\nu}(z, y)\,dz = -\delta_{\mu\nu}\delta(x-y),$$

where $F(x)$ is the Dirac operator and $\bar{A}_\mu$ is the averaged screened potential (averaged over the ensemble) related to the Bose Green's function $\mathfrak{D}_{\mu\nu}(x, y)$. M and P are the temperature dependent mass and polarization operators defined in a way similar to the usual case. These equations are formally identical to the usual Schwinger [Proc. Nat. Acad. Sci. U.S.A. **37** (1951), 452–455, 455–459; MR **13**, 520] equations for the Green's functions and can be extended to the case of two function temperature Green's functions also.

Taking a bilocal structure of the Green's functions as

$$K(x, y) = \sum_n p_n(\Phi_n\, T\{B(x)\, C(y)\}\Phi_n)$$

(x, y are four-vectors) where B and C contain an equal number of elementary Fermi operators, Φ_n's are the eigenfunctions of H of the system, and p_n is given by $p_n = \exp\{\beta\Omega+\mu N_n-\beta E_n\}$; the author shows that the Fourier transform of $K(x, y)$ with respect to (x_0-y_0) can be written as

$$K(\mathbf{x}, \mathbf{y}; p_0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dp_0'\left\{\frac{\rho_1(\mathbf{x}, \mathbf{y}; p_0')}{p_0'-p_0-i\varepsilon} \pm \frac{\rho_2(\mathbf{x}, \mathbf{y}; -p_0')}{-p_0'+p_0-i\varepsilon}\right\}$$

which is the familiar type of Lehman expansion, where ρ_1 and ρ_2 are defined in a proper fashion. In the case of $B=\bar{C}$ and $y=x$,

$$\operatorname{Im} K(\mathbf{x}, \mathbf{x}; p_0) = \tfrac{1}{2}\{\rho_1(\mathbf{x}, \mathbf{x}; p_0) \pm \rho_2(\mathbf{x}, \mathbf{x}; -p_0)\},$$

and the excitation spectrum is determined by the poles of ρ_1 and ρ_2 and their analytic continuations in the lower p_0 plane. In the case of $B(x)=\bar{C}(x)=\psi(x)$, K is the one particle Green's function for the fermions, and

$$G(\mathbf{p}, p_0) = \frac{1}{\pi}\int_{-\infty}^{+\infty} dp_0'\,\frac{\operatorname{Im} G(\mathbf{p}, p_0')}{1-e^{\mu-\beta p_0'}}\left\{\frac{1}{p_0'-p_0-i\varepsilon}+\frac{e^{\mu-\beta p_0'}}{p_0'-p_0+i\varepsilon}\right\}.$$

The boson Green's function can also be arrived at. Relating $G(x, y)$ to the "one-particle density matrix" defined as

$$R_1(\mathbf{x}, s, \mathbf{y}, s'; t) = i \lim_{\substack{x_0\to y_0=t \\ (x_0<y_0)}} G_{ss'}(x, y)$$

(s, s' being spin variables), the author obtains the distribution function in momentum space

$$f(\mathbf{p}, s) = 2(2\pi)^3\int_{-\infty}^{+\infty} dp_0\,\frac{\operatorname{Im} G_{ss}(\mathbf{p}, p_0)}{e^{\beta p_0-\mu}-1}$$

The thermodynamic potentials are also derived in terms of G and of the interaction terms in the Hamiltonian.

If these results are applied to the case of a non-degenerate electron gas in a semi-conductor (using the effective mass method), $G(p)$ and $\mathfrak{D}(k)$ are obtained in a simple form, and the poles of $\mathfrak{D}(k)$ determine the spectrum of collective oscillations, $\omega(\mathbf{k})$, given by

$$\omega^2(k) = \omega_0^2 + (3\kappa T/m)k^2,$$

ω_0 being the Langmuir frequency. The individual excitations are investigated under certain restrictions necessary for the applicability of the theory, and a dispersion law induced by the interaction is given, for small momenta, by

$$W_r = \frac{\mathbf{p}^2}{2m_r} - \frac{1}{2}\frac{\hbar\omega_0^2}{\kappa T},$$

where $m_r = m[1+\frac{1}{3}(\hbar\omega_0/\kappa T)^2]^{-1}$. The damping and the change in the distributions are also analyzed.

K. Sawada (Princeton, N.J.)

8769:

Szépfalusy, P. On the statistical treatment of the Fermion gas. I, II. Acta Phys. Acad. Sci. Hungar. **9** (1958/59), 203–216, 335–342. (Russian summary)

It is usually considered expeditious to use orthogonal one-particle wave functions in the approximate quantum-mechanical methods which are based on the statistical (Thomas-Fermi) model. The author investigates the advantages to be gained by a deliberate choice of non-orthogonal one-particle states. Interactions among the particles are not considered and all particles move in the same potential field. In paper I the motion is in one space dimension, and in paper II the extension to motion in a spherically symmetric 3-dimensional potential field is treated.

The basis of the theory is the choice of a family of normalized one-particle wave functions $(\varphi_1, \varphi_2, \cdots, \varphi_n)$ each of which is an everywhere-positive (nodeless) function of the coordinate of a particle. In paper II this condition applies only to the radial part of the wave function. Furthermore, the particle density functions, φ_l^2, are to be chosen in an optimal way which at first is not clearly defined, but which is characterized by the later development in paper I. {The argument as given is somewhat heuristic in nature, and involves semi-quantitative considerations based on earlier developments of the statistical model by various authors.} A set of differential equations is obtained from which the one-particle eigenvalues and density functions are to be found; since the wave functions are strictly positive by hypothesis, they can be found from the latter functions if desired.

The determination of the density function of the system, and of the total energy from the density function, are discussed in detail.

In paper II the wave functions are first expressed in terms of spherical harmonics, giving the resolution into angular momentum states, the radial wave functions being then treated by the method developed in paper I.

{The method given in these papers may well be, in fact, a considerable improvement over earlier treatments of the statistical model. Unfortunately, the mathematical development itself does not seem to be such as to offer a basis for the formation of quantitative estimates of the degree of approximation to a more rigorous quantum-mechanical theory.} *E. L. Hill* (Minneapolis, Minn.)

8770:

Kopylov, G. I. A method for the calculation of the statistical weights and distributions in theories of multiple production. Ž. Èksper. Teoret. Fiz. **39** (1960), 1091–1098 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 761–765.

Author's summary: "An effective method for obtaining the distributions and correlations in arbitrary models of multiple production has been developed. The formulas for the covariant model are especially simple. It is shown how a study of this model may facilitate the calculations for more complicated models."

8771:

Hill, Terrell L. Exact definition of quasi-thermodynamic point functions in statistical mechanics. J. Chem. Phys. **30** (1959), 1521–1523.

The present note provides two examples where quasi-thermodynamic point functions can be defined exactly in a transition region. In the case of a classical, one-component, homogeneous, monatomic fluid, with pairwise intermolecular potential energy, one can define the chemical potential at any point in the neighbourhood of a molecule at a distance r from it. This potential reduces to the usual expression as $r \to \infty$. One can also do the same at a plane interface between two phases. Other quasi-thermodynamic point functions like the Helmholtz free energy, internal energy and entropy can also be given exact definitions in the transition regions.

F. C. Auluck (Delhi)

8772:

De Dominicis, C. Thermodynamical variables in terms of distribution functions of quasi-particles. Discussion remark by R. Brout. Physica **26** (1960), supplement, S 94–S 102.

A procedure for the calculation of the thermodynamic functions of a system of fermions (or, with appropriate modifications, of bosons) is described. The formulas for the quantities to be calculated are expressed as power series in the interaction function of the particles, and are then expressed in terms of interaction diagrams of the Breuckner-Goldstone type. The discussion is purely methodological. The remark by R. Brout indicates a variant of this procedure.

E. L. Hill (Minneapolis, Minn.)

8773:

Azbel', M. Ya. Quantum oscillations of thermodynamic quantities for an arbitrary Fermi surface. Ž. Èksper. Teoret. Fiz. **39** (1960), 878–887 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 608–614.

Author's summary: "Thermodynamic quantities of an electron gas in a constant magnetic field **H** are calculated in the general case of a non-convex Fermi surface. It is shown that the presence of self-intersecting trajectories leads to quantum oscillations of these quantities as functions of **H**. It is pointed out that the experimentally observed oscillations corresponding to "anomalously weakly filled" bands may be due either to separated small surfaces, and are then described by the Lifshitz-Kosevich theory, or to small bulges or depressions in the main large band, in which case they are described by the present theory."

8774:

Hecht, Charles E. Thermodynamic potentials for systems at negative absolute temperatures. Phys. Rev. (2) **119** (1960), 1443–1444.

8775:

Mayer, J. E. Approach to thermodynamic equilibrium. J. Chem. Phys. **34** (1961), 1207–1223.

Author's summary: "The properties of a macroscopic classical system consisting of some 10^{20} molecules are determined by a probability density function W of the complete Γ space of moments and coordinates of all the molecules. This probability density function is that of the ensemble representing the totality of all experimental systems prepared according to the macroscopic specifications. The entropy is always to be defined as the negative of k times the integral over the distinguishable phase space of $W \ln Wh\Gamma$. However, the total probability density function W, even for a thermodynamically isolated system, does not obey the Liouville equation, $\partial W/\partial t = LW$, since small fluctuations due to its contact with the rest of the universe necessarily 'smoothes' W, by smoothing the direct many-body correlations in its logarithm. This smoothing is the cause of the entropy increase, and in systems near room temperature and above, in which there is heat conduction or chemical species diffusion, the smoothing keeps the true entropy numerically equal to that inferred from the local temperatures, pressures, and compositions. This, however, is by no means necessarily general. The criterion of thermodynamic isolation is not that the complete probability density function W is unaffected by the surroundings, but that reduced probability density functions w_n in the Γ space of $n = 2, 3, \cdots$ molecules evolve in time as if the system were unaffected by the surroundings. This criterion is sufficient to give a mathematically definable method of 'smoothing' the complete probability density function. The smoothing consists of replacing the direct many-body correlations in $\ln W$ by their average n-body values, $n = 2, 3, \cdots$, such that the smaller reduced probability density functions w_n are unaffected."

8776:

Ono, Syû. Variational principles in thermodynamics and statistical mechanics of irreversible processes. Advances in Chemical Physics, Vol. III, pp. 267–321. Interscience, New York-London, 1961.

This is a very extensive and complete review of the variational principles presently known for the description of irreversible processes. These principles cover completely the fields of linear irreversible thermodynamics, of the theory of the Boltzmann and of the Enskog equations (in the kinetic theory of gases). These principles all describe the behavior of systems close to thermal equilibrium.

The author first describes Onsager's principle of least dissipation of energy, which is, historically, the first one discovered. He then discusses various variational principles related to the Boltzmann equation and which provide an elegant way of finding approximate solutions of this equation. Murakami's recent and especially useful extension of such principles to Enskog's equation is described in detail.

All these kinetic principles have a clear physical interpretation in terms of the principle of minimum entropy production due to Prigogine, which is discussed in its general thermodynamical context. This principle amounts to a characterization of the stationary non-equilibrium states under external constraints by the minimum value of the entropy production compatible with the constraints. The author also discusses a generalization, due to Glansdorf and Prigogine, which covers the field of non-linear thermodynamics, but is less useful, because it is generally not integrable.

A statistical discussion of the minimum entropy production follows, but this problem is not yet solved in the most general case.

This review is likely to be very useful in collecting all known results in this field. It is especially worth mentioning that the author states very clearly the differences between various principles which are sometimes confused in the literature. *R. Balescu* (Brussels)

ELASTICITY, PLASTICITY

See also A7914, 8718, 9277.

8777a:

★Проблемы механики сплошной среды. [Problems of continuum mechanics]. In honor of the seventieth birthday of Academician N. I. Muskhelishvili. Izdat. Akad. Nauk SSSR, Moscow, 1961. 578 pp. (1 plate) 2.53 r.

8777b:

★Problems of continuum mechanics. Contributions in honor of the seventieth birthday of Academician N. I. Muskhelishvili, 16th February 1961. English edition. Society for Industrial and Applied Mathematics, Philadelphia, Pa., 1961. xx + 601 pp. (1 plate) $10.50.

Simultaneous publications, with identical contents; all 50 papers will be reviewed individually.

8778:

Truesdell, C.; Toupin, R. The classical field theories. With an appendix on tensor fields by J. L. Ericksen. Handbuch der Physik, Bd. III/1, pp. 226–793; appendix, pp. 794–858. Springer, Berlin, 1960.

This important contribution to mechanics of continua has the character of a treatise rather than that of a handbook article. The authors state their aims as follows: "We present the common foundation of the field viewpoint. We aim to provide the reader with a full panoply of the tools of research, whereby he himself, put into possession not only of the latest discoveries but also of the profound but all too often forgotten achievements of previous generations, may set to work as a theorist.—This treatise is intended for the specialist, not the beginner. Necessarily it presents the foundations of the field theories, not as they appeared in the last century and linger on in textbooks, nor as the experts in some other domains may think they ought to be presented, but as they are cultivated by the specialists of today." The fact that a large part of the treatise is concerned with kinematics of continua is justified as follows: "That over one half of the work is devoted to kinematics, the mathematical description of

motion, is not malapropos. As the need for more and more general field theories has grown, the preliminary light which kinematics unencumbered by physical restrictions can provide, always appreciated by virtuosi of mechanics, has become a necessity. In presenting here as our chapters B and C the first general treatise on the kinematics of continua, we believe that we look toward the future course of the field theories."

The enormous wealth of material is organized in seven chapters and an appendix. The brief introductory chapter A on The Field Viewpoint in Classical Physics (15 pp.) maps the authors' approach to their subject. In chapter B on Motion and Mass (251 pp.) the authors claim to have collected and organized all the researches they have been able to find on the kinematics of continuous media. Part I of this chapter is concerned with a single finite deformation; it contains sections on deformation gradients, strain, rotation, special deformations, small deformations, and oriented bodies (with application to strain in rods and shells). Continuous motions are treated in part II, which has sections on velocity, material systems, stretching and spin, acceleration, special developments concerning vorticity, special motions, and relative motion. Part III deals with the conservation of mass and momentum; its sections are labelled definition of mass, solution of the equation of continuity, momentum.

Chapter C on Singular Surfaces and Waves (40 pp.) is concerned with those properties of surfaces of discontinuity that are common to all media. The five parts of this chapter are headed geometry of singular surfaces, the motion of surfaces, kinematics of singular surfaces, singular surfaces associated with a motion, discontinuous equations of balance.

Chapter D on Stress (77 pp.) begins with a part on the balance of momentum. This is followed by an exposition of the properties of the stress tensor which the authors characterize as follows: "While every textbook on elasticity or plasticity makes some show of presenting the general theory of stress, no reasonably comprehensive exposition has ever been published before." The four parts of this exposition are concerned with the stress principle from which follow Cauchy's laws of motion, the application of these laws to the determination of power consumption and mean values of stress, the general solution of Cauchy's laws by stress functions, and variational principles.

Chapter E on Energy and Entropy (54 pp.) attempts to collect "everything of a general nature concerning the balance of energy in continuous media and the mathematical properties of entropy". The first part presents the differential equations and jump conditions expressing the energy balance. In the second part, the entropy is defined as a static parameter entering a caloric equation of state, which is supposed to regulate the specific internal energy, regardless of deformation and motion. The third part is concerned with definitions of equilibrium and stability.

Chapter F on Charge and Magnetic Flux (40 pp.) begins with an introductory part largely concerned with invariant integral and differential equations that are independent of a metric or connection. This is followed by parts on the conservation of charge and magnetic flux, the Maxwell-Lorentz aether relations, and conservation of energy and momentum.

The final chapter G on Constitutive Equations (45 pp.) does not attempt an exhaustive treatment of this subject. A brief discussion of principles used in formulating constitutive equations is followed by specific examples classified as kinematical, energetic, mechanical, thermo-mechanical, electromagnetic, and electromechanical constitutive equations. Reference is made to Vol. VIII, Part 2 of the handbook, *The non-linear field theories of mechanics*.

There follows a list of works cited, which is supplemented by four special bibliographies on kinematics of special motions, non-relativistic kinematics and mechanics in generalized spaces, principles of mechanics, and relativistic continuum theories (all together 49 pp.). The work concludes with an appendix (by J. L. Ericksen) on Tensor Fields (65 pp.), which the author too modestly describes as "a heterogeneous collection of results on tensor fields". The six parts of this appendix are labelled preliminaries, dimensions and physical components, double tensor fields, integrals of tensor fields, vector fields, and tensors of order two.

This brief description of the contents unavoidably fails to convey an idea of the scholarship and industry that went into the writing of this magnificent treatise. The authors' frequent assertion of the unique qualities of their exposition and their preference for crediting primarily very early or very recent contributions to the field are minor irritants that should not be allowed to interfere with the reader's appreciation of the work. A more serious blemish is the authors' predilection for unnecessary generality, which requires the use of complicated notations thus making the exposition inaccessible to many who could otherwise have profited from it.

W. Prager (Providence, R.I.)

8779:

Green, A. E.; Adkins, J. E. ★**Large elastic deformations, and non-linear continuum mechanics.** Clarendon Press, Oxford, 1960. xiii+348 pp. $8.80.

This book is a natural sequel to Green and Zerna's *Theoretical elasticity* [Clarendon, Oxford, 1954; MR **16**, 306], providing much more information concerning developments in nonlinear elasticity theory. It is reasonably self-contained, but does not reproduce all pertinent background material. The authors have put together an impressive collection of accomplishments in a difficult subject. A brief summary of the contents is given by the chapter titles, which are, respectively, The strain energy function, General solutions, Theory of successive approximations, Approximate solution of two-dimensional problems, Reinforcement in inextensible cords, Thermodynamics of deformation, Stability, Experimental applications and rheological equations of state. The latter treats the formulation of rather general theories of viscoelasticity and hypo-elasticity.

The treatment is sound. The authors take some pains to point out questions of existence and uniqueness, though they do not include complete coverage of existence and uniqueness theory. Some topics such as dynamic elasticity and molecular elasticity theories are largely ignored.

{The authors have supplied the reviewer with the following Corrigendum: Chapter II, §§ 2.8, 2.10, 2.14–2.18. In some parts of these sections, particularly in § 2.8, the wrong impression has been given that the strain energy w can have a completely general form. Restrictions on w are, however, necessary to ensure that τ^{12}, τ^{13} are zero; otherwise the equations of equilibrium are not satisfied. Since e_{12}, e_{13} ($\gamma'_{(12)}$, $\gamma'_{(13)}$ where appropriate) are zero it is

sufficient to exclude the first power of these quantities from w. All the results given are then satisfactory. Alternatively, if we wish w to be unrestricted, we must consider the more general deformation mentioned in the footnote on p. 71, with suitable modifications for the rectilinear cuboid. For further details see the paper mentioned on p. 71 of the book under review.}

J. L. Ericksen (Baltimore, Md.)

8780:

Коренев, Б. Г. [Korenev, B. G.]. ★Некоторые задачи теории упругости и теплопроводности, решаемые в бесселевых функциях. [Some problems in the theory of elasticity and heat conduction solved by Bessel functions]. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1960. 458 pp. 16.90 r.

The object of this book is to demonstrate, by selected examples, the application of Bessel functions to the solution of problems in the fields mentioned in the title. To this end, it provides a useful collection of such solutions, and a worthwhile exercise in the use of Bessel functions for both the advanced student and research worker. To a certain extent the choice of problems reflects the author's own interests, but this should not be regarded as a criticism, since it results in a unified text, whereas selection from a wider field would have resulted merely in a collection of unrelated examples.

The introductory chapter provides an introduction to Bessel functions, and derives many of the results required in subsequent chapters. Thereafter the book is divided into two parts, dealing respectively with applications to the theories of elasticity and heat conduction.

The first part, which is the longer by a factor of five, is concerned with problems involving plates, shells and bars. The problems considered in chapter I deal with free and forced vibrations of circular plates. Cases are considered where the configuration is, or is not, damped or axisymmetric. The instability of a circular plate under a compressive radial pressure is also considered. Chapters II and III deal respectively with the axisymmetric and non-axisymmetric deformation of a circular plate resting on an elastic foundation. The fourth chapter treats shells of revolution, the main applications being to a conical shell under conditions of axisymmetric loading and uneven heating. Problems of torsion, bending and vibration of bars are considered in chapter V, the bars being of variable cross-section in some examples and sometimes resting on elastic foundations. Chapter VI treats briefly the vibration of systems with one degree of freedom and the following chapter applies the method of images to the solution of certain problems in the theories of non-circular plates and membranes. The final chapter of this first part deals with certain problems of plates, rods and punches on elastic foundations.

The second, shorter, part of the book is made up of two chapters dealing respectively with stationary and dynamic thermal problems. The selected problems again involve plates, bars and shells of revolution. There are two appendices concerned with tables of Bessel functions.

F. J. Lockett (Sevenoaks)

8781:

Parkus, Heinz. ★Mechanik der festen Körper. Springer-Verlag, Vienna, 1960. viii + 264 pp. $6.90.

Wie der Verfasser im Vorwort darlegt, enthält das Buch im wesentlichen seine Vorlesungen über technische Mechanik für die Studierenden des Maschinenbaus, der Elektrotechnik und der Technischen Physik. Der Stoff umfaßt die allgemeine Mechanik und die Elastizitätslehre.

Der Verfasser bemüht sich, die grundlegenden, allen Problemen gemeinsamen Fragen und Prinzipien herauszuarbeiten. Alle Sätze werden in voller Allgemeinheit für den beliebig bewegten verformbaren Körper hergeleitet und dann erst auf Sonderfälle spezialisiert.

In 21 Kapiteln werden behandelt: die allgemeinen Grundlagen der Mechanik und Elastizitätstheorie gerader und gekrümmter Stäbe, die Kreisplatte, Rotationsschalen, Sätze über Formänderungsarbeiten, Wärmespannungen, einige Stabilitätsprobleme und Stoßvorgänge. Die Behandelten Probleme sind durch Beispiele und Literaturhinweise ergänzt.

W. Zerna (Hannover)

8782:

Szabó, István. ★Repertorium und Übungsbuch der Technischen Mechanik. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. vii + 273 pp. DM 24.00.

Das Buch enthält eine Sammlung der wichtigsten Sätze und Formeln der Mechanik mit Übungsbeispielen aus den Gebieten der Statik starren Körper, Festigkeitslehre elastischer Tragwerke, Kinematik und Kinetik, Dynamik der Flüssigkeiten und Gase.

Die Lösungen zu den Aufgaben sind in knapper Form angegeben. Bei der Auswahl der Aufgaben kam es dem Verfasser in erster Linie daraufan, die wesentlichen Gedankengänge der Mechanik zu verarbeiten und nicht nur Beispiele aus der Praxis zu geben.

W. Zerna (Hannover)

8783:

★Proceedings of the Fourth Congress on Theoretical and Applied Mechanics, Howrah (Calcutta)—1958, December, 28–31. Indian Society of Theoretical and Applied Mechanics, Kharagpur. xi + 307 pp.

A set of papers under three headings: (1) Elasticity, plasticity and rheology; (2) Fluid mechanics, aerodynamics, hydrodynamics, mechanics of solids, ballistics, vibrations, friction and lubrication; (3) Statistical mechanics, mathematics of physics, statistics and computation.

Those of mathematical interest will be reviewed separately.

8784:

Todhunter, Isaac. ★A history of the theory of elasticity and of the strength of materials, from Galilei to Lord Kelvin. Edited and completed by Karl Pearson. Dover Publications, Inc., New York, 1960. Vol. I: xvi + 936 pp.; Vol. II, Part I: xiii + 762 pp. (4 plates); Vol. II, Part II: iv + 546 pp. $12.50.

This new edition is an unabridged and corrected republication of the work originally published by the Cambridge University Press, volume I in 1886, volume II, parts I and II, in 1893.

8785:

★Proceedings of the Fifth Congress on Theoretical and Applied Mechanics & the Symposium on Non-Linear

Physical Problems, Roorkee-1959. Indian Society of Theoretical and Applied Mechanics, Kharagpur. xi+381 pp. (1 insert)

The mathematical papers in this volume will be reviewed separately.

8786:

Lehmann, Th. Einige Betrachtungen zur Beschreibung von Vorgängen in der klassischen Kontinuumsmechanik. Ing.-Arch. **29** (1960), 316–330.

This is an expository paper on continuum kinematics.

W. Noll (Pittsburgh, Pa.)

8787:

Babuška, Ivo. Lineare Theorie der inneren Reibung. II. Apl. Mat. **5** (1960), 371–380. (Russian. Czech and German summaries)

In an earlier paper [Apl. Mat. **4** (1959), 177–202; MR **21** #4640], the author gave a general definition of an internal friction operator. In this paper he considers a special class of these operators ("realizable holomorphic operators") and for these operators discusses the problem of force oscillations. *J. P. LaSalle* (Baltimore, Md.)

8788:

Truesdell, C. Modern theories of materials. Trans. Soc. Rheology **4** (1960), 9–22.

The author gives an illustrative account of three phases of theoretical work in rheology. The first phase, consisting of work prior to 1945, receives very brief, unflattering treatment. The second period (1945–1955) is accorded a theme: Ideal materials exhibiting nonlinear response are defined explicitly as exact mathematical realizations of simple ideas. More recent work is described as determining the most general constitutive equations consistent with observed phenomena. *J. L. Ericksen* (Baltimore, Md.)

8789:

Bernstein, Barry. Hypo-elasticity and elasticity. Arch. Rational Mech. Anal. **6**, 89–104 (1960).

This paper is concerned with solids whose constitutive law is an isotropic relation between the strain-rate, stress, and rigid-body stress-rate. (These solids were called hypo-elastic by Truesdell, who did not, however, originally restrict the relation to be isotropic.) Necessary and sufficient conditions are formulated for any such material to be merely Cauchy-elastic, and still more particularly, Green-elastic. It is also shown that a hypo-elastic material is just Green-elastic unless there exist closed cycles of strain in which the net work is negative. Finally, it is shown that anisotropic Green-elastic solids exist which are not hypo-elastic in the author's restricted sense.

R. Hill (Nottingham)

8790:

Bramble, James H. Continuation of solutions of the equations of elasticity. Proc. London Math. Soc. (3) **10** (1960), 335–353.

This paper deals with the continuation of solutions to the field equations of three-dimensional and two-dimensional classical elastostatics. Continuations across a spherical surface (circular arc) are established on the assumption that the displacements vanish on this boundary. In two dimensions the case of vanishing surface tractions is also disposed of. The present investigation is a sequel to the author's earlier work on continuation of biharmonic functions across circular arcs [J. Math. Mech. **7** (1958), 905–924; MR **20** #6614] and generalizes analogous continuations across plane boundaries, due to Duffin [J. Rational Mech. Anal. **5** (1956), 939–950; MR **18**, 613].

E. Sternberg (Providence, R.I.)

8791:

Hill, R. Bifurcation and uniqueness in non-linear mechanics of continua. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 155–164. SIAM, Philadelphia, Pa., 1961.

This is a cryptic account of some of the author's researches on uniqueness theory and related questions for a rather wide class of nonlinear continuum theories.

J. L. Ericksen (Baltimore, Md.)

8792:

Gurtin, M. E.; Sternberg, Eli. On the first boundary-value problem of linear elastostatics. Arch. Rational Mech. Anal. **6**, 177–187 (1960).

The boundary-value problem referred to is that in which the vector displacement is prescribed everywhere on the surface of a homogeneous isotropic elastic mass in equilibrium and satisfying the classical Navier field equations. It has been shown (at least implicitly) by several writers that Kirchhoff's uniqueness theorem is unduly restrictive, in that it is unnecessary for the strain-energy density to be always positive; in fact Poisson's ratio may have any value outside the interval $(\frac{1}{2}, 1)$. The first explicit statement and proof is probably that of T. Boggio [Atti Accad. Lincei Rend. Cl. Sci. Fis. Mat. Nat. (5) **16** (1907), 248–255], not referred to by the authors. They detail a new version of the proof for bounded or unbounded regions in terms of vector analytic identities. They then show that the theorem of minimum potential energy also holds for the same extended range of Poisson's ratio; this result was already indicated by R. Hill [J. Mech. Phys. Solids **5** (1957), 229–241; MR **19**, 1103; pp. 233–234].

Finally, an equivalent minimum principle is proved for a related functional, together with a complementary (but only relative) maximum principle. Actually, these results are only very special cases of general theorems for a wide class of solids (not necessarily elastic) more recently proved by R. Hill [#8793 below]. The authors' vector method itself is, however, not suited to generalization.

R. Hill (Nottingham)

8793:

Hill, R. Uniqueness in general boundary-value problems for elastic or inelastic solids. J. Mech. Phys. Solids **9** (1961), 114–130.

The author discusses uniqueness and nonuniqueness theory of solutions of boundary value problems involving prescribed velocities or stress rates for a class of ideal materials for which suitably defined stress and strain rates are linearly related. The coefficients may depend on past deformations. Equations of linear elastic type are used for illustrating the general theory.

J. L. Ericksen (Baltimore, Md.)

8794:

Signorini, A. Questioni di elasticità non linearizzata e semilinearizzata. Rend. Mat. e Appl. (5) **18** (1959), 95–139.

This paper consists of three lectures given by the author to a meeting of the I.N.A.M. in 1959. The first two parts are a resumé of the theory of finite elasticity in the author's own notation and without reference to the work of other authors, except to one paper by G. Ferrarese. The third part is a discussion of the problem of the indentation of an elastic half-space by a rigid body with related minimum energy and uniqueness theorems.

D. R. Bland (Manchester)

8795:

John Fritz. Plane strain problems for a perfectly elastic material of harmonic type. Comm. Pure Appl. Math. **13** (1960), 239–296.

In finite planar strain the strain energy W reduces to a function of two arguments r, s, where

$$r = [(u_x+v_y)^2+(v_x-u_y)^2]^{1/2}, \quad s = u_xv_y-u_yv_x.$$

Here particles in the unstrained state have coordinates (x, y) and in the strained state coordinates (u, v), where $u=u(x, y)$, $v=v(x, y)$. The author considers a special material of "harmonic type" for which

$$W(r, s) = 2\mu[F(r)-s],$$
$$F'(2) = 1, \quad F''(2) = (\lambda+2\mu)/2\mu,$$

and λ, μ are the Lamé constants of the linear theory. The fundamental equations then reduce to four first-order equations for u, v, A, B, where two of the equations are the Cauchy-Riemann equations for A and B, and the other two are inhomogeneous Cauchy-Riemann equations for u, v with right-hand sides involving A and B. Restrictions on the function $F(r)$ are obtained by requiring that the behaviour of solutions of the equilibrium equations conforms with physical intuition. Further insight into this is gained by considering the propagation of elastic waves in a medium previously strained by a pure homogeneous deformation.

Two planar strain problems for harmonic media are considered in detail. Problem I consists of finding equilibrium positions for prescribed boundary displacements. Necessary and sufficient conditions are given for the existence of at most one proper solution of the equilibrium equations. In problem II the existence of a proper equilibrium solution under certain specified boundary conditions involving stresses and displacements is proved, although the solution is not necessarily unique. Both problems are solved by reduction to a variational problem.

This paper is a valuable contribution to the theory of large elastic deformations.

A. E. Green (Newcastle-upon-Tyne)

8796:

Seth, B. R. Finite strain in engineering design. Les mathématiques de l'ingénieur, pp. 386–390. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

Some results of the theory of finite deformations are given, which can easily be interpreted physically.

W. Urbanowski (Warsaw)

8797:

Barenblatt, G. I. On finiteness conditions in the mechanics of continuous media. Static problems of the theory of elasticity. Prikl. Mat. Meh. **24** (1960), 316–322 (Russian); translated as J. Appl. Math. Mech. **24**, 453–461.

8798:

Tan, Li-min'. Stresses in an elastic plane with adjacent circular holes. Sci. Record (N.S.) **3** (1959), 449–461. (Russian)

8799:

Mohan Gaind, Madan. A mixed boundary value problem for a thin elastic plate with a triangular hole. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 73–84. Indian Soc. Theoret. Appl. Mech., Kharagpur.

From the author's summary: "A mixed boundary value problem is considered for an infinite region with a hole the boundary of which is given by the parametric equations $x=R(\cos\theta+m\cos 2\theta)$, $y=R(\sin\theta-m\sin 2\theta)$, $R>0$, $0<m<\frac{1}{2}$. This curve for $m=\frac{1}{4}$ is of curvilinear shape resembling a triangle . . .

"By conformal representation the region occupied by the elastic body is transformed into the exterior of a unit circle . . .

"The problem is then reduced to the solution of the well-known non-homogeneous Hilbert problem.

"Numerical results are given when a portion L' of the boundary is subject to displacements $u+iv=lz$ where $z=x+iy$ and l is a real constant; and the remaining portion L'' is stress free."

W. Urbanowski (Warsaw)

8800:

Ugodčikov, A. G. A case of analogy on investigating stress concentration around holes. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 429–434. (Ukrainian. Russian and English summaries)

8801:

Bhowmick, Smriti Kana. Note on the effect of a centre of dilatation in a semi-infinite elastic solid with its plane face fixed. Z. Angew. Math. Phys. **11** (1960), 315–318. (German summary)

The title problem is solved by a semi-inverse method and the results are obtained in a closed form.

S. C. Das (Madras)

8802:

Choudhury, P. On the effect of indentation of sandy soil by a flat-ended cylinder. J. Sci. Engrg. Res. **2** (1958), 233–238.

The indentation of sandy soil by a flat-ended circular cylinder is considered on the assumption that for such a soil Young's modulus E, Poisson's ratio σ and the shear modulus μ satisfy the inequality $E>2\mu(1+\sigma)$. The author uses Hankel transforms to show the problem is governed by a pair of dual integral equations which can be solved exactly. Various quantities of interest are calculated.

W. D. Collins (Newcastle-upon-Tyne)

8803:

Aleksandrov, V. M.; Vorovič, I. I. The action of a die on an elastic layer of finite thickness. Prikl. Mat. Meh. **24** (1960), 323–333 (Russian); translated as J. Appl. Math. Mech. **24**, 462–476.

The authors consider the action of a rigid die on an elastic layer of finite thickness h resting on a rigid frictionless foundation. By means of Fourier transforms an integral equation for the pressure between the die and the layer is set up and, provided the pressure under the die is known when the die acts on an elastic half-space, this integral equation can be solved approximately by iteration to give the pressure as a series in powers of h^{-1}. The method can be used even when the cross-section of the die is non-circular. Specific results are given for a flat-ended die of elliptic cross-section.

W. D. Collins (Newcastle-upon-Tyne)

8804:

Gubenko, V. S.; Mossakovskiĭ, V. I. Pressure of an axially symmetric circular die on an elastic half-space. Prikl. Mat. Meh. **24** (1960), 334–340 (Russian); translated as J. Appl. Math. Mech. **24**, 477–486.

The authors consider the action of a rigid die of annular cross-section upon an elastic half-space. This problem is essentially that of determining the electrostatic potential due to a thin circular annulus maintained at a given potential, and the method of this paper gives a new approach to this problem. It is shown that, by means of integral transforms previously used by Mossakovskiĭ, the problem can be reduced to the solution of a Fredholm integral equation. This equation can be solved approximately, this being done for a die of plane base.

W. D. Collins (Newcastle-upon-Tyne)

8805:

Deresiewicz, H. The half-space under pressure distributed over an elliptical portion of its plane boundary. J. Appl. Mech. **27** (1960), 111–119.

The stresses and displacement in an elastic half-space arising from the application of a uniform pressure over an elliptic area of its surface are found using Boussinesq's potentials. Numerical results are given for the normal displacement on the surface of the half-space and for stress components on the normal to this surface through the centroid of the elliptic area.

W. D. Collins (Newcastle-upon-Tyne)

8806:

Eringen, A. C.; Dunkin, J. W. The elastic half plane subjected to surface tractions with random magnitude or separation. J. Appl. Mech. **27** (1960), 701–709.

The analysis is confined to first and second moment properties of the stress tensor, due to the following surface tractions: (a) a white-noise Gaussian load distribution; (b) concentrated loads of random magnitudes at equal intervals; (c) a concentrated load acting at a random location; (d) concentrated loads of equal magnitudes at random intervals. Numerical examples are given for each case.

W. Freiberger (Providence, R.I.)

8807:

Grioli, Giuseppe. Sullo stato tensionale dei continui in equilibrio e sulle deformazioni nel caso elastico. Confer. Sem. Mat. Univ. Bari **35–36**, 19 pp. (1958).

In suo precendente lavoro [Ann. Mat. Pura Appl. (4) **33** (1952), 239–246; MR **14**, 514] l'autore a messo in evidenza che dalle equazioni di Cauchy dell'equilibrio di un qualunque sistema continuo seguono le relazioni integrali che legano le caratteristiche di tensione alle forze esterne. Su tali relazioni l'autore fonda in questo lavoro un procedimento d'integrazione che permette, nel caso elastico, di determinare gli sforzi direttamente, anzichè per derivazione dagli spostamenti con la meglia rapidità di convergenza numerica. Questo procedimento d'integrazione e adatto al integrazione del problema fondamentale dell'elastostatica isoterma utilisando i polinomi P_i ($i=0, 1, \cdots$) ortogonali in la regione C occupata da un sistema continuo tridimensionale nella sua configurazione di equilibrio sotto l'azione di un sistema di forze di massa e forze superficiali. La regione C può indentificarsi comme la configurazione di equilibrio assunta da un corpo elastico poco deformabile nelle vicinanze di uno stato di equilibrio naturale. Ogni sestupla polinomiale $X_r^{(m)}$ ($r=1, \cdots, 6$) del uno tipo determinato di classe M verifica le condizioni di integrabilità di de Saint-Venant allora e soltanto allora che gli valori medii $(\overline{X_r^{(m)}} P_t)$ minimizzano l'energia potenziale elastica totale (V_m) dove $X_r^{(m)}$ è la caratteristica di tensione, che costituisce un'approssimazione polinomiale di ordine m della soluzione del problema.

Il procedimento d'integrazione esposto in questo lavoro è adatto allo studio del problema della flessione di una piastra isotropa quadrata uniformemente caricata ed ovunque appoggiata, approssimando con polinomi di quarto grado i momenti flettenti.

D. P. Rašković (Belgrade)

8808:

Schiel, Friedrich. ★Statik der Pfahlwerke. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. viii + 148 pp. DM 28.50.

Die Statik der Pfahlwerke wird auf der Grundlage der Elastizitätstheorie behandelt. Es finden sich aber darüberhinaus auch einige Bemerkungen über nichtelastische Pfahlwerksberechnungen.

Es werden die verschiedenartigen Pfahlwerkssysteme eingehend untersucht, das Knickverhalten von Pfahlwerken betrachtet und der Entwurf von Pfahlwerken besprochen. Die allgemeinen theoretischen Ableitungen werden durch Beispiele ergänzt. Die Ergebnisse werden mit Hilfe der Matrizenschreibweise in übersichtlicher Form dargestellt. Jedoch ist das Buch auch verständlich für den Leser, der die Matrizenrechnung nicht beherrscht, da alle wesentlichen Ergebnisse auf elementare Weise abgeleitet werden.

W. Zerna (Hannover)

8809:

Stampf, Walter. ★Der durchlaufende Bogenträger auf elastischen Stützen mit und ohne Versteifungsträger. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. viii + 196 pp. DM 37.50.

Für die Anwendung im Brückenbau werden die durchlaufenden Bogenträger auf elastische Stützen mit und ohne Versteifungsträger ausführlich behandelt, wobei vorhandene Pendelsäulen und Pendelwände als gelenkig angeschlossen betrachtet werden. Für die Anwendung im Hochbau werden ein- und mehrschiffige Hallenbögen

ohne Zugband, sowie durchlaufende Rahmen auf elastischen Säulen besprochen. Für die erörterten Tragwerke wird ein Näherungsverfahren erläutert, das für Zwecke der Projektbearbeitung ein Abschätzen der Abmessungen und Beanspruchungen erlaubt.

Als Grundlage für die statische Berechnung wird die Deformationsmethode verwendet. Es werden Verschiebungsgleichungen aufgestellt, in der die Deformationen dreier Knotenpunkte eingehen. Die zweckmäßige Lösung dieser Gleichungen wird vorgeführt.

W. Zerna (Hannover)

8810:

Leipholz, H. Knickung verwundener Stäbe unter Druck einer konservativen, kontinuierlich und gleichmässig verteilten Belastung. Ing.-Arch. **29** (1960), 262–279.

Bending of twisted bars is considered under uniformly distributed loads with various conditions at the end points. The problem is reduced to solving two linear differential equations with variable coefficients. The boundary conditions can be satisfied only for certain values of the parameters of the differential equations. Thus it is an eigenvalue problem and it is pointed out that for a given load the limiting length of the rod can be calculated and vice versa.

Numerical calculation is done for a particular case.

S. C. Das (Madras)

8811:

Brandt, Andrzej. The design of prestressed beams by equalizing the extremal stresses. Rozprawy Inż. **8** (1960), 231–252. (Polish. Russian and English summaries)

The paper discusses the determination of shape of prestressed concrete isostatic beams, in two states of loading.

In the case of the first load, the beam supports the prestressing force and its own weight. In the second case, the beam carries also a prescribed useful load.

The shape of the beam is determined from the condition of equal extreme values of the normal stresses along the span of the beam. This makes four conditions which are often not all needed for the determination of the beam shape. This depends to a considerable degree on the ratio of its own weight to the useful load. It is frequently sufficient to use only one or two of them. It is also shown that for beams of considerable span, it is reasonable to determine the shape from the unloaded state, since then its own weight plays the principal role.

The author solves several problems, in which the four functions sought are the following: (1) the beam height $a(x)$ and ordinate of the prestressing force; (2) the beam height $a(x)$ with constant position of the prestressing force; (3) the variable width of the beam $b(x)$ and the ordinate of the force constituting a given function; (4) the beam width $b(x)$ and the ordinate of the prestressing force $y(x)$. This last case gives as a result a constant width and a parabolic formula for $y(x)$. This case is practically useless, but it illustrates the convergence of both methods proposed by the author. One of them consists of equalling the normal stresses, on both upper and lower surfaces of the beam, and the second is based on the consideration of minimum potential energy of elastic strain for a given volume of the material. This agreement is often less distinct because of additional limitations on the shape, imposed by practical considerations. The author does not assert that the method of equalling extreme stresses is the only one possible for the determination of the prestressed beam shape.

A description is given only of the application of this method and its relation to the question of strength verification.

J. S. Naleszkiewicz (Warsaw)

8812:

Serebrennikova, I. I. On the bending of hollow rods. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 311–318. (Ukrainian. Russian and English summaries)

Author's summary: "The author considers the problem of the bending of an isotropic hollow cantilever, a force P being applied to its free base.

"The method used for the investigation was that of a complex variable and conformal transformation under the assumption that the function, which effects the conformal transformation of the circular ring on region S of the cantilever cross-section, is known.

"The solution of the problem of the bending of a round rod weakened by a triangular prismatic cavity is considered as an example."

8813:

Franeck, H.-J. Die Inversion der Torsionsspannungen prismatischer Stäbe in allgemeinen Koordinaten. Ing.-Arch. **29** (1960), 428–435.

Starting from the basic equations of the theory of elasticity in generalised coordinates, the corresponding basic equations of torsion theory are derived in such coordinates. The partial differential equations for the stress potential and its conjugate function are determined and the boundary conditions given. The resultant force and moment on a cross section are found in terms of these functions and their derivatives with respect to curvilinear coordinates. Transformations of coordinates from cartesian to curvilinear coordinates are then used and specified values of the stress potential in these coordinates are written down. The boundary condition for the stress potential then gives the possible boundary curves. This method is often referred to as the inverse method of solution of the torsion problem. Two particular cases, namely, cross sections of the inverse of a circle, and curvilinear squares, are given.

R. M. Morris (Cardiff)

8814:

Meškov, A. I. General solution of the torsion problem for an oblique-angled parallelepiped. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 3, 43–46. (Russian)

8815:

Das, A. K. Note on twisting of an isotropic circular cylinder embedded in a resisting medium acted upon by couple at one end, the other end being fixed. Indian J. Theoret. Phys. **7** (1959), 61–63.

An exact solution of the title problem is obtained in terms of Bessel functions. The presence of a number of typographical oddities does not make for smooth reading.

J. H. Wilkinson (London)

8816:

Abramyan, B. L.; Babloyan, A. A. Torsion of circular rods having longitudinal notches or teeth and a central cavity. Prikl. Mat. Meh. **24** (1960), 341–349 (Russian); translated as J. Appl. Math. Mech. **24**, 487–499.

The paper is concerned with the isotropic torsion problem for (i) a circular prismatic rod with symmetrically-placed grooves and teeth in the shape of an annular sector, and with a central circular hole; and (ii) a circular rod having a central circular hole whose contour consists of symmetrically-placed grooves and teeth. In both cases the usual stress-function approach leads to construction of an exact solution involving infinite systems of linear equations. Numerical calculations are made for a rod possessing (a) six external teeth together with a circular hole, (b) one external notch and a central hole, and (c) a central circular hole with six internal grooves; in each case upper and lower bounds for the stresses and the torsional rigidity are evaluated. *J. H. Wilkinson* (London)

8817:

Bassali, W. A. Some problems in the small deflexions of clamped thin isotropic plates. Z. Angew. Math. Mech. **40** (1960), 493–507. (German and Russian summaries)

Closed expressions are obtained for the complex potentials and small deflexions of clamped curvilinear plates mapped on the unit circle γ by

$$z = c\rho/(1+\lambda_1\rho^n+\lambda_2\rho^{2n}), \quad c > 0,$$

where n is an integer and λ_1, λ_2 are real coefficients such that roots of $z'(\rho)=0$ do not lie inside γ. The plates are either symmetrically loaded over a circular area or acted on by isolated forces or couples at arbitrary or specified points. Plates bounded by inverses of an ellipse with respect to internal or boundary points are discussed. The special cases included are: (a) a clamped circular plate symmetrically loaded over an eccentric circle; (b) an infinite plate clamped in one half of a straight line; and (c) a clamped half-plane with a curvilinear notch (or boss).

In the particular cases, the agreements with previous results obtained by Das, Woinowsky-Krieger and the author (references given in the paper) are shown. *S. C. Das* (Madras)

8818:

Teodorescu, P. P. Sur le problème de la demibande plane élastique. Arch. Mech. Stos. **12** (1960), 313–331. (Polish and Russian summaries)

An infinite elastic strip under symmetric as well as asymmetric loads along two parallel sides is considered. The infinite set of algebraic conditions can be solved by method of successive approximation when the loads are expressed in Fourier's series. *S. C. Das* (Madras)

8819:

Raasch, Jürgen. Die Spannungen in der nur am Rande belasteten, um einen Durchmesser verbogenen Kreisringplatte. Ing.-Arch. **29** (1960), 419–424.

A ring plate subjected to bending moments inside and outside is considered under various boundary conditions according to Kirchoff's plate theory. *S. C. Das* (Madras)

8820:

Wilde, Piotr. Rectangular anisotropic plate with clamped edges. Arch. Mech. Stos. **12** (1960), 241–258. (Polish and Russian summaries)

8821:

Hanuška, A. Die Anwendung der Methode der Störungsrechnung für die Untersuchung des Einflusses der Querdehnungszahl auf den Spannungszustand dünner platten. Z. Angew Math. Mech. **40** (1960), 570–571.

8822:

Bhowmick, Smritikana. Effect of an isolated force acting at a point near a rigid circular inclusion in a plate. Z. Angew. Math. Mech. **40** (1960), 511–512.

8823:

Grigorenko, Ya. M. Antisymmetrical bending of a disk of conical profile allowing for tensile forces. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 88–94. (Ukrainian. Russian and English summaries)

Author's summary: "This paper discusses the antisymmetrical bending of a disk of conical profile in a field of forces acting in a median plane.

"The paper gives an exact solution of the fundamental equation obtained in the form of series. An example of the calculation of a disk under definite boundary conditions is presented in the paper."

8824:

★Calcolo delle piastre munite di nervature. Consiglio Naz. Ricerche Pubbl. Ist. Appl. Calcolo No. 300. Manuali per Applicazioni Tecniche del Calcolo, II. Casa Editrice Perrella, Rome, 1951. 208 pp. L. 1600.

8825:

Kan, S. N.; Škol'nil, P. A. Strength of open prismatic shells. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **5** (1959), 353–370. (Ukrainian. Russian and English summaries)

Authors' summary: "A method of calculating open thin-walled shells used in building practice is described in this paper.

"Calculating formulae and graphs are given for the determination of internal strains in the structure. The effect of some parameters on the work of the system is presented in an illustrative form. A solution of the problem is presented by the energy method using Euler's differential equations of the variational problem. The method of calculation is illustrated by numerical examples."

8826:

Bakievič, N. I. Some boundary value problems for equations of mixed type arising in the study of infinitesimal bending of surfaces of rotation. Uspehi Mat. Nauk **15** (1960), no. 1 (91), 171–176. (Russian)

Es werden in der Arbeit mittels Fourierentwicklung

drei Randwertaufgaben bezüglich der Fundamentalgleichung

$$\frac{\partial}{\partial v}\left\{\frac{\sqrt{(E(v))}}{N(v)}\frac{\partial\varphi}{\partial v}\right\}+\frac{\sqrt{(E(v))}}{L(v)}\frac{\partial^2\varphi}{\partial u^2}+\left\{N(v)\sqrt{(E(v))}+\frac{L(v)}{\sqrt{(E(v))}}\right\}\varphi=0$$

(L und N Koeffizienten der zweiten und E der ersten Fundamentalform) der unendlich kleinen Verbiegung erster Ordnung einer Drehungsfläche gelöst. Die Randbedingungen sind mit dem Abkleben durch Ebenen der Drehungsfläche längs einem oder zwei Parallelkreiss verbunden. Verf. gibt ferner (ohne Beweis) hinreichende und notwendige Bedingungen (für allgemeine Flächen) an, die die Stetigkeit des Rotationfeldes längs einer parabolischen Linie sichern. *L. Tamássy* (Debrecen)

8827:

Mossakovs'kiĭ, V. I.; Kuaša, O. M. Design and calculation of hatches not giving rise to stress concentration in spherical casings. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **5** (1959), 371–378. (Ukrainian. Russian and English summaries)

Authors' summary: "A spherical casing is loaded with an internal pressure and weakened by a circular aperture which is reinforced by a ring and closed by a cover. It is shown that the reinforced ring and cover parameters may be selected in such a way that there is no stress concentration near the casing notch.

"The casing with the apertures reinforced in the given way are equivalent to an unweakened casing irrespective of the number of operations.

"A dependency for determining the sizes of the ring cross-section is obtained. The determination of the cover parameter is reduced to the application of a family of curves plotted for a thin aluminium rolled sheet."

8828:

Arhangorods'kiĭ, O. G.; Bilen'kiĭ, L. M. Certain questions of the similarity of thin-walled profiles. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **5** (1959), 421–427. (Ukrainian. Russian and English summaries)

Authors' summary: "The presented paper gives the results of an investigation of the similarity of two thin-walled profiles (unsymmetrical double-T and channel bar) in respect to the geometrical characteristics of the sections, different forms of stresses and certain forms of loss of stability."

8829:

Seth, B. R. Finite deformation of cylindrical shells. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 355–362. North-Holland, Amsterdam, 1960.

Finite components of strain are used to analyse a circular cylindrical shell under an axially symmetrical load applied normal to the surface. The load varies along a generator in such a way that the final strained state is a truncated cone with spherically deformed end caps. A linear stress-strain relationship is assumed.

L. S. D. Morley (Farnborough)

8830:

Obolašvili, E. I. A boundary problem for a momentless ellipsoidal shell. Soobšč. Akad. Nauk Gruzin. SSR **22** (1959), 393–400. (Russian)

The author considers an ellipsoidal shell in the membrane state. In the assumed parametric system of coordinates the equilibrium equations lead to a system of two non-homogeneous equations of first order. It is proved that by introduction of a complex variable the equilibrium equations are transformed to the non-homogeneous Cauchy-Riemann system.

A detailed examination is carried out for a truncated ellipsoidal shell, assuming that one of the edges is subject to no tangential forces and the second to no normal forces. By the conformal mapping of the truncated shell onto a ring the problem is reduced eventually to the solution of a Fredholm integral equation of second order. A rigorous solution of the latter equation can be obtained only for some particular types of loading, two of which have been examined. *Witold Nowacki* (Warsaw)

8831:

Hieke, Max. Ein Beitrag zur exzentrisch belasteten Kreismembran. Z. Angew. Math. Mech. **39** (1959), 180–192. (English, French and Russian summaries)

The author investigates a circular membrane loaded eccentrically over that part of a circle lying inside the membrane. Two types of loading are examined, namely a sudden loading and a loading harmonic in time. The particular solution of the equation is given in a polar coordinate system, the origin of which is located at the center of the loading circle. The full solution has the form of a double series of trigonometric and Bessel functions. The author indicates that the presented method of solution can be applied to obtain an approximate solution for the case in which the loading is not axisymmetrical with respect to its centre. *Witold Nowacki* (Warsaw)

8832:

Vaĭnberg, D. V.; Zaruc'kiĭ, V. O.; Itenberg, B. Z. Stressed state of cylindrical shells reinforced with ribs. Akad. Nauk Ukraïn. RSR Prikl. Meh. **6** (1960), 375–384. (Ukrainian. Russian and English summaries)

Authors' summary: "This paper deals with the methods of investigating the stressed state of a circular closed cylindrical shell reinforced with a regular network of annular and rectilinear ribs, taking into account the discrete nature of their disposal. The problem of the action on the shell of excess pressure of constant intensity and a longitudinal load is solved proceeding from a system of differential equations of the theory of thin shells, as well as from integro-differential equations derived by applying the theorem of reciprocal work."

8833:

Grigorenko, Ya. M. Antisymmetrical stressed state of a conical shell of variable thickness. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 385–392. (Ukrainian. Russian and English summaries)

Author's summary: "The author considers the antisymmetrical stressed state of a conical shell of variable thickness. A system of basic differential equations of the

fourth order is derived. An approximate equation in complex form is also obtained, and its solution is given in the form of hypergeometric functions."

8834:
Matsunaga, Syōgo. On the stress distribution of the rotating circular ring type shell. Z. Angew. Math. Mech. **40** (1960), 566–568.

8835:
Schnell, W. Zur Stabilität dünnwändiger längsgedrückter Kreiszylinderschalen bei zusätzlichem Innendruck. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 167–188. North-Holland, Amsterdam, 1960.

The author reviews work performed at the Institut für Festigkeit of the D.V.L. on buckling of thin cylinders subjected simultaneously to axial load and internal pressure, including his own and others' linear and nonlinear theories. Experimentally, as the internal pressure rises, the axial buckling load increases markedly and the usual checkerboard ("diamond shaped") wave form changes to a spiral row of waves and finally, if the test is made with great care, to narrow axisymmetrical rings. The theories give a good explanation of the effects of internal pressure and of anisotropy but, like other theories which neglect imperfections, they overestimate the strength. *L. H. Donnell* (Ann Arbor, Mich.)

8836:
Pian, T. H. H. Dynamic response of thin shell structures. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 443–452. Pergamon, Oxford, 1960.

A brief account is given of experimental work concerning the dynamic behaviour of hemispherical shells and multi-web beams in the plastic range. An energy method of approach is suggested for the estimation of the permanent deformation produced in such thin-walled structures under conditions of dynamic applied load. However, the difficulties inherent in such a general procedure that are due to the complex nature of collapse modes and their dependence upon the rate-of-loading appear to be underestimated by the author.
H. G. Hopkins (Sevenoaks)

8837:
Stojanovitch, Rastko. On the stress-strain relations for non-homogeneous isotropic elastic solids. Arch. Mech. Stos. **12** (1960), 281–285. (Polish and Russian summaries)

The author considers the stress-strain relations for a non-homogeneous isotropic elastic solid. Both the finite and infinitesimal strain cases are considered. First, the finite strain tensor is introduced in the usual manner [A. E. Green and W. Zerna, Philos. Mag. (7) **41** (1950), 313–316; MR **11**, 627]. Then, the usual fourth-order Hooke's tensor is defined. The stress tensor is defined as the contracted deformed Hooke's tensor. It is shown that for infinitesimal deformations the usual stress-strain relations of Lamé for a non-homogeneous material are obtained. *N. Coburn* (Ann Arbor, Mich.)

8838:
Knops, R. J. The use of Poisson's ratio in studying certain non-homogeneous elastic inclusions. Z. Angew. Math. Mech. **40** (1960), 541–550. (German and Russian summaries)

8839:
Lee, Ho Chong. Forced lateral vibration of a uniform cantilever beam with internal and external damping. J. Appl. Mech. **27** (1960), 551–556.

The steady state response problem with a sinusoidal shaking force at the base is discussed when the material of the beam is the general linear substance with stress-strain relationship $\sigma+\alpha(d\sigma/dt)=\beta\varepsilon+\gamma(d\varepsilon/dt)$, where $\alpha=c/k$, $\beta=E$ and $\gamma=(E+k)c/k$. The relationship is taken to be valid only for the longitudinal strain of the beam while the shear stress-strain relationship remains as in the case of elastic material. Neglecting the shear and rotatory inertia effects, the solutions for a simpler case are obtained. Agreements with the results derived by a tabular method are shown numerically.
S. C. Das (Madras)

8840:
Shimogo, Taro. On the beam under random loading. Bull. JSME **3** (1960), 60–65.

The mean-square values of deflections and stresses in a structure under random loading are evaluated. The typical power spectrum of stationary isotropic three-dimensional turbulence is used as the forcing function.
W. Freiberger (Providence, R.I.)

8841:
Beer, F. P. On the response of linear systems to time-dependent, multidimensional loadings. J. Appl. Mech. **28** (1961), 50–56.

The methods of communication engineering are, in this interesting paper, applied to the analysis of the response of an arbitrary linear mechanical system subjected to random loadings which are functions of space and time. The aim is to determine the analogue of the transfer function from the response of the system to given excitations. The method is generalized multivariate harmonic analysis, which is briefly explained.
W. Freiberger (Providence, R.I.)

8842:
Piszczek, Kazimierz. Parametric combination resonance (of the second kind) in non-linear systems. Rozprawy Inż. **8** (1960), 211–230. (Polish. Russian and English summaries)

This problem is considered for a model of a homogeneous beam with constant I-cross-section, whose vibration is excited parametrically by a harmonic pulsating concentrated transversal force $P(t)=P_0+P_1\cos\omega t$, acting in the middle of the length and in the plane of maximum flexural rigidity, normally to the longitudinal axis. The geometrical non-linearity is taken in account, by assuming that the ends cannot approach each other. The axial reaction is assumed to be due to both the transverse displacement of the points of the axis and the rotation of the cross-sections. For the exciting force it is assumed that: (1) it does not change its direction; (2) it rotates together

with the cross-section, its relative direction remaining unchanged.

It is found that the parametric resonance appears in the neighbourhood of the shaking frequency equal to the sum of free frequencies in the case where the force preserves its direction in each instant during the vibration. In case of parametric resonance with a pulsating load which rotates so as to be always in the cross-sectional plane in which it acts, the shaking frequency is equal to the difference of the frequencies of free vibrations.

From these considerations it also follows that with the same value of P_1 the width of the resonance region is less in case of rotating force than the width of the resonance region for a load of constant direction.

J. S. Naleszkiewicz (Warsaw)

8843:

Tyutekin, V. V. Flexural oscillations of a circular elastic plate loaded at the center. Akust. Ž. **6** (1960), 388–391 (Russian); translated as Soviet Physics. Acoust. **6** (1961), 389–392.

8844:

Rosenstock, Herbert B. Nature of vibrational modes in ionic crystals. Phys. Rev. (2) **121** (1961), 416–424.

The principal purpose of the paper is to discuss the nature of the vibration in a solid. The following is found. (1) Waves in lattices are in general neither transverse nor longitudinal; in particular, they need not be transverse or longitudinal when the propagation vector $\mathbf{k}$ is very small. (2) The relationship $\omega_l/\omega_t = (\Sigma_0/\Sigma_\infty)^{1/2}$ for "longitudinal" and "transverse" modes in ionic crystals applies, if at all, in a region of a small, but nonzero, wave vector $\mathbf{k}$. (3) The derivation of this relationship is based, at least implicitly, on the use of cyclic boundary conditions. (4) The use of cyclic boundary conditions is valid in statistical problems for crystals without long-range forces, but has never been justified for systems with Coulomb forces. (5) If cyclic boundary conditions are nonetheless used, it can be simply shown that for $\mathbf{k} = 0$, $\omega_l/\omega_t = 1$.

Werner Nowacki (Bern)

8845:

Holt, Maurice; Strack, Sainsbury L. Supersonic panel flutter of a cylindrical shell of finite length. J. Aerospace Sci. **28** (1961), 197–208.

The present investigation of the title problem may be regarded as an extension of that given by R. D. Stepanov [Prikl. Mat. Meh. **21** (1957), 644–647; MR **19**, 1213]. The aerodynamic forces are calculated in static approximation, allowing for the cylindrical geometry but neglecting unsteady flow effects; Stepanov used piston theory. Following Stepanov, the authors use Donnell's equation for axially symmetric motion of the shell and Goldenveizer's equation for non-axially symmetric motions. Actual solutions of the resulting integro-differential equation are based on a straight-line approximation to the aerodynamic kernel, however, which is tantamount to the assumption of a static, piston-theory approximation to the aerodynamic forces in the calculation of the critical flutter speed. This implies that the authors' results should approximate those of Stepanov for small reduced frequencies, but that those of Stepanov should be superior for large reduced frequencies; in fact, the two sets of results are in reasonable agreement for the assumed range of parameters.

The authors suggest that their model should be more realistic than that of the reviewer [same J. Aero. Sci. **24** (1957), 107–118; **26** (1959), 81–93, 107; MR **18**, 839; **20** #5631], which dealt with short traveling waves on an infinite shell and yielded much more critical design conditions. {The reviewer believes that this question can be settled only by observation but remarks that the authors' neglect of unsteady flow effects implies the a priori exclusion of traveling-wave flutter, in consequence of which the results of the two analyses are not directly comparable. See also the reviewer's remarks [ibid. **27** (1960), 476; MR **22** #3340] on Stepanov's paper.}

J. W. Miles (Los Angeles, Calif.)

8846:

Kollár, L. Stability of bent shell-arches. Acta Tech. Acad. Sci. Hungar. **32** (1961), 267–297. (German, French, and Russian summaries)

8847:

Barta, J. Schranken für die Knicklast des einseitig eingespannten dünnen Stabes von veränderlichem Querschnitt. Acta Tech. Acad. Sci. Hungar. **32** (1961), 321–323. (English, French, and Russian summaries)

8848:

Babuška, Ivo. Note on the theory of the rail. Arch. Mech. Stos. **12** (1960), 305–311. (Polish and Russian summaries)

8849:

Haywood, J. H. Post-buckling analysis of a circular elastic ring or a long cylindrical shell. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 122–136. North-Holland, Amsterdam, 1960.

The buckling referred to in the title is supposed due to a uniform radially inward load, so that the problem involves only one independent variable. Linear moment-curvature and membrane force-strain relations are used, and membrane strain is assumed small. The equations of equilibrium lead to a first order equation for the bending moment. A numerical method for finding the shape and stress of the cylinder for given load is described, with examples.

J. W. Craggs (Newcastle-upon-Tyne)

8850:

Chakraborty, Sakti Kanta. On compressional waves in the direction of the axis of a cylindrical hole in an anisotropic medium having hexagonal symmetry. Bull. Calcutta Math. Soc. **51** (1959), 142–144.

Axisymmetric periodic eigenfunction solutions are found for the problem of an elastic anisotropic medium bounded internally by a circular cylindrical hole; mechanical isotropy is assumed in planes perpendicular to the symmetry axis.

The dispersion relation (defining phase velocity as a function of wavelength) is obtained implicitly as the eliminant of two lengthy equations and some consideration is given to the short wavelength limiting case.

S. C. Hunter (Sevenoaks)

8851:

Green, A. E. Torsional vibrations of an initially stressed circular cylinder. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 148–154. SIAM, Philadelphia, Pa., 1961.

The author considers the propagation of small torsional vibrations along a solid circular cylinder of isotropic material which is subjected to a uniform finite extension. Taking thermal stresses into account, it is found that these vibrations are possible under isothermal conditions. The problem is solved for a general form of strain energy function. For a Mooney material the speed of propagation of the waves is increased when the rod is under tension and decreased when under compression.

J. E. Adkins (Providence, R.I.)

8852:

Mitra, Manindra. Propagation of elastic waves in an infinite plate of cylindrically aelotropic material. Z. Angew. Math. Phys. **10** (1959), 579–583. (German summary)

The axisymmetric vibrations of an aelotropic plate of finite thickness ($2a$) are investigated assuming radial (u) and axial (w) displacement functions of the form

$$u \sim J_1(\xi r)e^{i\xi ct+i\gamma z}, \quad w \sim J_0(\xi r)e^{i\xi ct+i\gamma z},$$

Imposition of boundary conditions at the stress-free surfaces ultimately yields two dispersion equations implicitly defining c as a multibranched function of ξ; one equation refers to motions symmetric about the centre of the plate, the second is for antisymmetric modes. Using assigned values of the elastic constants, the author has investigated numerically the two lowest branches for each dispersion curve. The final results are displayed in a non-dimensionless form with $\rho^{1/2}c$ plotted against $a\xi$. Since the author fails to state in what units he is measuring stress, the numerical work would appear to be of limited value.

S. C. Hunter (Sevenoaks)

8853:

Gazis, Denos C.; Herman, Robert; Wallis, Richard F. Surface elastic waves in cubic crystals. Phys. Rev. (2) **119** (1960), 533–544.

Surface waves in cubic solids are discussed from both the standpoints of bulk elasticity theory and of atomic vibrations. In the former case, phase velocities are computed as functions of the elastic constants. It is shown that ordinary Rayleigh waves exist for some values of the elastic constants, while "generalized" Rayleigh waves (waves with a complex attenuation constant) exist for others. Detailed calculations of phase velocity vs. direction of propagation are given for various materials. In the discrete particle theory (simple cubic lattice model), surface waves (as well as bulk ones) are shown to be dispersive.

H. B. Rosenstock (Washington, D.C.)

8854:

Farnell, G. W. Elastic waves in trigonal crystals. Canad. J. Phys. **39** (1961), 65–80.

In non-isotropic single crystals the normals to the wavefronts of elastic waves are not collinear with the vectors representing either the energy flow or the particle displacement. Calculations have been carried out on the propagation characteristics of sound waves in two particular trigonal crystals, α-quartz and sapphire. The development of the eigenvalue equation for the velocity and the formulae for the components of the displacement and energy-flow vectors are summarized. The assumption that the wave has a plane wavefront normal to a given direction leads to three solutions, one representing a quasi-longitudinal wave and the other two representing quasi-transverse waves. The velocities of propagation, directions of displacement, directions of energy flow for the three waves have been calculated for many orientations of the wave normal. Detailed results for propagation near one of the pure-mode axes are presented.

Werner Nowacki (Bern)

8855:

Miles, J. W. On the response of an elastic half-space to a moving blast wave. J. Appl. Mech. **27** (1960), 710–716.

This paper is concerned with the axisymmetric dynamic elasticity problem in which the free surface of a half space $z>0$, is subject to a radially expanding pressure distribution $P(r, t)$ for which $P(r, t)$ is non-zero within a finite radius $r_0(t)$. It proves convenient to work with the function inverse to $r_0(t)$, i.e., with the function $t_0(r)$, the arrival time of the blast wave at radius r. In terms of $t_0(r)$ the blast wave advances at a velocity $U(r)=[t_0'(r)]^{-1}$.

A formal solution is easily obtained using integral transform methods. The time coordinate is subjected to a Fourier transform and the radial coordinate to a Fourier Bessel (or Hankel) transform with kernel $rJ_0(kr)$. Thus the formal solution to the problem may be written in terms of double integrals; the bulk of the paper is devoted to an attempt to evaluate these integrals using approximations based primarily on physical arguments.

By replacing the Bessel function kernels by their asymptotic expressions, assuming the function $t_0(r)$ to be reasonably smooth and imposing somewhat severe restriction on the form of the pressure profile, the author shows (equation 34) that the Fourier transforms of the dilational and shear potentials are proportional to the Fourier transform of the pressure pulse, viz.,

$$\bar{\phi}(r, z, w) = \bar{P}(r, w)\bar{h}(r, z, w),$$

where the functions $\bar{h}$ are defined by definite integrals which are evaluated approximately, using the method of stationary phase. Thus finally the general solution of the problem, within the approximational framework employed, can be written in terms of convolution integrals over the pressure profile and explicitly defined Green's functions.

The author concludes by discussing various examples in which $U(r)$ is less than, of the same order as, or greater than the Rayleigh wave velocity.

{Because of the large number of approximations employed, the area of validity of the final result is not clear to the present reviewer.} *S. C. Hunter* (Sevenoaks)

8856:

Sternberg, Eli.; Chakravorty, J. G. On the propagation of shock waves in a nonhomogeneous elastic medium. J. Appl. Mech. **26** (1959), 528–536.

An infinite plate of arbitrary thickness has a transverse cylindrical hole at which uniform shearing tractions acting parallel to the faces of the plate are suddenly applied and steadily maintained thereafter. Young's modulus and the

shear modulus of the material are functions of the plate radius.

The governing linear differential equation is analytically tractable provided the shear modulus is of the form $\mu=\mu_0\rho^\alpha$, where ρ is a nondimensional radius and α an arbitrary constant. A Laplace transform in connection with a method employed previously by Kromm leads to Volterra integral equations for the unknown quantities. Three cases, $\alpha\leq 2$, $\alpha>2$, have to be distinguished and are discussed in detail. *H. Parkus* (Vienna)

8857:

Datta, S. K. Note on forced torsional vibration of a visco-elastic circular cylinder. Bull. Calcutta Math. Soc. **51** (1959), 167–170.

The forced torsional behaviour is derived for a visco-elastic (Maxwell) cylinder, fixed at one end and subject to an exponentially decaying shearing stress distributed linearly across the radius of the other end. The method of solution, which is particular to the assumed exponential time dependence of the forcing function, entails combining free damped oscillatory eigenfunctions with a particular integral, and thereby satisfying the various boundary conditions. *S. C. Hunter* (Sevenoaks)

8858:

Chakraborty, Sakti Kanta. Rotational waves in a visco-elastic medium due to a twist on the surface of a spherical cavity. J. Sci. Engrg. Res. **2** (1958), 195–200.

The equations of motion of a (Voigt) visco-elastic medium are solved in the form $u_r=u_\theta=0$ and $u_\phi=-\sin\theta f(r)e^{-int}$, where r, θ, ϕ denote the usual spherical polar coordinates. Using Fourier transform methods these results are applied to the problem of an internal cavity surface subjected to an applied shear $\widehat{r\phi}=-\sin\theta P(t)$, where P is an arbitrary function of time. An explicit (approximate) solution is obtained when $P(t)$ assumes the form $P_0e^{-\xi t}$ and some consideration is given to the limiting case of step-function loading ($\xi\to 0$).

S. C. Hunter (Sevenoaks)

8859:

Sharma, S. K. Propagation of sound waves in visco-elastic compressible fluids. J. Sci. Engrg. Res. **2** (1958), 253–258.

The propagation of small amplitude disturbances is investigated for a visco-elastic fluid whose properties are characterised by a bulk viscosity coefficient and a constitutive shear equation of the form $\Sigma+\tau\dot{\Sigma}=2\mu\dot{\varepsilon}$, where, in the reviewer's non-tensor notation, Σ denotes shear stress, ε is shear strain, τ is a relaxation time and μ is the shear viscosity coefficient.

In section 2 the linearised equations are solved for periodic disturbances for the cases of (a) spherical waves, (b) plane waves in which the velocity vector is parallel to the direction of propagation; for this second case the dispersion relation, connecting phase velocity with circular frequency, is obtained in a first non-trivial approximation. Section 3 is concerned with the modifications entailed in the plane wave case if allowance is made for thermo-mechanical effects.

{There are numerous misprints in equations (2.11) and (2.12) which should read (2.11) $s=Ae^{-iK(r-a)}/r$, and (2.12) $\phi=-ice^{-ik(r-a)}/K^2r$.} *S. C. Hunter* (Sevenoaks)

8860:

Cristescu, N. European contributions to dynamic loading and plastic waves. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 385–442. Pergamon, Oxford, 1960.

This paper aims to provide a detailed summary of the literature concerning the investigations that have been made of the dynamic anelastic behaviour of solids: particularly metals, but also polymers and soils. The title of the paper implies that American contributions to the development of the study of the subject are not to be discussed. Perusal of the 212 references listed at the end of the paper shows that this is in fact mainly the case. This division of the subject is not to be endorsed, and it leads, in any event, to anomalies arising, for example, from the extensive collaboration between European and American investigators. The main value of the paper is that a comprehensive summary of the research done by Russian investigators is provided in the English language.

The content of the paper is, of necessity, mainly descriptive, and emphasis is more often placed upon mathematical aspects, rather than upon physical ones, of the investigations considered. Work concerning the complications in mechanical behaviour due to effects associated with temperature, rate-of-strain, and high pressure is included in the paper.

The paper comprises the following sections: Introduction; Longitudinal waves in thin bars; The finite bar; Non-homogeneous media; The influence of the strain-rate; The propagation of the perturbations in strings; Plane plates; and Different problems.

The apparently limited objective of the author in this paper renders further comment on his development and treatment of the present subject inappropriate.

H. G. Hopkins (Sevenoaks)

8861:

Cristescu, N. Some observations on the propagation of plastic waves in plates. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 501–510. Pergamon, Oxford, 1960.

A plate of infinite extent is subjected to combined transverse and rotational impact by a rigid circular cylinder. The boundary conditions are supposed prescribed at the circle of contact in such a way that only shear strains occur. It is assumed that the radial component of displacement in the plate may be neglected, and attention is then given to axisymmetrical, elastic-plastic, stress-wave propagation on the basis of deformation theory. The preliminary analysis involves the use of the characteristics theory of the relevant wave-equations, and expressions are thereby found for the wave velocities. Following detailed discussion of the pattern of stress waves, specialization is made to perfectly-plastic and linear work-hardening materials. Finally, for work-hardening materials, attention is given to approximate methods of solution of the boundary-value problems that arise.

The paper is concerned mainly with the qualitative aspects of the nature of stress-wave propagation under

mathematically prescribed conditions, and little discussion is given of the physical significance of the results obtained.
H. G. Hopkins (Sevenoaks)

8862:

Kaliski, Sylwester. The propagation of a non-linear loading wave in a magnetic field for a perfect conductor. Proc. Vibration Problems No. 5 (1960), 11–26. (Polish and Russian summaries)

"One considers the propagation of one-dimensional elastic-plastic loading waves in the semi-space of a magnetic field. Perfect conduction is considered. The accurate solution is obtained for an elastic-linear work-hardening material (called by the author: bodies with angled $\sigma-\varepsilon$ characteristic) and also the approximate solution for an arbitrary work-hardening law. The values of the electro-magnetic field in vacuum and in the body, the displacements and the strain in the body and also the surface currents are obtained." (From the author's Russian summary)
N. Cristescu (Bucharest)

8863:

Kaliski, Sylwester. The unloading wave in a magnetic field, in a body with rigid unloading characteristic, assuming perfect electric conductivity. Proc. Vibration Problems No. 5 (1960), 27–54. (Polish and Russian summaries)

"The propagation of unloading waves in a magnetic field is considered, assuming perfect electric conductivity. The paper is a continuation of another one [see previous review] for the unloading case. The general solution for a plastic-rigid body is given, when the suddenly applied loading decreases monotonically to zero or when the applied loading increases and then decreases monotonically. The loading is applied at the surface of a semi-space, limited by vacuum." (From the author's Russian summary)

{Reviewer's observation: The figures 2 and 3 from this paper correspond to the previous paper and vice-versa.}
N. Critescu (Bucharest)

8864:

Bodner, S. R.; Symonds, P. S. Plastic deformations in impact and impulsive loading of beams. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 488–500. Pergamon, Oxford, 1960.

A summary is given of experimental work concerning the dynamic plastic deformation of cantilever beam specimens made of mild steel and of aluminum alloy. The tentative conclusion drawn from the discussion of the experimental results is that rate-of-strain effects are the most important single factor responsible for the significant discrepancies between experiment and the simple engineering perfectly-plastic, rigid theory. Thus, the assumption that the yield moment is independent of rate-of-strain is generally invalid. Following earlier work, this dependence is incorporated into a plastic, rigid theory, applicable to the present test conditions, that is based upon an empirical power law relation between strain-rate and over-stress. The predictions of this rate-of-strain dependent theory are found to agree well with experimental results for one of the tests made on mild steel specimens. The maximum experimental values of rates-of-strain are of the order of 1 and 10 sec^{-1} for the mild steel and the aluminum alloy (cf. the value of 10^{-3} sec^{-1} at which static tests are normally made).
H. G. Hopkins (Sevenoaks)

8865:

Lin, T. H. On the associated flow rule of plasticity based on crystal slips. J. Franklin Inst. **270** (1960), 291–300.

The paper begins with some theorems of Bishop and Hill (1951) which culminate in a proof that the yield function and flow potential coincide for an aggregate of rigid/plastic crystals deforming by multislip according to the Schmid law. The open question whether this coincidence holds also when the crystals are elastic/plastic is then examined. The author claims to establish this when the crystals are elastically isotropic but not otherwise.
R. Hill (Nottingham)

8866:

Olszak, Waclaw; Urbanowski, W. Plane problems of the theory of plasticity of non-homogeneous and anisotropic bodies. J. Sci. Engrg. Res. **2** (1958), 201–208. (1 insert)

A general quadratic form in the components of the stress deviator is first assumed as the yield function and flow potential for a rigid/plastic anisotropic solid, and is then specialized for the case of orthotropic symmetry. Although the authors do not say so, the resultant expression (2.5) is then identical with that given by R. Hill, *The mathematical theory of plasticity* [Clarendon, Oxford, 1950; MR **12**, 303; p. 318]. Further specializations to plane stress and plane strain are next made; again the results (3.1) and (6.3) were already given by Hill [op. cit., pp. 321 and 334, respectively]. The corresponding equations satisfied by an Airy-type stress function are obtained when the orthotropy is curvilinear and inhomogeneous. Only one problem is solved; this is the trivial one of a thick cylinder with cylindrical orthotropy under internal pressure.
R. Hill (Nottingham)

8867:

Sokolovsky, V. V. Axial plastic flow between non-circular cylinders. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 215–224. Pergamon, Oxford, 1960.

The paper deals with plastic flow in the annulus between coaxial non-circular rough cylinders which are in relative axial motion. The deformation everywhere is simple shear. A particular non-linear relation between shear stress and strain is assumed, such that the solution can be transformed into that for a linear relation. The method is worked out in detail for cylinders whose sections are confocal ellipses.
R. Hill (Nottingham)

8868:

Greenberg, H. J.; Dorn, W. S.; Wetherell, E. H. A comparison of flow and deformation theories in plastic torsion of a square cylinder. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 279–296. Pergamon, Oxford, 1960.

The problem considered is that of a solid bar of square section infinitesimally strained in torsion by end couples. It is solved for two contrasted types of constitutive law, namely when the relation between the stress, stress-rate, and strain-rate tensors is holonomic or non-holonomic. Under monotonic loading both types are capable of simulating the behaviour of a metal in the plastic range. The particular laws assumed contain a parameter controlling the rate of hardening.

Numerical solutions are obtained on an electronic computer for three values of this parameter. Graphs are presented comparing the stress-histories at selected points of the section and also the torque-twist relations. For the same value of the parameter, negligible difference is found between the results for the two types of law.

R. Hill (Nottingham)

8869:

Dobrovol'skiĭ, V. L. The problem of plane strain of a perfectly plastic body in terms of complex variables. Prikl. Mat. Meh. **24** (1960), 367–369 (Russian); translated as J. Appl. Math. Mech. **24**, 525–529.

A complex variable form is given to the equation satisfied by the Airy stress function in a two-dimensional (plastic) state of equilibrium with constant maximum shear stress. By this means some familiar simple stress fields are re-derived, along with a few new ones of an academic kind.

R. Hill (Nottingham)

8870:

Lensky, V. S. Analysis of plastic behavior of metals under complex loading. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 259–278. Pergamon, Oxford, 1960.

This paper summarizes and reviews recent Russian work on a postulational approach to the theory of plasticity which is apparently different from the flow theories which have been widely developed in the United States and England. Since the strain tensor ε_{ij} is subject to symmetry ($\varepsilon_{ij}=\varepsilon_{ji}$) and incompressibility ($\varepsilon_{ii}=0$), it can be represented in a suitably chosen five-dimensional strain space E_5 as a vector ε_α (in this review latin indices run from 1 to 3, greek indices from 1 to 5) such that $\varepsilon_\alpha\varepsilon_\alpha=\varepsilon_{ij}\varepsilon_{ij}$. The strain history of a material point is then represented as a curve in E_5 called the strain trajectory.

In similar fashion a stress vector σ_α may be defined from the stress deviator tensor s_{ij} such that $\sigma_\alpha\sigma_\alpha=s_{ij}s_{ij}$. Once the material properties are known, such a stress vector may be associated with each point on the strain trajectory. The isotropy postulate states that the stress vector σ_α at a given point on the strain trajectory depends only upon the intrinsic properties of the strain trajectory (arc length, curvature, etc.) and not upon its orientation in E_5. Tests on thin-walled copper tubes in complex loading paths in tension and torsion appear to corroborate this postulate.

A second postulate, called the principle of delay, asserts that in fact not all of the strain trajectory is necessary to define σ_α, but only a preceding finite portion of it of length λ. A particular consequence of this postulate is that if proportional straining (which results in a linear strain trajectory) has been carried out for a finite arc length λ, then σ_α must be directed along the strain trajectory regardless of its earlier shape. This postulate, too, appears to be verified by some experimental results.

P. G. Hodge, Jr. (Chicago, Ill.)

8871:

Hodge, P. G., Jr. The Mises yield condition for rotationally symmetric shells. Quart. Appl. Math. **18** (1960/61), 305–311.

Author's summary: "The stress state in a rotationally symmetric shell under small displacements is characterized by the direct stresses and moments in the circumferential and longitudinal directions. If the shell material is perfectly plastic, it is desirable to express the material yield condition in terms of these stress resultants. Previous investigations have obtained this yield condition in certain special cases and for the maximum shear stress criterion in the general case.

"Here a derivation is given based on the Mises or octahedral-shear-stress criterion for both uniform and idealized-sandwich shells. The flow law relating extension and curvature rates of the middle surface to the stress state is also obtained. The general equations are obtained in closed parametric form and various special cases are explicitly presented."

R. Hill (Nottingham)

8872:

Hodge, Philip G., Jr. Boundary value problems in plasticity. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 297–337. Pergamon, Oxford, 1960.

This paper contains a selection of some recent work in connection with bars, thin plates and shells of revolution. Not unnaturally in such an article the author has dealt mostly with problems in which he himself has been interested. The first part of the paper contains a comparison of the effect of the choice of the plastic stress-strain relations on the final solution for the problems of the three bar truss and the bending of a simple supported circular plate with the Tresca yield condition. The remainder of the paper is devoted to a variety of problems concerning shells of revolution with the Tresca yield condition. Problems considered are the spherical cap under external pressure, the simply supported annular plate under combined tension and bending, and the conical shell under concentrated load at the vertex parallel to the axis of the shell.

G. Eason (Newcastle-upon-Tyne)

8873:

Szczepinski, W. Recent advances in the theory of drawing of thin shells. Appl. Mech. Rev. **14** (1961), 173–176.

8874:

Haythornthwaite, R. M. Stress and strain in soils. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 185–193. Pergamon, Oxford, 1960.

During the last ten years a number of theoretical studies in soil mechanics have been based upon the assumption that, so far as its bulk behaviour is concerned, a soil may be regarded as an ideally plastic solid conforming to the Coulomb law of failure at yield and in subsequent flow. This assumption has certain consequences which are known to disagree with experimental results. In particular, the flow rule associated with the Coulomb yield condition predicts dilatations far in excess of those observed. Attempts have been made to improve the theory by introducing strain hardening and softening and by adopting a non-associated flow rule, but these additional assumptions have lacked detailed experimental support. The present paper is a preliminary report of experimental work aimed at a detailed examination of the assumptions

underlying the plasticity theory of soil mechanics and is particularly valuable in being written primarily from the viewpoint of the applied mathematician.

The results have been obtained by subjecting specimens of a remoulded silt having highly reproducible properties to combined axial compression and torsion in the presence of a hydrostatic pressure. The author's conclusions are the following. (i) The yield stresses and the directions of flow appear to be independent of the loading path. (ii) For complex stress states the results indicate substantial departures from the predictions of the Coulomb yield condition. (iii) Except in the case of simple compression superimposed upon a hydrostatic pressure, the deformation of the specimens can be accounted for more satisfactorily by the Tresca flow rule (which implies zero dilatation) than by the flow rule associated with the Coulomb condition. *P. Chadwick* (Sheffield)

8875:

Bolotin, V. V. Equations for the non-stationary temperature fields in thin shells in the presence of sources of heat. Prikl. Mat. Meh. **24** (1960), 361–363 (Russian); translated as J. Appl. Math. Mech. **24**, 515–519.

An uncoupled, non-stationary thermo-elastic problem for thin shells has been investigated. The author assumes a continuous distribution of heat sources, convection boundary conditions at the surfaces and linear variability of temperature in the direction normal to the surfaces of the shell. The heat-conduction equation derived with the aid of the tensor calculus and variational principles differs from that given by Melan and Parkus. As an example, the case of a circular conical shell has been discussed. *M. Sokolowski* (Lawrence, Kans.)

8876:

Youngdahl, C. K.; Sternberg, Eli. Transient thermal stresses in a circular cylinder. J. Appl. Mech. **28** (1961), 25–34.

The surface temperature of an infinitely long elastic circular cylinder undergoes a sudden uniform temperature change over a finite band between two cross sections and is steadily maintained thereafter. Temperature distribution and thermal stresses within the body are calculated using the quasi-static uncoupled equations of linear thermoelasticity.

The problem has been solved previously with the aid of the thermoelastic potential. The authors use the Neuber-Papkovich approach and arrive at a representation for the displacement functions and stresses in the form of improper integrals and infinite series which they show to be convergent together with all derivatives needed for the verification of the solution. Extensive numerical results are presented. *H. Parkus* (Vienna)

8877:

Sternberg, Eli.; Chakravorty, J. G. On inertia effects in a transient thermoelastic problem. J. Appl. Mech. **26** (1959), 503–509.

If a semi-infinite elastic body is exposed to a uniform sudden temperature change of its entire plane boundary the influence of inertia effects on the stresses and displacements in the body is considerable. In contradistinction the authors show that if the physically unrealistic assumption of a sudden temperature change is dropped and is replaced by a ramp-type increase of the surface temperature, inertia effects become practically negligible. *H. Parkus* (Vienna)

8878:

Deresiewicz, H. Effect of boundaries on waves in a thermoelastic solid: reflexion of plane waves from a plane boundary. J. Mech. Phys. Solids **8** (1960), 164–172.

In an infinite isotropic elastic medium the coupled linear thermoelastic equations predict the existence of two interacting dilatational waves, one predominantly elastic (E wave) and the other predominantly thermal (T wave), together with an independent plane shear wave (S wave). The E and T waves are subject to both dispersion and attenuation: the S waves have constant phase velocity and amplitude but may be polarized either in the plane of incidence (SH wave) or in the plane containing the propagation vector perpendicular to the free surface (SV wave). This paper considers the reflection of E and SV waves from a plane, traction free, thermally radiating boundary.

An incident E wave is found to give rise to a reflected SV wave the phase velocity of which is reduced and the angle of reflection increased as a result of thermoelastic coupling. The phase velocity and angle of reflection of the reflected E wave produced by an incident SV wave are both reduced. For physically realizable material properties and frequencies the phase velocities and angles of reflection of the T waves generated by incident E and SV waves are extremely small. The effect of thermal conductivity on the amplitude of the reflected E and SV waves is found to be greater for an incident SV wave than for an incident E wave; the amplitude of the reflected T wave is of the same order of magnitude in each case. In addition the amplitudes of the reflected E and T waves arising from a normally incident E wave are sensibly independent of the emissive power of the surface for frequencies well into the megacycle range. *P. Chadwick* (Sheffield)

STRUCTURE OF MATTER

See also 8743, 8769, 8844, 8853, 8854, 8966, 9107, 9180, 9181, 9182, 9183.

8879:

Seitz, Frederick; Turnbull, David (Editors). ★Solid state physics: Advances in research and applications, Vol. 7. Academic Press, New York, 1958. xiv + 525 pp. $12.00.

The first of the wide range of articles in this volume is on "Thermal conductivity and lattice vibration modes" by P. G. Klemens (pp. 1–98). It is mainly theoretical and begins with a comprehensive discussion of the theory of thermal conduction by lattice vibrations, including the scattering processes. There follow two sections in which the particular cases of non-metals and the lattice component for metals are discussed and the theory compared with experimental results. This article is followed by one on "Electron energy bands in solids" by J. Callaway (pp. 99–212). He begins with a brief review of the theoretical bases for calculating energy bands, much of which has been reviewed in more detail in earlier articles in this

series. The main part of the paper is then devoted to the results for a great range of substances, both elements and compounds.

A very comprehensive review of "The elastic constants of crystals" is given by H. B. Huntingdon (pp. 213–351). While many results are quoted, the principal emphasis is on the fundamental aspects of elastic properties and the interpretation of observed behaviour. There are sections dealing with theoretical calculations of elastic constants, and their dependence on temperature through an idealized equation of state, and also discussion of properties that cannot be treated through an idealized equation of state (e.g., properties depending on dislocations and other defects). There is a useful discussion of the current question of the role of body torques in elasticity, and special consideration of third-order elastic constants.

In an article entitled "Wave packets and transport of electrons in metals" (pp. 353–378), H. W. Lewis reviews the basic theory of electron transport, giving special attention to the problem of reconciling the apparently unfulfilled assumption, in deriving the transition probability from the scattering matrix, that the level widths of the final states are small compared to thermal energies. J. A. Becker reviews progress in the "Study of surfaces by using new tools" (pp. 379–424); with the use of ion gauges and field emission microscopes, remarkable information has been accumulated concerning the adsorption and migration of atoms and molecules on tungsten surfaces. Finally, A. F. Wells considers "The structure of crystals" (pp. 425–503) "from the standpoint of topology rather than the conventional one of lattices and symmetry". Thus he considers the various networks and frameworks that arise when an atom or molecule is always connected equidistantly to a specified number of other atoms or molecules and demonstrates how a great number of known crystal structures can be understood in this way.

M. S. Paterson (Canberra)

8880:

Seitz, Frederick; Turnbull, David (Editors). ★Solid state physics: Advances in research and applications, Vol. 8. Academic Press, New York-London, 1960. xiv + 519 pp. $13.50.

The eighth volume of Advances in Solid State Physics begins with "Electronic spectra of molecules and ions in crystals. I. Molecular crystals" by D. S. McClure (pp. 1–47). As a background to the experimental results that follow, a survey of the theory is given for the simple model of a lattice of fixed molecules, including energy levels, interaction potentials, transition probabilities, etc., with special attention to the use of symmetry.

Two articles are of interest to workers in the field of semiconductors. R. Newman and W. W. Tyler discuss "Photoconductivity in germanium" (pp. 49–107), dealing in general terms with the electronic structure of germanium and the theory of photoconductivity, and giving an extensive review of experimental results; special attention is given to the role of impurities that introduce deep levels. The second paper is a longer one on "Electronic processes in zinc oxide" by G. Heiland, E. Mollwo and F. Stöckmann (pp. 191–323). After a good deal of background information on zinc oxide, including its physical properties, preparation, the diffusion of imperfections in it and the optical properties, an extensive review is given of experimental observations on the various aspects of its electrical conductivity and the physical interpretation of these observations.

"Interaction of thermal neutrons with solids" by L. S. Kothari and K. S. Singwi (pp. 109–190) is essentially a theoretical paper. The general theory of neutron scattering is developed, both for the Debye model and for the more detailed approach needed for successful calculation of the dispersion relations and frequency distribution functions in crystal dynamics. Some comparisons with experimental results are given. There is also a section dealing with the slowing down of neutrons in the thermal energy region.

The final article in this volume is on "The structure and properties of grain boundaries" by S. Amelinckx and W. Dekeyser (pp. 325–499). A great range of experimental observations on the properties of grain boundaries and phenomena concerned with them are described and discussed, but the chief theoretical interest lies in the authors' review of the treatment of grain boundaries as arrays of dislocations, both in respect to the geometry of the boundaries and orientation relations across them and to the questions of the energy and stability of such dislocation arrays and the interaction of other dislocations with them.

M. S. Paterson (Canberra)

8881:

Houstoun, R. A. Note on the theory of optical activity. Opt. Acta **6** (1959), 216–221. (French and German summaries)

It may be possible to give a simple explanation of natural optical activity in terms of classical dispersion theory if one allows for the magnetic effects of a class of spinning electrons.

A. A. Blank (New York)

8882:

Takeno, Shozo. Energy spectrum of lattices with defects. I. General theory. Progr. Theoret. Phys. **25** (1961), 102–120.

The author shows that the equations governing the time independent behavior of lattice vibrations can formally be cast as a Schrödinger equation. Many methods which are useful in scattering theory can then be used to find the frequency spectrum, particularly for defect problems. Many of these methods have already been applied to this problem in a slightly different form and most of the results are already known. Specific applications are promised for a future paper.

G. Weiss (Washington, D.C.)

8883:

Kopeć, Zbigniew. Effective mass method in the case of non-quadratic dispersion formula. Acta Phys. Polon. **19** (1960), 295–317.

The concept of an effective mass is extended to the case of a non-parabolic energy band in a semiconductor. It transpires that several independent quantities are required to describe the electron motion, instead of the single quantity for a parabolic band of spherical symmetry. The results are applied to indium antimonide.

S. Simons (London)

8884:

Hrivnák, L'. New derivation of Bardeen-Shockley formula for mobility of electrons in homopolar semi-conductors. Czechoslovak J. Phys. **10** (1960), 633–644. (Russian summary)

Author's summary: "The Bardeen-Shockley formula for the mobility of an electron or hole in a homopolar semi-conductor is derived in a different way from that in which its authors obtained it. The interaction energy of the electron with the acoustic lattice oscillations is derived in an original way. A new possibility for determining the energy gap is given."

8885:

Avak'yanc, G. M. Some generalizations of the kinetic equations for the carrier of current in semiconductors. Izv. Akad. Nauk UzSSR. Ser. Fiz.-Mat. **1958**, no. 2, 5–24. (Russian. Uzbek summary)

8886:

Okazaki, Makoto. A theory of the magneto-optical absorption. Progr. Theoret. Phys. **25** (1961), 163–177.

Author's summary: "A theory of the magneto-optical absorption is developed by means of the Luttinger and Kohn theory of the effect of magnetic field on the energy bands in the case of a simple band model. The main purpose is to study the absorption line shape. As the width of the absorption line we get a value which agrees with experiment. It does not exhibit appreciable dependence on the magnetic field and temperature. The absorption coefficient in the absence of a magnetic field is also discussed."

8887:

Yakovlev, V. A.; Kalyuš, A. V. Excitation spectrum of a system of electrons and ions situated in a homogeneous magnetic field. Ž. Èksper. Teoret. Fiz. **39** (1960), 308–310 (Russian. English summary); Soviet Physics. JETP **12** (1961), 219–220.

Authors' summary: "The effect of a magnetic field on the spectrum of the acoustic branch of excitations of a system of electrons and ions is studied with the aid of the quantum kinetic equation."

8888:

Mills, R. L.; Kenan, R. P.; Korringa, J. Perturbation theory of antiferromagnetism. Physica **26** (1960), supplement, S 204–S 214.

Many-body perturbation techniques are applied to the calculation of the properties of an antiferromagnetic crystal lattice. A model unperturbed Hamiltonian is chosen so that the spin interactions can be treated as a perturbation. The partition function is expressed as a formal power series in the interaction Hamiltonian, the members of which can be interpreted in terms of connected diagrams. Certain fictitious diagrams are inserted to assist in the formation of sums over sets of diagrams. It is shown that the theory is related to the Néel theory of antiferromagnetism. Methods of performing summations over infinite sets of terms are discussed. The treatment is programmatic in nature and actual calculations are not attempted. *E. L. Hill* (Minneapolis, Minn.)

8889:

Byčkov, Yu. A. The quantum theory of the electrical conductivity of metals in strong magnetic fields. Ž. Èksper. Teoret. Fiz. **39** (1960), 689–702 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 483–491.

Author's summary: "We consider quantum mechanically galvanomagnetic phenomena in strong magnetic fields at very low temperatures in the limit where the lifetime of an electron is much larger than the period of its revolution in the magnetic field. We investigate metals with a quadratic dispersion law. We obtain formulae for the scattering of an electron by an impurity when a magnetic field is present."

8890:

Kaplan, T. A. Classical theory of spin configurations in the cubic spinel. Phys. Rev. (2) **119** (1960), 1460–1470.

Bekannterweise befinden sich die zweiwertigen Metallionen in den im Spinelltyp kristallisierenden ferrimagnetischen Stoffen (in den sogenannten Ferriten) in den von den O-Ionen gebildeten flächenzentrierten Gittern in tetraedrischen (A) Hohlräumen, die dreiwertigen dagegen (die in doppelter Zahl vorhanden sind), in oktaedrischen (B) Hohlräumen. Nach der Néelschen Theorie sind die Spinmomente in jedem Untergitter der Metallionen zueinander parallel orientiert, in den zwei verschiedenen Untergittern jedoch einander entgegengesetzt und auf diese Weise entsteht der scheinbare makroskopische Ferromagnetismus dieser Körper. Ziel der vorliegenden Arbeit ist die Frage zu untersuchen, unter welchen Bedingungen diese Néelsche Anordnung die stabilste ist und wann Abweichungen davon zu erwarten sind. Es wird angenommen, dass antiferromagnetische Wechselwirkungen erstens nur zwischen den unmittelbaren Nachbaren und zweitens nur zwischen den Gitterplätzen $A-B$ und $B-B$ auftreten. (Die sogenannten Superaustauschwechselwirkungen bleiben unberücksichtigt.) Für die entstehende Energie hat man dann

$$(1) \qquad E = 2F_{AB}S_AS_B\{\sum_{(i,j)}\sigma_i^A\cdot\sigma_j^B+y\sum_{(i,j)}\sigma_i^B\cdot\sigma_j^B\},$$

wo $y=F_{BB}S_B/F_{AB}S_A\geq 0$ ist. F_{AB} und F_{BB} bedeuten die Heisenbergschen Austauschenergien, die hier (Antiferromagnetismus) positiv sind und welche der Verfasser die klassischen Austauschenergien nennt, S_A und S_B sind die Spins und es wird angenommen, dass an einem Gitterpunkt immer nur ein Spinmoment vorhanden ist. Wenn bei der Néelschen Anordnung die Spinmomente entlang der Z-Achse orientiert sind, so kann man kleine Abweichungen in der Form

$$\sigma_i^{Ax}=[1-(\sigma_i^{Ax})^2-(\sigma_i^{Ay})^2]^{1/2}=1-\tfrac{1}{2}[(\sigma_i^{Ax})^2+(\sigma_i^{Ay})^2]\cdots$$

(und analog für σ_j^{Bx}) schreiben. Setzt man diese Werte in (1) ein, so kann die Energie in eine Reihe $E=E_0+E_2+E_4+\cdots$ entwickelt werden. Die weiteren Berechnungen zeigen, dass die Néelsche Anordnung stabil ist, solange nur $y<\frac{2}{3}$ ist. (E_2 ist in diesem Gebiet positiv definit.) Wird jedoch $y-\frac{2}{3}=\eta$ positiv, so treten im Raume sinusoidale Abweichungen von der Néelschen Anordnung auf. Für kleine η wird dann, um den kleinsten Energiewert finden zu können, eine Störungstheorie angewandt. Als Resultat folgt, dass es Gleichgewichtsverteilungen gibt, bei denen auch die an den A und auch die an den B Plätzen sich

befindenden Spins von Null verschiedene Winkel miteinander einschliessen und dass es entgegen der Behauptung von Anderson Winkelverteilungen von grosser Reichweite gibt. Es wird ausserdem bewiesen, dass die Dreieckskonfiguration der Spins in den zwei Untergittern von Y. Yafet und C. Kittel [Phys. Rev. (2) **87** (1952), 290–294] keinem Energieminimum entspricht. Die erhaltenen Resultate werden zuletzt noch mit der Erfahrung verglichen. Die von L. Corliss und J. Hastings mit Hilfe von Neutronenstrahlen an $MnCr_2O_4$ erhaltenen Resultate stimmen qualitativ mit den Ergebnissen dieser Arbeit überein. Zur Erreichung einer quantitativen Übereinstimmung müsste man wahrscheinlich noch höhere Näherungen berücksichtigen. *T. Neugebauer* (Budapest)

8891:

Kociński, Jerzy. On the theory of spin waves in ferrimagnetics. Acta Phys. Polon. **19** (1960), 565–572.

Author's summary: "We investigate a ferrimagnetic with spins pointing in the same direction, while at absolute zero temperature. As the model of this ferrimagnetic we take simple cubic and body centered cubic lattices with two kinds of spins s, S, attached to the lattice points of two magnetic sublattices in such a way that a small spin s has as its nearest neighbours large spins S and vice versa.

"Spin waves are introduced in a manner analogous to that of Dyson for the ferromagnetic case. As in Dyson's theory, the Hamiltonian of the system is explicitly separated into a diagonal part representing the energy of free spin waves and a nondiagonal part determining the dynamical spin-wave interaction. A quadratic dispersion law with two frequency branches of spin wave propagation has been obtained."

8892:

Hedin, Lars T. A microscopic derivation of the Born-Huang relations between the atomic force constants. Ark. Fys. **18**, 369–378 (1960).

Nimmt man mit M. Born an, dass die atomare potentielle Energie Φ in einem festen Körper nur von den Verschiebungen (u) der Gitterbausteine abhängt, so hat man bei einer kleinen Deformation

$$(1)\qquad \Delta\Phi = \frac{1}{2}\sum_{\substack{ll'\\ kk'\\ \alpha\beta}} \Phi_{\alpha\beta}\begin{pmatrix} ll' \\ kk' \end{pmatrix} u_\alpha\begin{pmatrix} l \\ k \end{pmatrix} u_\beta\begin{pmatrix} l' \\ k' \end{pmatrix} + \cdots,$$

wo α und β cartesische Koordinaten, die l die Gitterzellen und die k die einzelnen Atome in diesen Zellen bedeuten. Die $\Phi_{\alpha\beta}$ sind die atomaren Kraftkomponenten, die man mit den elastischen Konstanten in Zusammenhang bringen kann.

Ausgehend von einigen von M. Born herrührenden theoretischen Arbeiten konnte K. Huang [Proc. Roy. Soc. London Ser. A **203** (1950), 178–194; MR **12**, 375] zeigen, dass man aus der Theorie der in einem unendlichen festen Körper auftretenden langen akustischen Wellen, wenn man sie mit der erwähnten Theorie vergleicht, zehn neue Zusammenhänge zwischen den atomaren Kraftkomponenten herleiten kann. Die sind die sogenannten Born-Huangschen Beziehungen. Es ist jedoch nicht möglich die in einem unendlichen Gitter aus einer rein mikroskopischen Betrachtungsweise herzuleiten. Ausführlich sind diese Fragen in dem Werke von M. Born und K. Huang [*Dynamical theory of crystal lattices*, Clarendon Press, Oxford, 1954] besprochen.

Ziel der vorliegenden Untersuchung ist zu zeigen, dass man diese Born-Huangschen Beziehungen auch aus einer mikrophysikalischen Betrachtungsweise herleiten kann, wenn man den fraglichen Körper von endlicher Ausdehnung annimmt, was so zu verstehen ist, dass man zuerst aus einem endlichen Gitter ausgeht und dann den Grenzübergang zum unendlichen vollführt, statt gleich von Anfang an ein unendliches Gitter zu betrachten. Beim Beweise wird erstens die Invarianz von Φ gegenüber einer Rotation benutzt und ausserdem werden plausible Annahmen über die Vernachlässigbarkeit von gewissen von der Oberfläche herrührenden Glieder eingeführt. Im letzten Teil wird noch gezeigt, dass die ausgearbeitete Methode es ermöglicht, den Zusammenhang zwischen den elastischen Konstanten und den atomaren Kraftkomponenten unter Berücksichtigung einer homogenen Deformation (statt langen Wellen) herzuleiten.

T. Neugebauer (Budapest)

8893:

Gubanov, A. I.; Čevyčelov, A. D. Calculation of the energy spectrum for strongly anisotropic crystals. Fiz. Tverd. Tela **2** (1960), 1379–1389 (Russian); translated as Soviet Physics. Solid State **2** (1961), 1253–1262.

Authors' summary: "A quantum mechanical calculation of the electron energy spectrum for two different models of an anisotropic crystal is carried out. It is assumed that along one of the crystallographic directions the electron moves freely, but that along the others the motion is significantly impeded. The calculation is carried out with the aid of the methods of strong and weak bonds. Deductions are made concerning the topology of the isoenergetic surfaces in the space of the inverse lattice. Within the framework of the models it is shown that the electron dispersion laws are different for lattices with identical crystallographic symmetry but different types of chemical bonds."

8894:

Koloskova, N. G.; Kopvillem, U. H. On the effect of the internal electric field in nonconducting paramagnetic single crystals of the two-particle spin-spin interactions. Fiz. Tverd. Tela **2** (1960), 1368–1378 (Russian); translated as Soviet Physics. Solid State **2** (1961), 1243–1252.

Authors' summary: "The coefficients of the spin Hamiltonian of dipole-dipole and anisotropic-exchange interactions between ions with anisotropic g factor and effective electron spin $\frac{1}{2}$ are used to calculate the second and fourth moments of the distribution curve of the nondiagonal matrix elements of the spin-spin interaction operator. The results are used to investigate the line shape in paramagnetic resonance and in free magnetic induction. The order of magnitude of the spin-spin paramagnetic relaxation time is estimated. Concrete calculations are made for ethyl sulfates and double nitrates of rare-earth ions."

8895:

Bir, G. L.; Pikus, G. E. Theory of the deformation potential for semiconductors with a complex band structure.

Fiz. Tverd. Tela **2** (1960), 2287–2300 (Russian); translated as Soviet Physics. Solid State **2** (1961), 2039–2051.

Authors' summary: "We develop a theory of the deformation potential for semiconductors with a complex band structure. We construct an operator for the interaction between electrons and the long wavelength acoustical and optical phonons, and calculate the transition probabilities. The obtained general expressions are used in order to calculate the matrix elements for the scattering of holes in *p*-type germanium."

8896:

Klyačkin, V. I. Theory of orientation ordering of dipole crystals. Fiz. Tverd. Tela **2** (1960), 1390–1398 (Russian); translated as Soviet Physics. Solid State **2** (1961), 1263–1271.

Author's summary: "The present paper considers the problem of the orientation ordering of dipole crystals, the molecules of which are bound both by short-range orientation forces and by long-range forces of the dipole type. On the basis of N. N. Bogolyubov's method of partial functions the phase transition point is determined, and the temperature behavior of the permittivity is investigated. An answer is provided for the question of the relative contribution of dipole forces in the correlation effects of molecular orientations."

8897:

Lang, I. G. Application of the method of the density matrix to the problem of conductivity electrons interacting with lattice vibrations. Fiz. Tverd. Tela **2** (1960), 2330–2340 (Russian); translated as Soviet Physics. Solid State **2** (1961), 2077–2086.

Author's summary: "It is shown that the method of the density matrix (method of Kohn and Luttinger), applied in the first approximation to the interaction of electrons with thermal lattice vibrations, leads to the usual system of kinetic equations, accounting for the entrainment of phonons by electrons, and electrons by phonons."

8898:

Merten, Ludwig. Über die Gitterschwingungen in Kristallen mit Wurtzitstruktur. I. Grenzschwingungen ohne Berücksichtigung der Coulomb-Kräfte. Z. Naturforschg. **15a** (1960), 512–523.

So im Zinkblende- wie im Wurtzitgitter sitzen die nächsten Nachbaren eines Atoms in den Ecken eines gleichseitigen Tetraeders. Im ersteren Gitter (Diamantstruktur) sind diese Tetraeder streng symmetrisch und bauen einen in dem regulären System gehörenden Kristall auf. Im letzteren sind dagegen die Tetraeder in der Richtung von einer Bindung meistens ein ganz wenig zusammengedrückt und auf diesem Wege entsteht ein hexagonales (mikroskopisch betrachtet eigentlich trigonales) Gitter. Aus den physikalischen Eigenschaften der im Wurtzittyp kristallisierenden Verbindungen folgt weiter, dass der Bindungstyp bei diesen vorwiegend homöopolar ist, doch werden auch Coulombsche Kräfte eine gewisse Rolle spielen. Der Verfasser berechnet zuerst die Orts- und Abstandsvektoren für die 4 nächsten und 12 weiteren Nachbaren der vier in der Elementarzelle enthaltenen Atome. Die erhaltenen Ergebnisse werden auch tabellarisch zusammengefasst. Danach stellt er die Grundgleichungen der Schwingungen auf, die eigentlich spezielle Fälle der mit Hilfe von Exponentialfunktionen integrierten elementaren Differentialgleichung des harmonischen Oszillators sind. Seine Resultate lauten:

$$m_k\omega^2\mathbf{v}(k) =$$

$$(1)\qquad \sum_{l'k'} \Phi\begin{pmatrix} ll' \\ kk' \end{pmatrix}\cdot\mathbf{v}(k')\exp\left\{i\mathbf{q}\left[\mathbf{r}\begin{pmatrix} l' \\ k' \end{pmatrix}-\mathbf{r}\begin{pmatrix} l \\ k \end{pmatrix}\right]\right\} = \sum_{k'} C\begin{pmatrix} \mathbf{q} \\ kk' \end{pmatrix}\cdot\mathbf{v}(k'),$$

wo ω die Frequenz, $\mathbf{q}$ den Wellenzahlvektor, die r die Abstandsvektoren und die v die Eigenvektoren bedeuten. Ausserdem ist Φ das Gitterpotential und

$$(2)\qquad \Phi\begin{pmatrix} ll' \\ kk' \end{pmatrix} = \frac{\partial^2\Phi}{\partial\mathbf{u}\begin{pmatrix} l \\ k \end{pmatrix}\partial\mathbf{u}\begin{pmatrix} l' \\ k' \end{pmatrix}}$$

Damit das Gleichungssystem lösbar sein soll, muss die Determinante

$$(3)\qquad \left| C\begin{pmatrix} \mathbf{q} \\ kk' \end{pmatrix} - m_k\omega^2\delta_{kk'}I \right|$$

(I = Einheitsmatrix) verschwinden. Diese Bedingung liefert einen Zusammenhang zwischen Frequenz und Wellenzahl, also eine Dispersionsgleichung. Da die Determinante $3n$-reihig (n = Zahl der Atome in der Elementarzelle) ist, so erhält man für ω 12 Lösungen, die als Schwingungszweige bezeichnet werden. Beim Grenzübergang $\lambda\to\infty$ geht von dreien die Frequenz zu Null, die werden als akustische Schwingungen bezeichnet, bei den übrigen dagegen nicht, die sind die optischen Schwingungen. Durch gewisse Symmetriebetrachtungen lässt sich die Determinantengleichung weitgehend vereinfachen. Danach werden unter Nichtberücksichtigung der Coulombschen Kräfte und unter Zuhilfenahme von einigen sich auf Determinanten beziehenden Sätze (wobei auch die von Laplace benutzt werden), die Frequenzen der Grenzschwingungen berechnet. Von den neun optischen Schwingungen sind nur sechs voneinander verschieden und nur zwei von ihnen sind ultrarotaktiv. Die Polarisationsverhältnisse werden ebenfalls berechnet.

T. Neugebauer (Budapest)

8899:

Neronova, N. N.; Belov, N. V. Ferro-magnetic and segneto-electric space groups. Akad. Nauk SSSR. Kristallografiya **4** (1959), 807–812. (Russian)

8900:

Gončarenko, A. M. On the approximative theory of properties of absorbing crystals. Akad. Nauk SSSR. Kristallografiya **4** (1959), 848–854. (Russian)

8901:

Rumanova, I. M. Symmetry of weighted projections of electron density of crystals of lower systems. Akad. Nauk SSSR. Kristallografiya **5** (1960), 180–193. (Russian)

8902:

Tihonov, L. V. Study of the fine crystallographic structure according to the intensities of the diffraction lines. Akad. Nauk SSSR. Kristallografiya 5 (1960), 194–199. (Russian)

FLUID MECHANICS, ACOUSTICS

See also A7914, A7924, A8190, 8782, 8783, 8785, 8845, 8862, 8863, 9015, 9022, 9061, 9093, 9095.

8903:

Goldstein, Sydney. ★Lectures on fluid mechanics. With special lectures by J. M. Burgers. Lectures in Applied Mathematics. Proceedings of the Summer Seminar, Boulder, Colorado, 1957, Vol. II. Interscience Publishers, London-New York, 1960. xvi+309 pp. $6.60.

To make "mature mathematicians familiar with fluid dynamics" in 12 lectures is a task on the border, at least, of the possible. With a large minority of the audience composed of distinguished specialists in fluid dynamics, the task is well beyond that border. And the requirement of publication of the lectures cuts off any retreat. These, we may speculate, were some of the reasons for asking the particular author to make the attempt. As would be expected, he shuns resolutely the superficially elegant and the sensational, and the resulting book is an unusual and valuable contribution to the literature.

The book may be divided roughly in three major parts. The first 100 pages are devoted to a discussion of the equations governing the motion of fluids and of topics directly related to them. A section on electromagnetic forces is included. The treatment is reasonably complete in many areas, but often brief. Its purpose is not to replace the existing monographs, to which the reader is referred freely, but rather to clarify what others have left obscure, to correct arguments which others have left in erroneous form, and not to shirk what has been carefully left unsaid by brinksmen like Lamb.

The second part is concerned with certain classes of solutions of the equations of fluid motion. The reader is offered a concise presentation, on the one hand, of the slender-body approximation for the flow of inviscid gases, and on the other hand, of the main exact solutions now known of the boundary layer equations for incompressible fluids and for gases. Moreover, the relation of the incompressible boundary layer equations to the equations of motion is discussed by explaining the great difficulties involved and describing a new attempt to construct an asymptotic expansion for the flat-plate problem.

The third part offers an introduction to four topics in magnetohydrodynamics. First, a derivation of the equations of electrodynamics of a moving continuum is presented in some detail for a moderately general class of such processes. Secondly, a simple treatment of longitudinal electrical waves of high frequency in a rarefied ionised gas is given both from the continuum and kinetic point of view. Special lectures by J. M. Burgers then deal with an approximation to the interaction between a shock wave and a magnetic dipole, with formation of a current sheet, and give an illuminating introduction to the tensorial character of electrical conductivity which arises in a partially ionised, rarified gas from cross-diffusion of the ions and electrons in an electromagnetic field. *R. E. Meyer* (Providence, R.I.)

8904:

★Advances in the Astronautical Sciences, Vol. 6. Proceedings of the 6th annual meeting of the American Astronautical Society, New York, January 18–21, 1960. The Macmillan Company, New York, 1961. lxix+898 pp. $25.00.

The papers in this volume which are of mathematical interest will be reviewed separately.

8905:

Tietjens, O. ★Strömungslehre. Physikalische Grundlagen vom technischen Standpunkt. Bd. I: Hydro- und Aerostatik. Bewegung der idealen Flüssigkeit. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1960. xvi+536 pp. DM 66.00.

Der vorliegende erste Band der "Strömungslehre" soll nach Verfasser im Sinne von L. Prandtl nicht nur Kenntnisse vermitteln, sondern beim Leser auch die Fähigkeit entwickeln, diese Kenntnisse in der Praxis anzuwenden. Daher ist Verfasser bemüht, immer vom Gegenständlichen, vom Experiment und Gedankenexperiment auszugehen und auf dieser Grundlage die Theorie zu entwickeln. Ferner bemüht sich Verfasser, den Stoff so darzustellen, daß der Leser nach dem Studium nichts rein äußerlich Angelerntes mitnimmt, sondern etwas, das er von "Grund aus" verstanden hat. Dieser Vorsatz bedingt naturgemäß eine gewisse Breite der Stoffbehandlung. Das zeigt bereits die Unterteilung der einzelnen Kapitel in aller Deutlichkeit. Kapitel I ist den Eigenschaften der Flüssigkeiten und Gase gewidmet. Behandelt werden: Zähigkeit, Dichte und Kompressibilität. Der Theorie der Zähigkeit geht eine eingehende Darstellung der experimentellen Grundlagen voraus, dann werden Einwirkungen von Temperatur und Druck behandelt und numerische Werte angegeben. Methodisch analog wird der Begriff Dichte behandelt. Wird die Dichte in einem Punkt P durch einen Differentialquotienten definiert, so versäumt Verfasser nicht, auf die eigentümlichen Schwierigkeiten hinzuweisen, welche sich bei dieser Definition einstellen können: ist P ein mathematischer Punkt, so kann die konvergente Folge der P enthaltenden Volumina ΔV_i unter Umständen aus solchen ΔV_i bestehen, welche zwischen den Molekülen liegen. Die Berücksichtigung der Kompressibilität kann in der Strömungslehre auf zwei Ursachen zurückgeführt werden: große Höhenunterschiede und große Geschwindigkeiten.

Kapitel II enthält die Gleichgewichts- und Stabilitätstheorie: Flüssigkeitsdruck, kommunizierende Gefäße, Manometer, Druckkräfte auf Begrenzungsflächen, Schweben und Schwimmen, Stabilität eines Ballons.

Mit der Behandlung der Potentialströmung einer inkompressiblen Flüssigkeit ohne freie Oberfläche in Kapitel IV beginnt (nach einer Darstellung der Kapillarität in Kapitel III) naturgemäß die zunehmende Verwendung mathematischer Hilfsmittel. Das Kapitel Potentialströmungen zerfällt in fünf Abschnitte; Eulersche und Bernoullische Gleichung, Beispiele von Potentialströmungen, physikalische Deutung der Ausdrücke $\partial v/\partial x - \partial u/\partial y$ usw., Integralsätze, Überlagerung von Lösungen

der Laplaceschen Gleichung, Alle diese Dinge werden dem Leser in unmittelbarer Verbindung mit der mathematischen Theorie stets durch typische Beispiele und eine Vielzahl ausgezeichneter Abbildungen nahegebracht. Das gilt genau so für die folgenden Kapitel V, VI und VII mit den Unterabschnitten: Beziehungen zwischen Potential- und Stromfunktionen, Beispiele verschiedener Strömungsfunktionen $F(z)$, Erfüllen der Randbedingungen, konforme Abbildung, Hodographenmethode, Strömungen über ein Wehr und Ausfluß aus seitlichen Öffnungen, Ausflußzeiten, Niveaueinspiegelungen, Strömungen in Kanälen, Flüssigkeitswellen, Strömungen mit Rotation, einzelne gerade Wirbel, Kinematik und Dynamik der Wirbelbewegungen, gegenseitige Beeinflußung gerader Wirbel, das Geschwindigkeitsfeld eines einzelnen Wirbels von beliebiger Gestalt, Wirbelschichten, Diskontinuitätsflächen, Entstehen und Vergehen von Wirbeln.

Nur wenig können im Rahmen einer Besprechung wie der vorliegenden Einzelheiten hervorgehoben werden. Von solchen erscheint vielleicht die genaue Diskussion des Begriffs "rotationsfrei" von besonderem Interesse, indem Verfasser etwa betont, daß bei den einzelnen Gliedern der Eulerschen Gleichung, jegliche Zuordnung zu bestimmten Flüssigkeitsteilchen aufgehoben ist. Gerade darin ist die große Leistung Eulers zu sehen, die ihn zum Begründer der Hydrodynamik werden ließ, daß er die Newtonsche Auffassung eines diskreten Flüssingkeitspunktes aufgab und statt dessen den Begriff des Geschwindigkeitsfeldes sowie des Druckfeldes einführte. In den Eulerschen Differentialgleichungen beziehen sich deshalb die einzelnen Glieder dieser Gleichungen nicht auf Flüssigkeitsteilchen, sondern auf Raumpunkte und auf die in ihnen vorhandenen Feldgrößen wie Geschwindigkeit, Druck und Dichte sowie auf deren räumliche und zeitliche Änderungen. Erwähnen wir noch die interessante Diskussion von Entstehen und Vergehen von Wirblen (VII/7) im Musterfalle eines anfahrenden Tragflügels im Rahmen der Helmholtzschen Wirbeltheorie.

Methodisch charakteristisch erscheint auch die Behandlung der Theorie der Auftriebsverteilung um einen Tragflügel. Der Weg zur elliptischen Auftriebsverteilung wird vom Verfasser über die von vornherein näherliegende parabolische Auftriebsverteilung geführt. Da es sich um den induzierten Widerstand handelt, erscheint dieser Weg didaktisch wertvoll, denn es zeigt sich, daß die parabolische Auftriebsverteilung gegenüber der optimalen elliptischen Auftriebsverteilung schlecht abschneidet, d.h. auf einen größeren induzierten Widerstand führt.

Im Schlußwort behandelt Verfasser die wichtige Frage, wann man die klassische Hydrodynamik auf tatsächliche Strömungsvorgänge anwenden kann, ohne mit der Erfahrung in Widerspruch zu geraten. Die Grenzen der Theorie sind durch Grenzschicht, Turbulenz und Kompressibilität gegeben. Damit sind auch die Hauptstücke des Inhalts des vom Verfasser in Aussicht gestellten zweiten Bandes der "Strömungslehre" vorgezeichnet.

M. Pinl (Cologne)

8906:

Goulard, R. High temperature aerodynamics. Appl. Mech. Rev. **14** (1961), 257–261.

8907:

★Oscillazioni dell'acqua in impianti idraulici dotati di pozzi piezometrici. Consiglio Naz. Ricerche Pubbl. Ist. Appl. Calcolo No. 250. Manuali per Applicazioni Tecniche del Calcolo, I. Casa Editrice Perrella, Rome, 1950. 87 pp. (11 page insert) L. 750.

8908:

Dergarabedian, Paul. The behavior of vortex motion in an emptying container. Proc. 1960 Heat Transfer Fluid Mech. Inst., pp. 47–61. Stanford Univ. Press, Stanford, Calif., 1960.

The setting is an initial axially symmetric tangential speed distribution in a liquid of finite depth and infinite radius. Viscosity effects during the emptying process are neglected. A plausible radial speed distribution with suitable radial end conditions is assumed. The bottom of the container is shaped so that the continuity equation is satisfied when the assumed radial speed is used. An initial tangential speed distribution is selected from an initial non-emptying rotation process during which viscosity is taken into account. The appropriateness of the required simplifying assumptions is discussed. The calculations show a downward motion of the central surface which is exponential with respect to time and contain model parameters that would be useful for verification experiments.

M. G. Scherberg (Dayton, Ohio)

8909:

Kito, Fumiki. On virtual mass of water contained in a rectangular tank whose side-walls are vibrating. II. Proc. Fac. Engrg. Keio Univ. **12** (1959), 121–130.

Author's summary: "The author has presented, in the previous paper [same Proc. **11** (1958), no. 40, 1–19; MR **22** #2206], theoretical formulae which give the value of virtual mass of water, with regard to vibration of side-walls of a rectangular water tank, full of water. The theoretical formulae were obtained by calculation of kinetic energy of water, which is in vibration induced by the vibration of side-walls. This was an approximate calculation, and not a rigorous one. So, there arose the question about the degree of approximation of the given formulae. In the present paper, this question is examined by taking into account higher modes of vibration of side-walls, and using the minimum principle of Lord Rayleigh. From the result of calculation, we see that the formulae for virtual mass of water in the previous paper are sufficiently accurate for practical use, at least for water tanks having usual proportions.

"The formulae for virtual mass, for the case in which the tank is only partially filled, are also given here."

8910:

Kito, Fumiki. On virtual mass of water contained in a rectangular tank whose side-walls are vibrating. III. Proc. Fac. Engrg. Keio Univ. **12** (1959), 177–183.

Author's summary: "When side-walls of a rectangular tank, which is filled with water, are vibrating, the inside water will also make vibratory motion. This motion of water lowers considerably the natural frequency of vibration of side-walls of the tank. This effect is conveniently expressed by 'virtual mass' of water. In the previous reports [see #8909], the author has made a theoretical study on the value of 'virtual mass' of water,

restricting himself to the case in which two opposite sidewalls (among the six walls) are vibrating. The present report is an outcome of continuation of this study, and treats a case in which four walls of the tank vibrate simultaneously. An approximate formula for natural frequency of vibration for a whole system consisting of four-walls and the inside water, is also given.

"The calculation throughout is made, regarding water to be an incompressible non-viscous fluid, and the amplitude of vibration to be infinitesimally small."

8911:

Stanišić, M. M. Concerning the integral equation of unsteady motion of a delta wing in supersonic flight by means of the acceleration-potential method. J. Aerospace Sci. **28** (1961), 343–345.

8912:

Laschka, Boris. Über das Abwindfeld hinter Tragflügeln bei Überschallgeschwindigkeit. Z. Flugwiss. **9** (1961), 33–45. (English and French summaries)

Author's summary: "A method of approximation for the calculation of the downwash field behind wings at supersonic speeds is presented. Moreover, a survey is given on theories employed hitherto to determine the induced downwash."

8913:

Malavard, L.; Lepage, L. Principes et résultats de calculs d'ailes à jet par analogies rhéoélectriques. Rech. Aéro. No. 77 (1960), 3–17.

Authors' summary: "Un schéma théorique simple, basé sur l'hypothèse des faibles perturbations, peut rendre compte de l'effet d'un soufflage au bord de fuite d'un profil mince ou d'une surface portante. La méthode de calcul rhéoélectrique permet d'établir un modèle analogique de ce schéma. Cette méthode a d'abord été appliquée au cas bidimensionnel des profils à jet; elle a été ensuite exploitée pour le calcul d'ailes à jet de différentes formes. Des résultats sont donnés pour des ailes de divers allongements, rectangulaires, en delta et en flèche avec des modes de soufflage variés: singulier, régulier, uniforme, partiel, etc."

8914:

Frankl', F. I. Approximate calculation of the potential flow of a fluid jet flowing in a thin layer over the surface of a solid body. Prikl. Mat. Meh. **24** (1960), 378–379 (Russian); translated as J. Appl. Math. Mech. **24**, 546–548.

The author treats the problem indicated in the title of the paper. He derives approximate expressions for the flow potential and for the pressure on the surface of the body. *L. E. Payne* (College Park, Md.)

8915:

Ball, F. K. Finite tidal waves propagated without change of shape. J. Fluid Mech. **9** (1960), 506–512.

The non-linear equations of motion, based on the assumption of hydrostatic pressure, are modified by the insertion of Coriolis terms. The external applied pressure is such that a state of uniform steady motion is possible. All solutions representing waves travelling at constant speed without change of form are investigated. (It is well known that there are no such solutions when the Coriolis terms are absent.) By considering the phase plane it is shown that certain solutions are periodic in space, and some of these are illustrated in the paper.

F. Ursell (Cambridge, England)

8916:

Čerkosov, L. V. On the development of surface waves excited by a periodic pressure. Dokl. Akad. Nauk SSSR **127** (1959), 774–776 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 776–778.

The mean free surface is taken as the x-y plane. When $t>0$, the surface pressure is $a\exp(ikx-i\omega t)$ in the strip $|y|<b$, zero outside the strip; when $t<0$, the fluid is at rest. Integral expressions are given for the three-dimensional wave motion generated by this pressure, and their asymptotic behaviour for large t is stated.

F. Ursell (Cambridge, England)

8917:

Rumyancev, B. N. On waves generated on the surface of an incompressible fluid by a shock wave. Prikl. Mat. Meh. **24** (1960), 240–248 (Russian); translated as J. Appl. Math. Mech. **24**, 344–356.

The effect of the shock wave is represented by a prescribed pressure distribution acting on the free surface of the fluid. The deformation of the free surface and gravity are neglected. A number of self-similar solutions are studied: (1) The plane problem of a shock wave moving perpendicular to an inclined wall, and encountering the free surface of a fluid. The fluid is bounded below by a sloping beach. The pressure behind the shock wave is assumed either constant, or varying according to a certain law. (2) The plane problem of a cylindrical region of pressure spreading from a horizontal axis lying above a fluid of infinite depth. (3) The corresponding three-dimensional problem of a spherical region of pressure spreading from a point, for two different laws of pressure.

F. Ursell (Cambridge, England)

8918:

Glauert, M. B. The method of images in shear flow. J. Fluid Mech. **9** (1960), 561–580.

Author's summary: "The method of images is used to determine the flow perturbation caused by a small point source of fluid in a stream containing a vortex sheet, a number of vortex sheets, or a continuous two-dimensional distribution of vorticity. Calculations are carried out for a variety of such shear flows, to illustrate the range of applicability of the method.

"The image source distribution provides an alternative approach to that given by Lighthill [same J. **3** (1957), 113–144; MR **19**, 1217] for calculating small disturbances to a parallel shear flow, and suggests a simple physical picture by which to interpret the complicated effects associated with the distortion and stretching of vortex lines." *K. Stewartson* (Durham)

8919:

Taylor, G. I. Deposition of a viscous fluid on a plane surface. J. Fluid Mech. **9** (1960), 218–224.

The theory of the deposition of a viscous fluid as a thin

film on a plane is developed, both for a "mathematical paint-brush", i.e., a block with semicircular grooves on the bottom and charged with a viscous liquid, and for a paint roller. Effects of surface tension are neglected. Experimental measurements of the thickness of the deposited film were in fair agreement with theoretical estimates except for the film of water deposited by the roller. It is shown that the effects of surface-tension are not negligible when the working fluid is water.

A. A. Townsend (Cambridge, England)

8920:

Bloom, Martin H. On moderately separated viscous flows. J. Aerospace Sci. **28** (1961), 339–340.

8921:

Kanwal, R. P. Slow steady rotation of axially symmetric bodies in a viscous fluid. J. Fluid Mech. **10** (1961), 17–24.

The author considers the Stokes flow problem for an axially symmetric body rotating at constant angular velocity about its axis of symmetry. The non-zero velocity components are related to the polarization potential due to a grounded conductor (of the same shape as the body) in a uniform field directed at right angles to the axis of symmetry. Solutions are given for the spindle, the torus, and the lens. The angular velocity and the drag torque are computed in each case.

L. E. Payne (College Park, Md.)

8922:

Jonáš, P. Die Integralbeziehungen bei der stationären laminaren Strömung mit geschlossenen Stromlinien in annähernd kreisförmigen Gebieten. Czechoslovak J. Phys. **10** (1960), 737–741. (Russian summary)

The author considers the following configuration corresponding to a steady flow at large Reynolds' number: a two-dimensional region bounded by a nearly circular streamline Γ, a boundary layer M bounded by Γ and by a nearly circular inner streamline Γ_0, and an inner core J bounded by Γ_0 in which the vorticity ω_0 is constant. For this configuration he derives the integral relations

$$\oint_\Gamma (\partial w/\partial n)_\Gamma \, ds = \omega_0 \oint_\Gamma ds,$$

$$\psi_0 = K \oint_\Gamma [U^2 - u^2] \, ds,$$

where ψ_0 = flux through M, U = speed on Γ_0, u = speed on Γ, and $K^{-1} = -2\omega_0 \oint_\Gamma ds$.

It seems to the reviewer that in a steady flow, the assumptions used to derive these equations will be verified only in the case that all streamlines are for all practical purposes circular. In this case the unique solution of the (exact) Navier-Stokes equations will be a rigid rotation of the region, a flow configuration which differs markedly from the one presupposed in this paper. The author's result might, however, serve as a good approximation near the initial time for the unsteady flow obtained by rapidly setting a circular boundary into rotational motion.

R. Finn (Stanford, Calif.)

8923:

Proudman, Ian. An example of steady laminar flow at large Reynolds number. J. Fluid Mech. **9** (1960), 593–602.

The flow considered takes place in a two-dimensional channel with porous walls through which fluid is either injected or extracted at a uniform rate. The governing equations thus have a solution in which v, the velocity component perpendicular to the walls, is a function of distance from the wall only. It is shown that in the limit of vanishing viscosity the flow corresponding to this solution can be divided into, at most, two inviscid cells. Boundary layers occur at any wall through which fluid is being extracted and a shear layer occurs in the fluid along a line where $v=0$ if, and only if, fluid is being injected through both walls.

The paper is an elegant illustration of the power of arguments based on limiting processes.

K. Stewartson (Durham)

8924:

Schmeidler, Werner. Die Navier-Stokesschen Gleichungen der Hydrodynamik als System quadratischer Integralgleichungen. Math. Z. **72** (1959/60), 279–293.

L'A. studia il problema misto per il sistema di Navier-Stokes:

$$v\Delta \mathbf{u} - \frac{\partial \mathbf{u}}{\partial t} = \frac{1}{\rho} \operatorname{grad} p + \sum_i \left(u_i \frac{\partial}{\partial x_i} \right) \mathbf{u}, \quad \operatorname{div} \mathbf{u} = 0.$$

Sia G_t un dominio tridimensionale (in generale variabile al variare di t); i valori delle funzioni incognite $\{\mathbf{u}, p\}$ vengono assegnati in G_0 per $t=0$, e sulla frontiera di G_t per $t>0$. L'A. premette la costruzione della funzione di Green per l'operatore (del calore) $v\Delta - \partial/\partial t$, quindi traduce il problema in un sistema integrale, che viene trattato col metodo delle approssimazioni successive. Al recensore non sembrano sufficientemente chiari questi punti: (a) la compatibilità tra i valori assegnati per $\mathbf{u}$ e per p, (b) se il procedimento è effettivamente atto a garantire l'esistenza e unicità "in grande" cioè in un intervallo $0 \leq t \leq \tau$ prefissato.

G. Prodi (Trieste)

8925:

Manohar, R. A characteristic difference method for the calculation of steady boundary-layer flow. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 135–144. Indian Soc. Theoret. Appl. Mech., Kharagpur.

A considerable part of this paper is devoted to discussing different methods of computing incompressible two-dimensional laminar boundary-layer flow ("approximate" methods based on momentum or energy theorems and "exact" methods based on finite difference methods).

The author's own method is based on using a stream function Ψ dependent on the coordinate x along the wall and y normal to the wall. By introducing a new coordinate normal to the wall $\eta = y(Re)^{1/2}/g(x)$ and expressing Ψ by F through $\Psi = G(x)F(x, \eta)$ the author obtains a nonlinear partial differential equation of third order for F. In this equation derivatives with respect to x are replaced by finite differences and a nonlinear ordinary differential equation for $F(x_r, \eta)$ is obtained in which the coefficients depend on known functions at stations x_{r-1}, x_{r-2} and so on. This nonlinear ordinary differential equation has to be solved by an iterative process, because boundary conditions are given at the wall and at the outer edge and the integration starts at the wall. More details are given in the author's Ph.D. thesis [Göttingen, 1957], which contains various examples.

I. Flügge-Lotz (Stanford, Calif.)

8926:
Nickel, Karl. Eine Vereinfachung des v. Kármán-Pohlhausen-Verfahrens für Grenzschichten an Rotationskörpern. Z. Angew. Math. Mech. **40** (1960), 510–511.

8927:
Petuhov, I. V. A numerical solution of the laminar boundary-layer equations. Dokl. Akad. Nauk SSSR **132** (1960), 307–310 (Russian); translated as Soviet Physics. Dokl. **5**, 489–492.

Consider a plane, steady boundary layer flow of an incompressible fluid along a surface. In the usual formulation of this problem, the asymptotic boundary condition for the transverse component of the flow can in general not be satisfied. If all asymptotic boundary conditions are to be satisfied exactly, one has to assume that the surface of the body is displaced proportionally to a finite boundary layer thickness. This approach is proposed by the author. By satisfying all asymptotic boundary conditions, the boundary layer equations far from the body will be asymptotically equal to the linear heat-conduction equation. This equation does not depend on conditions at the surface. After employing a variable-transformation which is essentially equivalent to one used by H. Görtler [J. Math. Mech. **6** (1957), 1–66; MR **18**, 843], the author points out how the appropriate solution of this heat conduction equation can be used as external boundary condition for a finite inner region of the boundary layer. No proofs, details or numerical applications are given.

W. C. Rheinboldt (Syracuse, N.Y.)

8928:
Hama, Francis R.; Nutant, John. Self-induced velocity on a curved vortex. Phys. Fluids **4** (1961), 28–32.

Authors' summary: "By means of examples the distribution of self-induced velocity along a curved vortex is investigated analytically as well as by means of an analog experiment of the magnetic field of a current-carrying cable. Results verify, at least for these examples, a theorem which was intuitively introduced by Hama in his study of detailed process of boundary-layer transition; the induced velocity on a curved vortex is largest in the region where the curvature is largest."

8929:
Hassan, H. A. On skin friction in the slip-flow regime. J. Aerospace Sci. **28** (1961), 335–336.

8930:
Bush, William B. A method of obtaining an approximate solution of the laminar boundary-layer equations. J. Aerospace Sci. **28** (1961), 350–351.

8931:
Spiegel, Edward A. The convective instability of a radiating fluid layer. Astrophys. J. **132** (1960), 716–728.

The stability of a thin horizontal layer of gray radiating fluid is explored. For a constant adverse temperature gradient, the author determines the integro-differential equations for marginal stability and a useful variational formulation for the effective critical Rayleigh number. The values found for this critical number determine the extent to which the onset of convection is inhibited by the radiation. *W. V. R. Malkus* (Los Angeles, Calif.)

8932:
Rota, Gian-Carlo. Endomorphismes de Reynolds et théorie ergodique. C. R. Acad. Sci. Paris **250** (1960), 2791–2793.

Soit R un opérateur linéaire continu dans un espace de Banach $L_\infty(S, \Sigma, \mu)$ sur un espace de mesure (S, Σ, μ) à mesure finie. Si $f \geq 0$ entraîne $Rf \geq 0$, et si $R(fg) = RfRg + R[(f - Rf)(g - Rg)]$, f, $g \in L_\infty$, R est dit opérateur de Reynolds. L'espérance mathématique conditionnelle fournit de tels opérateurs. On en obtient d'autres par l'intermédiaire de certaines transformations de la forme $f(s) \to (Vf)(s) = f(\varphi(s))$, en posant $R = z(zI - \log V)^{-1}$, où z est réel et où e^z n'appartient pas au spectre de V dans L_∞. On examine plus particulièrement le cas où R a un spectre ponctuel et où les combinaisons linéaires des fonctions propres généralisées (solutions de $(2I - R)^n f = 0$) sont faiblement denses. On démontre qu'on peut décomposer l'espace S en deux ensembles disjoints S_1 et S_2 tels que: (a) si R_1 est l'opérateur défini par $f \to Rf$ dans S_1, $f \to 0$ dans S_2, la restriction de R_1 à $L_\infty(S_1)$ soit un opérateur de Reynolds; (b) si $f = 1$ dans S_1, $f = 0$ dans S_2, on ait $R_1 f = f$; (c) si $f \geq 0$, $Rf = 0$, le support de f soit contenu dans S_2, et réciproquement. Enfin il y a sur S_1 une mesure μ_1, invariante par R_1 et équivalente à μ. *J. Bass* (Paris)

8933:
Rota, Gian-Carlo. Représentation des opérateurs de Reynolds. C. R. Acad. Sci. Paris **250** (1960), 2831–2833.

Suite de l'étude des opérateurs de Reynolds à spectre ponctuel [voir l'analyse ci-dessus]. L'opérateur de Reynolds R est dit simple quand la seule fonction invariante est la fonction égale à 1 presque partout. Tout opérateur R dans $L_2(S, \Sigma, \mu)$ est une intégrale directe d'opérateurs simples. On peut d'ailleurs toujours se ramener au cas où R est un opérateur normal dans $L_2(S, \Sigma, \mu)$. On peut alors décomposer S en deux ensembles disjoints S' et S'' tels que: (a) $L_2(S')$ et $L_2(S'')$ soient invariants par R; (b) la restriction de R à $L_2(S')$ soit une espérance mathématique conditionnelle; (c) $S'' = I \times S'''$, où I est l'intervalle $[0, 1]$ avec la mesure de Lebesgue et où S''' est un espace à mesure finie. Si $f(t, s)$ est une fonction à valeurs complexes, $t \in I$, $s \in S'''$, on peut l'étendre en une fonction périodique de t de période 1, et il existe un nombre réel r tel que

$$Rf(t, s) = \frac{r}{e^r - 1} \int_t^{1+t} e^{r(u-t)} f(u, s)\, du.$$

J. Bass (Paris)

8934:
Rajaratnam, N. Direct solution for diameter of pipe in rough turbulent flow. Houille Blanche **15** (1960), 714–719. (Text also in French)

A semi-graphical inversion of the empirical formulae relating pressure loss to pipe diameter in turbulent flow.

W. V. R. Malkus (Los Angeles, Calif.)

8935:

Kraichnan, Robert H. A theory of turbulence dynamics. 2nd Sympos. Naval Hydrodynamics (Washington, D.C., 1958), pp. 29–44. U.S. Govt. Printing Office, Washington, D.C., 1960.

This paper is a qualitative study of the foundations of the author's approximation technique for dealing with non-linear equations. It is a useful adjunct to his paper on isotropic turbulence [J. Fluid. Mech. **5** (1959), 497–543; MR **21** #4697]. *W. V. R. Malkus* (Los Angeles, Calif.)

8936:

Kraichnan, Robert H. Dynamics of nonlinear stochastic systems. J. Mathematical Phys. **2** (1961), 124–148.

Random couplings are introduced among an infinite collection of similar systems. The true non-linear problem corresponds to the limit where these couplings vanish. The technique of solution consists of reducing the coupling between the systems in a formal sequence which appears to converge rapidly. The first step in this sequence is identical with the "direct interaction approximation" introduced by the author in his study of isotropic turbulence [Phys. Rev. (2) **109** (1958), 1407–1422; MR **19**, 1221]. In contrast to "cumulant discard" approximation techniques, this procedure retains at each step the positive definite character of such quantities as energy and probability density. The procedure is applied to a linear oscillator with a random frequency parameter, to the Schrödinger equation of a particle in a random potential and to Burgers' analog for turbulent dynamics.

W. V. R. Malkus (Los Angeles, Calif.)

8937:

Ledoux, P.; Schwarzschild, M.; Spiegel, E. A. On the spectrum of turbulent convection. Astrophys. J. **133** (1961), 184–197.

The energy input from the buoyancy forces in an unstable layer is assessed by deriving the growth rates from the linearized equations. The exchange of energy between modes is assumed to follow Heisenberg's heuristic decay law. A closed solution is found for the spectrum in a thin layer of constant temperature gradient.

W. V. R. Malkus (Los Angeles, Calif.)

8938:

Friedlander, S. K.; Topper, Leonard (Editors). ★Turbulence: Classic papers on statistical theory. Interscience Publishers, New York-London, 1961. ix + 187 pp. $6.00.

A set of 12 papers by G. I. Taylor, L. F. Richardson, T. Von Kármán, L. Howarth, H. L. Dryden, A. N. Kolmogoroff and C. C. Lin published between 1921 and 1949.

8939:

Yuan, S. W.; Brogren, E. W. Turbulent flow in a circular pipe with porous wall. Phys. Fluids **4** (1961), 368–372.

By the use of the mixing length formalism, and a similarity assumption, the Reynolds equations for the mean velocity of isothermal turbulent flow in a circular pipe with exponentially varying injection or suction at the wall have been reduced to a non-linear ordinary differential equation of third order. The first order perturbation solution to this equation has been obtained, perturbing about the condition of zero suction. This solution is simplified by recourse to experimental data, and is then used to obtain a second order perturbation solution. The simplified form of this solution is compared with experiment, permitting the calculation of the mixing length proportionality constant, and constants related to the axial pressure gradient. Using these constants, good agreement between calculated and measured axial velocity profiles is obtained over a range of injection ratios. For small values of injection or suction velocity, or close to the origin, the exponential behavior is close enough to constant to permit comparison with experiments for which the suction or injection was constant.

J. L. Lumley (University Park, Pa.)

8940:

Липман, Г. В. [Liepmann, H. W.]; Рошко, А. [Roshko, A.]. ★Элементы газовой динамики. [Elements of gasdynamics]. Translated from the English by V. P. Šidlovskiĭ; edited by È. I. Grigolyuk. Izdat. Inostr. Lit., Moscow, 1960. 518 pp. 23.20 r.

For the original [Wiley, New York, 1957] see MR **19**, 1121. The present edition contains a brief preface and footnotes by the editor, and the bibliography has been supplemented by references to Russian and recent literature.

8941:

Kleĭman, Ya. Z. On the method of characteristics in the theory of motion of a multicomponent medium. Prikl. Mat. Meh. **23** (1959), 391–394 (Russian); translated as J. Appl. Math. Mech. **23**, 551–555.

The characteristics and the compatibility relations across the characteristic surfaces for a multi-component medium are derived. The medium considered is a fluid mixture consisting of several components, each of which possesses a velocity and density of its own and occupies a part of the space of the gross mixture, but all subjected to the same pressure. The following assumptions are made. (1) Each component behaves like an inviscid fluid. (2) Interaction of any two components of the mixture takes place through an interacting force which depends on the relative velocities of the two components. (3) The density of each component is a function of pressure only. Depending on the densities and relative velocities of the different components, the characteristics may be either all real and without multiplicity or all real with some multiplicity or partly real and partly imaginary. The case of a two-component mixture is discussed in detail. {Reviewer's remark: The third assumption is, strictly speaking, incompatible with the second assumption. The results of the paper are, therefore, applicable only in those instances where these assumptions are approximately compatible, e.g., in an isothermal flow where the conductivity of the various components is very high, or in an isentropic flow when the entropy production per unit mass due to the interacting forces and conduction is very small in comparison with the specific heats of the various components.}

B. T. Chu (Providence, R.I.)

8942:

Karlikov, V. P. The general theory of axially symmetric motion of a gas. Dokl. Akad. Nauk SSSR **133** (1960), 1049–1052 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 694–697.

The author considers the axial-symmetric non-steady motion of a perfect gas. Solutions of the equations of motion, energy and continuity are sought in terms of powers of the radial distance, r, of spherical coordinates. The coefficients of r^n are functions, f_n, g_n, h_n, q_n, of time, t, and the colatitude, θ; the dependent variables are the velocity components, v_r, v_θ and p, ρ, the pressure and density, respectively. Recurrence relations among f_n, g_n, h_n, q_n are studied. In particular, f_0, g_0, h_0, q_0 and f_1, g_1, h_1, q_1, are explicitly determined for a general class of flows. *N. Coburn* (Ann Arbor, Mich.)

8943:

Szaniawski, Andrzej. Certains problèmes de la dynamique du gaz parfait contenant des molecules qui peuvent se decomposer. Arch. Mech. Stos. **12** (1960), 483–496. (Polish and Russian summaries)

8944:

Gadd, George E. The possibility of normal shock waves on a body with convex surfaces in inviscid transonic flow. Z. Angew. Math. Phys. **11** (1960), 51–58. (German summary)

By assuming certain regularity properties on the derivatives of flow quantities it has been shown that in an inviscid flow with Mach number close to 1 a shock normal to a convex surface cannot occur. The author shows that if some reasonable logarithmic singularities are admitted the argument breaks down. This suggests that by an appropriate asymptotic expansion one might determine the local behavior of a normal shock.

C. S. Morawetz (New York)

8945:

Bïibosunov, I. An example of plane-parallel transonic flow with a curved density step terminating within the flow and having a stream function of the form $\psi_{2/3} = \rho^{2/3} f_{2/3}(\theta/\rho)$. Dokl. Akad. Nauk SSSR **126** (1959), 951–954 (Russian); translated as Soviet Physics. Dokl. **4**, 532–536.

The author generalizes Frankl's flow, representing an almost sonic shock locally in the hodograph plane. He claims that the flow can be mapped into the physical plane by generalizing Frankl's proof.

C. S. Morawetz (New York)

8946:

Ryzhov, O. S. On gas flow in Laval nozzles. J. Appl. Math. Mech. **22** (1958), 549–553 (396–398 Prikl. Mat. Meh.).

Some simple analytical solutions of the transonic equation are found. They are interpreted in terms of flow near the sonic line in nozzles with two planes of symmetry. *D. C. Pack* (Glasgow)

8947:

Lighthill, M. J. Dynamics of a dissociating gas. II. Quasi-equilibrium transfer theory. J. Fluid Mech. **8** (1960), 161–182.

Part I [same J. **2** (1957), 1–32; MR **19**, 352] gave a valuable introduction, from the aerodynamicist's viewpoint, to the dynamics of a pure dissociating gas in equilibrium. Here the same service is performed for gas flows in near equilibrium. (A projected part III treating large departures from equilibrium is abandoned as no longer necessary.) Diffusive transport properties are discussed in the first half, and radiative transport in the second. The application always in mind is to the incandescent boundary layer on a projectile in high-speed flight. The two papers provide an admirable point of departure for the gasdynamicist concerned with real gas effects. *M. D. Van Dyke* (Stanford, Calif.)

8948:

Marini, Marino. Corrente supersonica linearizzata in un getto o in un tubo quasi cilindrico. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **28** (1960), 802–806.

8949:

Reyn, J. W. Differential-geometric considerations on the hodograph transformation for irrotational conical flow. Arch. Rational Mech. Anal. **6**, 299–354 (1960).

In a conical flow the velocity and physical state of the gas depend only on two angular space coordinates, and in general there is a relation between the three velocity components u, v, w. When the flow is isentropic and irrotational this relation is expressible as a non-linear equation of the second order (hodograph equation) for w as a function of u, v. This paper studies the differential geometry of solution-surfaces of this equation, in relation to concepts that are of physical interest and in relation to the corresponding surfaces in the physical space; for example the relations between conical limit-lines, streamlines and characteristics are discussed, and the limiting case of conical simple waves is considered. The investigation is essentially of local properties only, and its significance for the problem of actually determining conical flow seems to be merely qualitative. This is illustrated by a discussion of supersonic flow past a delta wing, in the case where the leading edges are supersonic.

T. M. Cherry (Melbourne)

8950:

Žigulev, V. N.; Žilin, Yu. L. On bodies of minimum wave drag. Prikl. Mat. Meh. **23** (1959), 1019–1029 (Russian); translated as J. Appl. Math. Mech. **23**, 1462–1475.

Linearized supersonic flow theory showed a flurry of renewed activity following Nikol'skiĭ's disclosure in 1956 of a method of calculating the minimum drag of a body under various isoperimetric conditions. (More precisely, a lower bound is found for the drag, which may not be attainable with a real body of positive thickness.) Whereas previous work was restricted to special shapes, the present analysis is relatively general. Application is made to ducted bodies of revolution and to wing-body combinations. *M. D. Van Dyke* (Stanford, Calif.)

8951:
Naylor, D. A non-linear boundary value problem in hypersonic gas dynamics. J. Math. Mech. **9** (1960), 665–680.

Plane flow is considered past a finite wedge followed by a parallel section of indefinite extent. The hypersonic small-disturbance approximation is adopted, and the adiabatic exponent γ taken as unity, which renders the flow effectively isentropic. (This is justified by appeal to the fact that flow past a cone is insensitive to the value of γ, but one could object that the sensitivity is much greater for the wedge.) Characteristic variables are introduced, and Riemann's method leads to a Volterra integral equation for the shape of the shock wave. An approximate solution is given, showing the ultimate decay to a Mach wave. *M. D. Van Dyke* (Stanford, Calif.)

8952:
Godunov, S. K. A difference method for the calculation of shock waves. Amer. Math. Soc. Transl. (2) **16** (1960), 389–390.

Translation of Uspehi Mat. Nauk **12** (1957), no. 1 (73), 176–177 [MR **19**, 353].

8953:
Hsu, Cheng-Ting. On the gradient functions for non-equilibrium dissociative flow behind a shock. J. Aerospace Sci. **28** (1961), 337–339.

8954:
Szaniawski, Andrzej. Certaines relations existant entre les dérivées partielles des deux côtés de l'onde de choc. Apl. Mat. **5** (1960), 341–351. (Czech and Russian summaries)

Resumé de l'auteur: "On trouve les équations pour les valeurs limites des dérivées partielles des paramètres du gaz des deux côtes de l'onde de choc, qui permettent de déterminer l'approximation du mouvement au voisinage de l'onde." *J. P. Guiraud* (Meudon)

8955:
Ram, G. S.; Ribner, H. S. ★The sound generated by interaction of a single vortex with a shock wave. Heat transfer and fluid mechanics institute, held at California Institute of Technology, Pasadena, Calif., June, 1957, pp. 1–22. Stanford University Press, Stanford, Calif. $7.50.

Authors' summary: "The passage of a columnar vortex 'broadside' through a shock is investigated. The vortex is decomposed (by Fourier transform) into plane sinusoidal shear waves disposed with radial symmetry. The plane sound waves produced by each shear wave-shock interaction, known from previous work, are recombined in the Fourier integral. The waves possess an envelope that is essentially a growing cylindrical sound wave, partly cut off by the shock. The sound wave is centered at the transmitted (and modified) vortex and attenuates inversely as the square root of the growing radius. The strength varies smoothly around the arc, from compression at one shock intersection to rarefaction at the other shock intersection. Comparison is made with results of a shock-tube investigation and heuristic theory by Hollingsworth and Richards in England." *H. C. Levey* (Nedlands)

8956:
Ôno, Yôrô; Sakashita, Shirô; Yamazaki, Hatuo. Propagation of shock waves in inhomogeneous gases. I. Progr. Theoret. Phys. **23** (1960), 294–304.

Les auteurs étudient la propagation rectiligne d'un choc dans un milieu homogène par une généralisation de la méthode de Chisnell [Proc. Roy. Soc. London Ser. A **232** (1955), 350–370; MR **17**, 426]. Cette méthode, qui consiste à séparer le fluide en régions de pression et masse spécifique constantes, séparées par une discontinuité infiniment petite de masse spécifique, est étendue au cas d'un fluide soumis à des forces de masse, l'état initial étant celui d'un gaz polytropique. L'intensité du choc qui se propage est calculée en fonction de la masse spécifique avant le choc; la relation obtenue est indépendante des forces de masse. Une application au cas de l'astrophysique est indiquée. *H. Cabannes* (Paris)

8957:
Ôno, Yôrô; Sakashita, Shirô; Yamazaki, Hatuo. Propagation of shock waves in inhomogeneous gases. II. Hydromagnetic shock. Progr. Theoret. Phys. **24** (1960), 155–160.

Utilisant la méthode établie dans la première partie [#8956], les auteurs étudient la propagation d'un choc hydromagnétique dans un milieu non homogène. Par comparaison avec le cas de l'aérodynamique classique, la présence d'un champ magnétique diminue l'accroissement de l'intensité du choc, dans certains cas la présence du champ magnétique peut même diminuer l'intensité du choc. *H. Cabannes* (Paris)

8958:
Ôno, Yôrô. Propagation of shock waves in inhomogeneous gases. III. Spherical shock waves. Progr. Theoret. Phys. **24** (1960), 825–828.

L'auteur étend au cas des ondes de choc sphériques les résultats antérieurs [voir ci-dessus]. La mème méthode conduit à une équation différentielle qui détermine l'intensité du choc en fonction de la distance au centre de la sphère. L'amortissement est surtout prépondérant près du centre ce qui fait que les modèles gazeux de ce type (étoiles) sont stables. *H. Cabannes* (Paris)

8959:
Fowles, G. R. Attentuation of the shock wave produced in a solid by a flying plate. J. Appl. Phys. **31** (1960), 655–661.

The shock wave in the plate is reflected as a rarefaction wave at the rear of the plate. The attenuation of the shock wave in the solid by this rarefaction wave is calculated by the method of Riemann invariants on the assumption that dissipative processes and the effects of material rigidity may be neglected. *D. C. Pack* (Glasgow)

8960:
Powell, Alan. Concerning the noise of turbulent jets. J. Acoust. Soc. Amer. **32** (1960), 1609–1612.

A similarity argument is given to explain the surprising precision with which the acoustic power radiated from an air or hot gas jet varies as the eighth power of jet exit velocity over a range extending well into the supersonic regime. Refraction effects across the jet and convection effects along it are considered.

W. W. Soroka (Berkeley, Calif.)

8961:

Carleton, Herbert R. Theoretical development of volume reverberation as a first-order scattering phenomenon. J. Acoust. Soc. Amer. **33** (1961), 317–323.

Author's summary: "A model for acoustic scattering of an arbitrary pressure wave is developed from the fundamental wave equation. A statistical distribution of scatterers in the medium is assumed and is described by a spatial correlation function. It is shown that the predominant features of volume reverberation can be derived from the model. Furthermore, under certain simplifying conditions, it is shown that the reverberation signal can be represented as a random noise passed through a narrow band filter whose properties are determined by the mode of transmission. The determination of the dependence of reverberation level on operating frequency, and the possibility of optimizing an acoustic system by the choice of transmission is suggested."

J. W. Miles (Los Angeles, Calif.)

8962:

Sherwood, J. W. C. Transient sound propagation in a layered liquid medium. J. Acoust. Soc. Amer. **32** (1960), 1673–1684.

A theoretical analysis is given for the transient sound propagation between a source and receiver located at arbitrary points in a horizontally stratified liquid medium. The analysis starts with expressing the well known retarded-time solution of the pressure field as an integral superposition of cylindrical waves. With the cylindrical axis taken perpendicular to the stratified boundaries, this integral representation of the pressure field provides a convenient evaluation of the reflection and transmission of the waves at an interface between two layered fluids, yielding Snell's law of reflection. The theory is further developed on the basis of a detailed ray propagation for the general case of multilayered medium. The pressure variation at the receiver, up to some finite time t, may be determined by evaluating all the reflections and refractions that have taken place preceding t. The employment of a large digital computer for the numerical computation of the problem is also discussed.

T. Yao-tsu Wu (Pasadena, Calif.)

8963:

Wenz, Gordon M. Some periodic variations in low-frequency acoustic ambient noise levels in the ocean. J. Acoust. Soc. Amer. **33** (1961), 64–74.

Time series are generated in which the variable is a noise level averaged over frequency and time and their spectral density functions are estimated by Tukey's techniques.

W. Freiberger (Providence, R.I.)

8964:

Schuster, K. Zur Theorie der Schallausbreitung in einem durchströmten Kanal. Acustica **10** (1960), 326–329. (French and English summaries)

Author's summary: "Solutions of Navier-Stokes equations are discussed which correspond to the superposition of sound waves with laminar flow in a duct with two plane, rigid walls. Sound waves with a spatial exponential increase of amplitude are found under certain conditions."

8965:

Tabuchi, Daisaku. Dispersion and absorption of sound in liquids in general chemical equilibrium and its application to chemical kinetics. J. Chem. Phys. **26** (1957), 993–1001.

8966:

Leibfried, G.; Ludwig, W. Zur Adiabasie der Schallausbreitung in Kristallen. Z. Physik **161** (1960/61), 475–485.

It is shown that the propagation of sound is an adiabatic process, and that the sound velocities are determined by the adiabatic elastic data and the density in the familiar way. The proof is analogous to that in simple monatomic gases and takes anharmonic effects fully into account.

Werner Nowacki (Bern)

8967:

Jones, J. R. Flow of a non-Newtonian liquid in a curved pipe. Quart. J. Mech. Appl. Math. **13** (1960), 428–443.

For an incompressible liquid with stress (p_{ik}) and rate-of-strain (e_{ik}) relation of the form

$$p_{ik} = -P\delta_{ik} + 2\eta e_{ik} + 4\zeta e_{ij}e_{jk},$$

where η, ζ are constants, the author discusses flow in a curved pipe whose central line has radius large in comparison with the radius of a cross-section of the pipe. An approximate solution is obtained and streamlines in the plane of symmetry and the projection of the streamlines on a normal section are compared with those of a Newtonian liquid.

A. E. Green (Newcastle-upon-Tyne)

8968:

Breus, S. N. Stability of a liquid cylinder carrying current for the case of finite conductivity. Ž. Tehn. Fiz. **30** (1960), 1030–1034 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 960–965.

Author's summary: "The stability of a current pinch in a magnetic field is investigated for the case in which no magnetic field is frozen inside the conductor. It is shown that in this case the instability is increased with an increase in the longitudinal magnetic field."

8969:

Ôki, Tosio. On the magnetohydrodynamic instability of a gaseous cylinder. Sci. Rep. Tôhoku Univ. Ser. I **43** (1959), 113–120.

In this paper the magnetohydrodynamic instability of an infinite gaseous cylinder, with a prevalent longitudinal magnetic field and a uniform electric current flowing parallel to the axis of the cylinder is investigated by using a perturbation method. The fluid is assumed to be infinitely conducting, but the gravitational effects and viscosity are neglected. The author has discussed the applicability of these results to the formation of star chains from inter-stellar gas clouds.

F. C. Auluck (Delhi)

8970:

Korobeĭnikov, V. P.; Ryazanov, E. V. Some solutions of the equations of one-dimensional magnetohydrodynamics and their application to problems of shock wave propagation. Prikl. Mat. Meh. **24** (1960), 111–120 (Russian); translated as J. Appl. Math. Mech. **24**, 144–158.

A survey is made of cases of integrability of the equations of plane and (in greater detail) cylindrically-symmetric unsteady motion of a perfect electrically-conducting gas. For infinite conductivity, a general solution is given for steady flow, and self-similar motions are calculated corresponding to motion of a piston according to a power of time. A class of solutions is considered for which the velocity varies linearly with radius, and a method given for joining to gas at rest through a shock wave. Specific examples are intended to represent gaseous discharge.

M. D. Van Dyke (Stanford, Calif.)

8971:

Germain, P. Contribution à la théorie des ondes de choc en magnétodynamique des fluides. O. N. E. R. A. Publ. No. 97 (1959), 33 pp. (2 plates).

A very general theory for magnetohydrodynamic shock structure is developed under the following hypotheses. (1) There is one fluid with a general equation of state satisfying Weyl's conditions. (2) The dissipative effects are produced by electrical conductivity, viscosity, and heat conductivity.

The problem is to find a steady solution of the equations of motion and Maxwell's equations which depend on only one space variable x and such that the fields and flow quantities are prescribed at, say, $x = -\infty$.

The equations are reduced to four ordinary differential equations with four singular points. From the general properties of the solution of these equations the author concludes the existence of fast, intermediate, and slow shocks. However, if a shock is defined as stable if the solution tends to a limit as the dissipation coefficients tend to zero independently then only the fast and slow shocks are stable.

C. S. Morawetz (New York)

8972:

Kanwal, R. P.; Truesdell, C. Electric current and fluid spin created by the passage of a magnetosonic wave. Arch. Rational Mech. Anal. **5**, 432–439 (1960).

The authors present a general and elegant theory of magnetohydrodynamic waves by means of the method of singular surfaces. A theorem corresponding to the Hugoniot-Hadamard theorem for a compressible non-magnetic fluid is derived for the case of Alfven waves. This theorem shows that in an incompressible conducting fluid only the following kinds of singular surfaces are possible: (a) material vortex-current sheets which are everywhere tangent to the lines of magnetic force; (b) transverse waves carrying jumps of current density and vorticity. A systematic classification of singular surfaces together with their interesting properties is given. As the main result the authors show that no wave carrying a condensation can propagate along the magnetic field. In the general case that the wave-front is not tangent to the magnetic field there are four possible waves: one pair travels at supersonic speed; the other at subsonic. The general theory also serves to exhibit a new means of generating vorticity in fluid motions.

R. S. B. Ong (Leiden)

8973:

Geršuni, G. Z.; Žuhovickiĭ, E. M. Flow of a conducting fluid around a sphere in a strong magnetic field. Ž. Tehn. Fiz. **30** (1960), 925–926 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 870–871.

The corresponding problem of flow in weak fields was solved by Chester [J. Fluid Mech. **3** (1957), 304–308; MR **19**, 801], who gave the drag force as a series in ascending powers of the Hartmann number M. In strong fields, $M \gg 1$, the solution takes a simple asymptotic form indicating the presence of a boundary layer. The drag force takes the simple form

$$\frac{F}{F_0} = \frac{4}{9} M,$$

where F_0 is the Stokes drag force in the absence of a field.

K. C. Westfold (Clayton)

8974:

Srivastava, A. C. The effect of magnetic field on the flow between two non-parallel planes. Proc. Sympos. Non-Linear Phys. Probl. (Roorkee, 1959), pp. S.79–S.84. Indian Soc. Theoret. Appl. Mech., Kharagpur.

A solution of a two-dimensional steady flow problem in a sector ($|\theta| \leq \alpha$). The fluid is resistive and viscous, the field purely azimuthal, $H_\theta = 2c/r$. It is shown that application of the magnetic field is stabilizing in the sense that purely divergent flow becomes possible for increased values of α.

A. A. Blank (New York)

8975:

Auluck, F. C.; Kothari, D. S. The influence of a magnetic field on the longitudinal stability of a gravitating cylinder. Z. Astrophys. **42** (1957), 101–113.

The paper deals with the problem of longitudinal stability of an infinitely long gravitating cylinder of incompressible and inviscid fluid having infinite electrical conductivity. The authors consider the following four initial magnetic field configurations in cylindrical coordinates (r, θ, z): (i) $\mathbf{H}^{(i)} = [0, 0, \frac{1}{2}K(r^2 - R_0^2)]$; $\mathbf{H}^{(e)} = (0, 0, 0)$; (ii) $\mathbf{H}^{(i)} = [0, 0, \frac{1}{2}Kr^2]$, $\mathbf{H}^{(e)} = [0, 0, \frac{1}{2}KR_0^2]$; (iii) $\mathbf{H}^{(i)} = [0, 0, H_1]$, $\mathbf{H}^{(e)} = [0, 0, H_2]$; (iv) $\mathbf{H}^{(i)} = [0, Kr, 0]$, $\mathbf{H}^{(e)} = [0, KR_0^2/r, 0]$; where the superscripts (i) and (e) denote internal and external magnetic field respectively, K, H_1, H_2 are constants and R_0 is the undisturbed radius of the cylinder. The case (iii) is a generalization of Chandrasekhar-Fermi model [S. Chandrasekhar and E. Fermi, Astrophys. J. **118** (1953), 116–141; MR **15**, 168] in which $H_2 = 0$. In discussing this model the authors take into account the surface currents arising due to the discontinuity of the magnetic field. Using the energy method they prove that in all these cases, the presence of the magnetic field increases the stability of the cylinder.

P. L. Bhatnagar (Bangalore)

8976:

Lu, Pau-Chang. Free-convection magnetohydrodynamic flow past a porous flat plate. J. Aerospace Sci. **28** (1961), 346–347.

8977:

Shmoys, J.; Mishkin, E. Hydromagnetic waveguide with finite conductivity and arbitrary cross section. Phys. Fluids **3** (1960), 473–475.

Authors' summary: "Low pressure magnetohydrodynamic waveguides of arbitrary cross section, having a dc axial magnetic field, exhibit both TE and TM modes. The longitudinal electric field vanishes when the conductivity of the plasma is infinite, converting the TM modes into principal ones. The propagation constants for both modes are derived from the solutions of the eigenvalue problems. Linearized magnetohydrodynamic equations are used."

8978:

Bhatnagar, P. L.; Kumar, Ram. Propagation of small disturbances in viscous and electrically conducting liquids in the presence of magnetic field. Indian J. Math. **2**, 41–66 (1960).

The author studies the propagation of a small disturbance in a viscous liquid or an electrically conducting liquid in a constant magnetic field. The disturbance is produced by a circular disc rotating slowly about an axis perpendicular to it. Particular cases are computed.

C. S. Morawetz (New York)

8979:

Ergun, A. N. Axisymmetric solutions of the incompressible magnetohydrodynamic equations. Quart. J. Mech. Appl. Math. **13** (1960), 408–427.

Davies [same J. **13** (1960), 168–183; MR **22** #6330] has studied the axially symmetric flows of electrically conducting and compressible fluids in the absence of the dissipative effects of viscosity, heat conductivity and electrical resistance. He has proved that the system of equations governing such flows reduces to two equations for two functions of position. One of them is Bernoulli's equation and the other is the vorticity equation.

The present author analyses these two equations for the case of incompressible fluids. The discussion is mainly confined to the equation of vorticity. This equation is a non-linear partial differential equation of the second order in the Stokes stream function. Many particular solutions of this equation are considered under various simplifying assumptions. The singularity of this equation is discussed in some detail. *R. P. Kanwal* (University Park, Pa.)

8980:

Bhutani, O. P. Flow over a wavy wall in magneto-gasdynamics. I. Equilibrium flow. Progr. Theoret. Phys. **24** (1960), 721–733.

The hydromagnetic plane steady flow of an inviscid, infinitely conducting gas over a thin, nonconducting, infinite, wave-shaped wall is discussed in two special cases. Case I: The applied magnetic field is parallel to the undisturbed flow, while the Alfvén speed A_e is much smaller than the sound speed a_0. A linear perturbation analysis shows that the resulting flow is (to first order in A_e/a_0) irrotational provided the flow speed $U_\infty \neq A_e$. The flow is then found for the "subsonic" situation $A_e < U_\infty < a_0$ and the "supersonic" situation $U_\infty > a_0$ by separation of variables. Case II: The applied field is parallel to the wall but perpendicular to the flow, while the flow speed satisfies $U_\infty^{-2} < a_0^{-2} + A_e^{-2}$. The flow is found to consist of waves of electric current and vorticity extending from the body. The drag is determined in both cases.

H. C. Kranzer (Garden City, N.Y.)

8981:

Yoshihara, Hideo. Plasma flow over a thin charged conductor. J. Aerospace Sci. **28** (1961), 141–144, 157.

The flow of a dense plasma over a thin conductor with cusped edges is studied in a situation where the effect of magnetic field is negligible compared to electrostatic influences. The approach is through a boundary-layer analysis of the linearized equations governing flow over an infinite, conducting, sinusoidal wall with a given surface-charge distribution; the cusped conductor is then taken as one wavelength of the infinite wall with suitable endpoint modifications. It is found that the Coulomb drag vanishes identically, while the fluid-pressure drag corresponds to the usual Ackeret value for a neutral particle gas at a reduced Mach number $M_p = V_0/a_p$, where V_0 is the undisturbed fluid velocity and $a_p^2 = (a^+)^2 + (\omega_p^+)^2 R_D^2$, with a^+ the ion sound speed, ω_p^+ the ion plasma frequency, and R_D the Debye length.

H. C. Kranzer (Garden City, N.Y.)

8982:

Chang, I-Dee. Stokes flow of a conducting fluid past an axially symmetric body in the presence of a uniform magnetic field. J. Fluid Mech. **9** (1960), 473–477.

The author investigates the low Reynolds number flow of an incompressible fluid past an axially symmetric body in the presence of a uniform magnetic field. The usual perturbation method is employed. This consists of two types of expansions: an "inner" expansion valid in the neighborhood of the body and an "outer" expansion valid at large distances from the body. At some finite distance where the two expansions are both valid the boundary conditions are replaced by matching conditions. For small Hartmann numbers the author gives an approximate formula for the drag. Also a comparison is made of the drag of various bodies in a Stokes flow of a conducting fluid and that in a similar flow with no magnetic effect. In general there is an increase in the drag of the body due to the effect of the magnetic field.

R. S. B. Ong (Leiden)

8983:

Agostinelli, Cataldo. Figure ellissoidali di equilibrio relativo magneto idrodinamico di masse fluide elettricamente conduttrici uniformemente rotanti e gravitanti. Ann. Mat. Pura Appl. (4) **48** (1959), 193–208.

Cet article fait suite à un travail anterieur du même auteur [Boll. Un. Mat. Ital. (3) **14** (1959), 95–101; MR **21** #5393] dans lequel est étudié l'équilibre magnétohydrodynamique relatif d'une masse ellipsoïdale de révolution d'un fluide incompressible, parfaitement conducteur, en rotation uniforme autour d'un axe barycentrique. On mettait en évidence l'existence d'une fonction V, s'annulant sur la surface de l'ellipsoïde, solution de l'équation

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial V}{\partial r}\right)+\frac{\partial^2 V}{\partial z^2}+k^2 V = 0$$

en fonction de laquelle peuvent être calculés le champ magnétique et la pression à l'intérieur de la masse.

Le but du présent article est le calcul de V par la méthode de la séparation des variables en utilisant "un procédé analogue à celui qui permet de résoudre l'équation de Mathieu". On étudie en particulier le cas d'un ellipsoïde de faible excentricité en rotation lente.

J. Naze (Marseille)

8984:

Golicyn, G. S. Fluctuations of the magnetic field and current density in a turbulent flow of a weakly conducting fluid. Dokl. Akad. Nauk SSSR **132** (1960), 315–318 (Russian); translated as Soviet Physics. Dokl. **5**, 536–539.

The theory of the fluctuations of magnetic field produced in a weakly conducting fluid by its turbulent motion in an external magnetic field is developed for conditions of negligible self-induction and influence of the magnetic field on the turbulent motion. The conditions for this are that the scale of the phenomena should be small compared with $l_m=(\nu_m{}^3/\varepsilon)^{1/4}$ (ν_m is the magnetic diffusivity and ε is the rate of energy dissipation by the turbulence) and that $l_m>(H_0{}^2/4\pi\rho)^{3/2}/\varepsilon$ (H_0 is the external magnetic field). The structure function of the field fluctuations is then related to the structure function of the velocity fluctuations in terms of their Fourier transforms by

$$F_{ij}=\frac{\mu^2E(k)}{4\pi k^4}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)=\frac{E_H(k,\mu)}{4\pi k^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right),$$

where μ is the scalar products of the direction vectors of field and wave-number, and $E(k)$ is the scalar function determining the velocity spectrum. In the inertial subrange, the magnetic spectrum decreases as $k^{-11/3}$. Further results are obtained for the current field, and the application of the results to the ionosphere and to flow of mercury is discussed.

A. A. Townsend (Cambridge, England)

8985:

Uflyand, Ya. S. On some cases of nonequilibrium flow of a conducting liquid in an annular tube. Ž. Tehn. Fiz. **30** (1960), 799–802 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 753–757.

8986:

Dokučaev, V. P. On the growth of magnetohydrodynamic waves in a plasma stream moving through an ionized gas. Ž. Èksper. Teoret. Fiz. **39** (1960), 413–415 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 292–293.

Author's summary: "A dispersion equation is obtained which describes the propagation of electromagnetic waves in a plasma stream moving through an ionized gas. The low-frequency case (below the ion gyrofrequency), when the waves degenerate into magnetohydrodynamic waves, is considered in detail. It is shown that the system becomes unstable if the flow velocity exceeds the velocity of Alfvén waves in the stream-plus-stationary-plasma system. In this case one of the normal waves builds up with time."

8987:

Passell, T. O.; Perry, C. L. Upstream diffusion in Hagen-Poiseuille flow. Phys. Fluids **4** (1961), 444–447.

Authors' summary: "A theoretical study is described for steady state diffusion of a contaminant from a large reservoir of static fluid into a cylindrical tube discharging fluid into the reservoir. Contaminant decay is considered. Numerical results are presented for the rate of decrease along the tube of the concentration of contaminant for Reynolds × Schmidt numbers >10 and either small or zero decay. The method used explicitly exhibits quantities of physical interest including a simple form for the asymptotic solution. These results are of practical use in estimating the contamination in the tube at points that are relatively close to the mouth of the tube. An estimate is made of the distance from the tube mouth at which the asymptotic solution given becomes valid. Used for the prediction of heat flow, the results are valid for Péclet numbers >10."

8988:

Nikolaevskiĭ, V. N. Convective diffusion in porous media. Prikl. Mat. Meh. **23** (1959), 1042–1050 (Russian); translated as J. Appl. Math. Mech. **23**, 1492–1503.

The diffusion of dynamically inert tracer particles added to the flow of a fluid through an isotropic, homogeneous porous medium is examined. Averages over ensembles of porous media are used. The central limit theorem is imposed to obtain normality for the concentration distribution for diffusion times large compared to a time characteristic of the motion through the pore structure. A general form for the diffusion coefficient thus defined is derived, as an axisymmetric second rank tensor about the direction of mean flow. By considering flows in which inertia can be neglected, a random tensor transformation from the mean to the local velocity is defined (independent of mean velocity). The mean square fluctuating velocity is expressed in terms of averages of products of this tensor, to which symmetry and isotropy conditions are then applied. The relation is then made between this construction, and the diffusion coefficient tensor previously derived, introducing an integral scale. This permits interpretation of the invariants of the diffusion tensor in terms of properties of the porous medium. Similar results (but dependent on Reynolds number) are mentioned for flows in which inertia is also important. The possibility of taking simultaneous account of molecular diffusion is mentioned, using the assumption of no correlation between the molecular and convective diffusion. A scaling relation (subject to test) is given for diffusion coefficients in different media of similar structure. Solutions for the appropriate diffusion equations for continuous point and line sources are presented, and the results compared with measurement. Numerical estimates of the diffusion coefficients and integral scales are made, and molecular diffusion is found to be negligible by comparison.

J. L. Lumley (University Park, Pa.)

8989:

Plesset, Milton S.; Hsieh, Din-Yu. Theory of gas bubble dynamics in oscillating pressure fields. Phys. Fluids **3** (1960), 882–892.

A spherical gas bubble is placed in a spherically symmetric pressure field in an infinite incompressible liquid. The resulting field problem has spherical symmetry. A harmonic oscillation in the external liquid pressure is initiated at time $t=0$, this pressure having previously been

constant. The subsequent state of the bubble and the surrounding medium is determined. In the hydrodynamic theory thermal diffusion is considered so the authors are able to conclude that the process is isothermal for low frequencies but adiabatic for high frequencies.

The problem is solved first assuming uniform conditions within the bubble and then without this assumption. The equations are linearized for small variations from the initial conditions and solved using the Laplace transform, and asymptotic expressions (t large) are obtained for the fields.

The results should have application to the problem of diffraction of sound waves by clouds of air bubbles.

E. Pinney (Berkeley, Calif.)

8990:

Hsieh, Din-Yu; Plesset, Milton S. Theory of rectified diffusion of mass into gas bubbles. J. Acoust. Soc. Amer. **33** (1961), 206–215.

The problem considered is the behavior of a gas bubble in a liquid saturated with dissolved gas when oscillating pressures are imposed on the system. Since gas diffuses into the bubble during the expansion half-cycle in which the pressure drops below its mean value, and diffuses out of the bubble during the compression half-cycle in which the pressure rises above its mean value, there is in second order a net inflow of gas into the bubble which is called rectified diffusion. The equations which determine the system include the equation of state of the gas in the bubble, the equation of motion for the bubble boundary in the liquid, and the equation for the diffusion of dissolved gas in the liquid. In the solution presented here, the acoustic approximation is made, and it is assumed that the gas in the bubble remains isothermal throughout the oscillations. Under these conditions one finds for the mean rate of gas flow into the bubble the (approximate) expression $(dm/dt) = (8\pi/3)DC_\infty R_0(\Delta P/P_0)^2$, where D is the diffusivity of the dissolved gas in the liquid, C_∞ is the equilibrium dissolved gas concentration for the mean ambient pressure P_0, R_0 is the mean radius of the bubble, and ΔP is the amplitude of the acoustic pressure oscillations. It follows that R_0 grows asymptotically as $t^{1/2}$; a stability discussion shows, however, that the bubble becomes unstable beyond a certain critical radius.

J. B. Serrin (Minneapolis, Minn.)

8991:

Mikhailov, G. K. On the problem of the influence of capillarity on the flow of liquid in a porous medium to a rectilinear slope of a porous mass. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 275–281. SIAM, Philadelphia, Pa., 1961.

After a brief reference to allied work, the title problem is solved for the particular case when the rectilinear boundary consists of the vertical wall and the porous mass occupies the half space. Due to the influence of capillarity, the free surface of the flow region near the vertical wall is depressed from the horizontal. The region is conformally represented on to a half space and the solution obtained in terms of elliptic functions. The magnitude of depression of the free surface is shown in the graphical form. {Less conciseness in the presentation of the paper would have helped towards smoother reading.}

G. Paria (Kharagpur)

OPTICS, ELECTROMAGNETIC THEORY, CIRCUITS

See also 8401, 8753, 8778.

8992:

Biot, A. Sur la lentille épaisse. Ann. Soc. Sci. Bruxelles. Sér. I **74** (1960), 106–119.

This article contains a discussion of the geometrical properties of a thick divergent lens, especially with regard to the focal and chromatic properties of such lenses and, for concentric systems as well as for equiradial arrangements. It is found that thick divergent lenses have achromatic focal length if their shape is of meniscus type. Moreover, the author shows that the necessary and sufficient conditions for vanishing lateral chromatic aberration of a concentric system are satisfied when the entrance pupil passes through the common center of the system of lens surfaces. [A general treatment on color correction for various optical systems (lens arrangements) is given in chapter III of Herzberger, *Modern geometrical optics*, Interscience, New York, 1958; MR **22** #4321.]

N. Chako (New York)

8993:

Linfoot, E. H. Contrast transmission and the image evaluation problem. Opt. Acta **7** (1960), 65–80. (French and German summaries)

Various quality criteria for optical systems are compared. The ray-theoretic moment of inertia seems practical for astrographic cameras. For images of the core-and-halo type in photographic applications a "photographic Strehl" definition appears to be best as indicated by contrast transmission calculations for systems having primary spherical aberration. *E. W. Marchand* (Rochester, N.Y.)

8994:

Tsujiuchi, Jumpei. Restitution des images aberrantes par le filtrage des frequences spatiales. Opt. Acta **7** (1960), 243–261. (English and German summaries)

A method based on spatial frequency filtering is used for restoring images which have been damaged by the aberrations of an objective. The method is applied to the case of an objective whose response function is a Gaussian curve. *E. W. Marchand* (Rochester, N.Y.)

8995:

Herzberger, Max. Colour correction in optical systems and a new dispersion formula. Opt. Acta **6** (1959), 197–215. (French and German summaries)

The author's interpolation formula for computing the dispersion of glass and other optical materials has been improved. If four data are given for a glass, the refractive indices for any wavelengths can be computed. Various methods of plotting the glass data are suggested for various applications. Three glasses on a straight line in the plot of infra-red partial dispersion against ultra-violet partial dispersion lead to a cemented triplet practically color-corrected from 0.365 μ to 1.01 μ.

E. W. Marchand (Rochester, N.Y.)

8996:

Lee, L.; Farnell, G. W. Phase singularities for microwave lenses with aberrations. J. Opt. Soc. Amer. **51** (1961), 273–277.

Authors' summary: "The dark rings of the Airy pattern in the focal region of a microwave lens form phase singularities. In an error-free lens these dark rings lie on the surface of a sphere. It is shown that in the presence of primary aberrations the line singularities remain closed loops and the phase behavior near them is similar to that in the ideal lens. Third-order astigmatism of relatively small amounts produces a marked distortion of the shape of the line singularity."

8997:

Heynacher, E. Über den Zusammenhang von Lichtgebirge und inkohärenter Übertragungsfunktion. Opt. Acta **6** (1959), 266–271. (English and French summaries)

A method is described as suitable for incoherent illumination, for evaluating the Fourier transform of a given intensity distribution function. The method involves expressing the given function as a sum of basic functions whose transforms are easily obtained.

E. W. Marchand (Rochester, N.Y.)

8998:

Parrent, G. B., Jr. Studies in the theory of partial coherence. Opt. Acta **6** (1959), 285–296. (French and German summaries)

Previous results on quasi-monochromatic optical fields are extended to polychromatic fields. It is shown that (1) an optical field is coherent if and only if it is monochromatic, and (2) an incoherent optical field cannot exist even if the illumination is polychromatic. However, an incoherent source can be defined.

Propagation theorems are generalized to the polychromatic case. *E. W. Marchand* (Rochester, N.Y.)

8999:

Smith, Luther W.; Osterberg, Harold. Diffraction images of circular self-radiant disks. J. Opt. Soc. Amer. **51** (1961), 412–414.

Authors' summary: "The diffraction image formed by an Airy-type objective when the object is a circular, uniformly self-radiant disk is found to exhibit the phenomenon of a secondary Airy disk when the radius of the object is near 1.2 Airy units. The time-averaged irradiance in the diffraction image is presented for radii up to 4 Airy units. A separate expression for the irradiance at the geometrical radius in the image shows that as the object becomes very large the value of this irradiance approaches one-half that at the center of the image."

9000:

Senior, T. B. A. Scalar diffraction by a prolate spheroid at low frequencies. Canad. J. Phys. **38** (1960), 1632–1641.

The scalar problem of diffraction by a perfectly conducting prolate or oblate spheroid can be solved exactly by infinite series of spheroidal functions. Very few calculations have been made with these series because of the difficulties of computing the spheroidal functions. And, at the shorter wave-lengths, the convergence of the infinite series is very slow.

The situation is simple for the longer wave-lengths. In particular, as is shown here, a sequence of terms in the Rayleigh series can be deduced from the exact solution. The terms are calculated up to the term in $(ka)^6$, where k is the wave-number and $2a$ is the interfocal distance.

The limiting cases of a sphere or a circular disc are considered, and the results agree with the known solutions.

E. T. Copson (St. Andrews)

9001:

Šorygin, P. P.; Krušinskiĭ, L. L. On the theory of the Raman spectra of light. Dokl. Akad. Nauk SSSR **133** (1960), 337–340 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 790–793.

9002:

Minin, I. N. Point source of light in an absorbing medium between parallel planes. Dokl. Akad. Nauk SSSR **133** (1960), 74–76 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 754–756.

9003:

Pekar, S. I.; Striževskiĭ, V. L. The theory of the temperature dependence of the dispersion and exciton absorption of light in crystals. Fiz. Tverd. Tela **2** (1960), 894–897 (Russian); translated as Soviet Physics. Solid State **2**, 816–819.

In einer vorangehenden Arbeit [Ž. Èksper. Teoret. Fiz. **36** (1959), 451–464; MR **21** #5398] hat S. I. Pekar die Theorie der Absorption und der Dispersion des Lichtes in Kristallen (unter der Berücksichtigung von Exzitonen) für den Fall, dass sich der betrachtete Kristall am absoluten Nullpunkt befindet, ausgearbeitet. Ziel der vorliegenden kleineren Veröffentlichung ist diese Theorie auf endliche Temperaturen zu erweitern. Es wird angenommen, dass in nullter Näherung der ganze Kristall aus zwei nicht in Wechselwirkung stehenden quantenmechanischen Systemen aufgebaut ist. Das erste System, in dem die Teile des Kristalles zusammengefasst werden, welche relativ hohe Anregungsenergien besitzen (also z.B. die sich im Felde der Atomkerne bewegenden Elektronen) wird durch die Hamiltonsche Funktion $H'(\cdots r_l \cdots)$ beschrieben und das zweite, welche die Teile enthält, deren Anregungsenergien nicht mehr gross im Verhältnis zu kT sind (also z.B. die Gitterschwingungen), durch die Funktion $H''(\cdots q_k \cdots)$. In erster Näherung tritt dann die Wechselwirkung zwischen diesen zwei Systemen $U(\cdots r_l \cdots q_k \cdots)$ auf und der ganze springende Punkt der Überlegung ist, dass jetzt angenommen wird, dass H'' thermisch Anregungen enthält.

Nach den in der Optik üblichen Methoden folgt für den Tensor der Polarisierbarkeit

$$\beta = \beta' + \frac{\mathscr{E}'(\mathbf{k}')V\mathbf{P}_m(0)\mathbf{P}_m^*(0)}{\hbar\omega(\mathscr{E}'(\mathbf{k}') - i\tfrac{1}{2}\gamma\hbar - \hbar\omega)}, \tag{1}$$

wo ω die Frequenz des Lichtes, $\mathscr{E}'(k')$ die Anregungsenergie der Exzitonen mit dem Quasimoment k', das gleich dem Wellenvektor des Lichtes ist, V das Fundamentalvolumen im Kristall und die $\mathbf{P}_m(0)$ die Matrizenelemente des Dipolmomentes von dem Einheitsvolumen des Kristalles bedeuten. Alle bezüglich der Betrachtungen unwesentlichen Teile von β sind in β' zusammengefasst. Für γ folgt

$$\gamma = \frac{2\pi}{\hbar}\sum_p \rho_p(\hbar\omega)|U_{mq}|^2_{\mathscr{E}_q=\hbar\omega} \tag{2}$$

ρ bedeutet hier die Dichte der Energiezustände und U_{mq} ist ein Matrizenelement der Wechselwirkungsenergie, von dem durch γ die ganze Temperaturabhängigkeit der Dispersion und Absorption bestimmt wird. Weiter wird angenommen, dass U linear in den Koordinaten (der Schwingungen) ist und dass man für die Anregung dieser Oszillatoren den Mittelwert der Planckschen Formel einsetzen kann. Es folgt dann, dass mit zunehmender Temperatur γ ebenfalls zunimmt und bei hohen Temperaturen proportional zu T wird. *T. Neugebauer* (Budapest)

9004:

Miyamoto, Kenro. The phase Fresnel lens. J. Opt. Soc. Amer. **51** (1961), 17–20.

A phase Fresnel lens inserted in the pupil of an optical system can deform the wave surface by $\phi(u, v)$. If λ is the wavelength and $0 \leqq \phi(u, v) < m\lambda$, the (u, v) region of the pupil is divided into m zones by Fresnel's condition $(k-1)\lambda \leqq \phi(u, v) < k\lambda$ $(k = 1, \cdots, m)$. If the phase Fresnel lens shifts the wave surface by $\phi(u, v) - (k-1)\lambda$ in each kth Fresnel zone, the deformation in each zone is smaller than λ but this phase Fresnel lens is quite equivalent to a lens which deforms the wave surface by $\phi(u, v)$ because of Fresnel's condition. Some properties of the phase Fresnel lens are discussed. *J. E. Rosenthal* (Passaic, N.J.)

9005:

Monaco, Sebastiano F. Reflectance of an inhomogeneous thin film. J. Opt. Soc. Amer. **51** (1961), 280–282.

Author's summary: "An exact formula for the intensity reflectance of light incident normally on a nonabsorbing nonhomogeneous thin film is derived for the case where the refractive index changes exponentially with film thickness and the film is bounded on either side by nonabsorbing homogeneous semi-infinite media. A formula for the phase change at reflection is also derived."

9006:

Seman, O. I. On the theoretical limits of the chromatic aberration coefficient in limited electron-optical magnetic fields. Radiotehn. i Èlektron. **4** (1959), 1702–1707 (Russian); translated as Radio Engrg. and Electronics **4**, no. 10, 235–244.

Author's summary: "The limits of the chromatic aberration coefficient of image shift and the corresponding limiting fields for given enlargement, length of system and magnetic field intensity, not exceeding a given maximum, are found by a variational method."

9007:

Solncev, V. A.; Tager, A. S. Electron waves and their interaction with the field of a waveguide system. Radiotehn. i Èlektron. **4** (1959), 1652–1659 (Russian); translated as Radio Engrg. and Electronics **4**, no. 10, 138–153.

Authors' summary: "Propagation of a small h.f. signal in a rectilinear electron beam, the velocity of which varies periodically in space, is considered. The velocities and amplitudes of electron waves of the space harmonics of the current and the conditions of their growth along the beam are found. The interaction of the space harmonics of the current with the slow and fast electromagnetic waves as well as the interaction of the space harmonics of the current of two electron streams are investigated."

9008:

Dnestrovskiĭ, Yu. N.; Kostomarov, D. P. Radiation of a modulated beam of charged particles in passage through a round opening in an ideally conducting plane screen. Radiotehn. i Èlektron. **4** (1959), 1644–1651 (Russian); translated as Radio Engrg. and Electronics **4**, no. 10, 122–137.

Authors' summary: "The electromagnetic radiation arising in the passage of a modulated electron beam through an opening in an ideally conducting plane screen is investigated. The problem is reduced to a Fredholm integral equation of the first kind, which is solved numerically on an electronic computer in the range of velocities from $\beta = 0.1$ to $\beta = 0.99$."

9009:

Cincadze, N. L. Motion of a charged particle and a charged disc through an electron-ion beam. Ž. Tehn. Fiz. **30** (1960), 913–919 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 858–864.

Author's summary: "The energy loss of a charged particle and a bundle of particles, having the form of a disc with a uniformly distributed charge density, which travel at a constant velocity along the axis of a relativistic electron-ion beam contained in a waveguide, is discussed.

"An expression is found for the Cerenkov radiation intensity, and the spectrum of the radiated frequencies is determined. The waveguide radius corresponding to the maximum value of the radiation intensity for a given frequency ω is found."

9010:

dePackh, D. C.; Ulrich, P. B. Brillouin flow in relativistic beams. J. Electronics Control (1) **10** (1961), 139–146.

Authors' summary: "A solution in closed form for Brillouin flow under relativistic conditions in a cylindrical geometry is derived in the limit of low beam temperature. Curves and formulae are given relating density, magnetic field, energy and tangential velocity throughout the beam. In particular it is found that the forward velocity is constant to all orders in the radial coordinate, that under ultra-relativistic conditions the beam of maximum perveance has a forward velocity approaching $c/\sqrt{2}$, and that the maximum current is proportional in the ultra-relativistic limit to the square of the energy of the outermost particles."

9011:

Osovec, S. M. Dynamic stabilization of a plasma ring. Ž. Èksper. Teoret. Fiz. **39** (1960), 311–316 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 221–224.

Author's summary: "A method is proposed for dynamic stabilization of long-wave instabilities of a plasma filament. The stability conditions are presented and the frequency range of the rapidly varying stabilizing magnetic fields is determined."

9012:

Beard, David B. Cyclotron radiation from magnetically confined plasmas. Phys. Fluids **2** (1959), 379–389.

Author's summary: "The cyclotron emission from a hot, completely ionized, magnetically confined plasma has been estimated by computing the absorption of an incident plane wave. The harmonics of the fundamental cyclotron frequency, which are emitted perpendicular to the magnetic field direction, were summed over and also treated individually. Because of the Doppler effect and the relativistic variation in mass the behavior of the electrons and the electromagnetic properties of the medium are functions of electron velocity.

"Assuming a Maxwell-Boltzmann distribution in electron velocity, the polarization of the plasma was computed by integrating over electron velocity. Solution of Maxwell's equations yielded the absorption of an incident wave, and thus, from Kirchhoff's relation, the emission of the plasma was determined. The entire calculation was carried out nonrelativistically to first order in v^2/c^2, and at temperatures too low for fluctuations to play a dominant role."

9013:

Trubnikov, B. A. On the angular distribution of cyclotron radiation from a hot plasma. Phys. Fluids **4** (1961), 195–198.

The author points out a mathematical error in the paper of W. E. Drummond and M. N. Rosenbluth [Phys. Fluids **3** (1960), 45–51; MR **22** #2296; see also "Comment" in ibid. **4** (1961), 277–278] and illustrates its significance by a numerical example. In the appendix he carries out the necessary integrations to derive his radiation formula for a thermal distribution of independently radiating electrons in Larmour orbits from his earlier equation for the radiation of a single typical electron.

J. E. Drummond (Seattle, Wash.)

9014:

Whitmer, R. F.; Barrett, E. B. Nonlinear interaction of an electromagnetic wave with a plasma layer in the presence of a static magnetic field. I. Theory of harmonic generation. Phys. Rev. (2) **121** (1961), 661–668.

The effects of the non-linear terms $n\mathbf{v}$, $(\mathbf{v}\cdot\nabla)\mathbf{v}$, and $\mathbf{v}\times\mathbf{B}$ in the equations governing the propagation of electromagnetic radiation in a quasi-neutral plasma of electron density n and electron velocity $\mathbf{v}$ in a uniform magnetic field are investigated. Thermal and pressure gradients and the motion of the positive ions are neglected, and only elastic collisions between electrons and neutral particles are considered.

Corresponding to an incident wave of frequency $\omega/2\pi$ it is assumed that all quantities are of the form

$$\mathbf{f}(\mathbf{r}, t) = \mathbf{f}_0 + \sum_{h=1}^{\infty} \mathbf{f}_h(\mathbf{r})e^{-ih\omega t},$$

where $\mathbf{f}_0$ is constant and $\mathbf{f}_1 \gg \mathbf{f}_2 \gg \mathbf{f}_3 \gg \cdots$. There results a single inhomogeneous equation for the electric vector harmonic $\mathbf{E}_h$ in terms of quantities of lower order. In principle these can be solved in successively higher order.

The particular problem considered is that of a wave incident normally on a slab of finite thickness in a transverse magnetic field, and the calculations are carried to the second harmonic.

It is found that the second harmonic is longitudinal, bearing no power in the absence of a uniform magnetic field. The effect of the latter is to couple the longitudinal wave to the incident field, giving rise to second-order effects. The resulting second-harmonic power flow is found in terms of appropriate dimensionless parameters, and calculations carried out for the electron gyro-frequency ω_e near resonance. The effect of collisions is, as usual, to reduce the peak value of power, but to broaden the resonance. The peak value is sensibly constant over the range of plasma frequencies $0.1 < \omega_p/\omega < 0.9$.

K. C. Westfold (Clayton)

9015:

Lehnert, B. Energy balance and confinement of a magnetized plasma. Ark. Fys. **18**, 251–271 (1960).

Under the assumption of negligible mass velocity of the neutral particles, a two-fluid model (ions and electrons) is sufficient to describe the macroscopic motion of a plasma. The author derived the fundamental equations of the plasma dynamics for this case by including the effects of ionization, charge exchange and elastic collisions with neutrals. Some approximations to the general equations for the gas discharge experiments on laboratory scale are given.

Then the author studies the confinement of a rotating plasma situated in the magnetic field of a current loop from the macroscopic theory. A very strong confinement has been found. This result agrees with the previous theoretical result based on the motion of a single particle by the same author. From energy balance, the energy loss by escaping particles for the case of infinite heat conductivity in the direction along the magnetic field is found to be several orders of magnitude less in the current loop configuration than in earlier described homopolar machines.

Some currents on the stability problems are also given.

S. I. Pai (College Park, Md.)

9016:

Hzardžyan, S. M. On the theory of non-linear oscillations of plasma in a constant external magnetic field. Akad. Nauk Armyan. SSR. Dokl. **29** (1959), 215–221. (Russian. Armenian summary)

9017:

Salz, Frank; Meyerand, Russell, G., Jr. Quasi-thermalization of a plasma beam. Phys. Fluids **4** (1961), 438–443.

Authors' summary: "A study of the trajectories of charged particles as they pass through a cusped magnetic field shows that a significant fraction of their original axial kinetic energy is transformed into kinetic energy of rotation around the guiding centers of the particles. The energy-exchange mechanism predicted by an idealized model agrees exactly with the analog computer solution of the equations of motion of a particle and agrees generally with the results of an experiment in which a plasma beam was transmitted through a cusped magnetic field."

9018:

Kritz, Arnold H.; Mintzer, David. Propagation of plasma waves across a density discontinuity. Phys. Rev. (2) **117** (1960), 382–386.

The plane wave solutions of the small-signal, linearized

plasma equations in two media of different electron density are matched at a plane boundary, and the transmission and reflection coefficients are evaluated for longitudinal and transverse waves. In addition, the coupling coefficients between these wave types are calculated. It is shown that an incident longitudinal wave can induce transverse reflected and transmitted waves only for a narrow cone of angles near normal incidence. Outside this range the energy is coupled into a surface wave propagating along the interface. Reciprocally, an incident transverse wave at any angle can only produce longitudinal waves propagating normal to the boundary. Numerical results are presented for a two-to-one density ratio at three different frequencies. *R. Kodis* (Providence, R.I.)

9019:

Stern, E. A.; Ferrell, R. A. Surface plasma oscillations of a degenerate electron gas. Phys. Rev. (2) **120** (1960), 130–136.

Nach den Berechnungen von R. H. Ritchie [Phys. Rev. (2) **106** (1957), 874–881; MR **19**, 210] treten in einem Plasma ausser den sogenannten Plasmaschwingungen, deren Frequenz in den gewohnten Bezeichnungen

$$\omega_p = (4\pi ne^2/m)^{1/2} \tag{1}$$

ist und die von den Pendelungen der freien Elektronen herrühren, an der Grenzfläche von Plasma und Vakuum noch andere Schwingungen auf, für deren Frequenz Ritchie $\omega_r = \omega_p/\sqrt{2}$ erhielt und die ähnliche Ursachen wie die Plasmaschwingungen haben. Theoretische Gründe sprechen weiter dafür, dass man das Elektronengas in einigen Metallen von diesem Standpunkt aus als frei betrachten kann und tatsächlich konnten C. J. Powell und J. B. Swan [Phys. Rev. (2) **115** (1959), 869–875; **116** (1959), 81–83] auf experimentellem Wege zeigen, dass bei Al and Mg diese zwei Frequenzen auftreten. Doch erhielten sie für ω_r etwas zu kleine Werte, andere Autoren haben noch kleinere gefunden und es lag deshalb der Gedanke nahe, dass diese Erfahrung davon herrührt, dass das Plasma nicht an ein Vakuum; sondern an eine Metalloxidschicht angrenzt. Die theoretische Untersuchung dieser Frage ist das Ziel der vorliegenden Arbeit. Die Verfasser bedienen sich dabei eines anderen Gedankenganges als Ritchie. Sie gehen aus der Formel für die Dielektrizitätskonstante des Plasmas

$$\varepsilon_p(\omega) = 1 - \omega_p^2/\omega^2 \tag{2}$$

aus. Weiter betrachten sie eine klassische Ladungswelle, die entlang der Grenzfläche von Plasma und Dielektrikum fortschreitet und zeigen, dass man in diesem Fall aus dem kontinuierlichen Übergange des Vektors der elektrischen Verschiebung

$$\varepsilon_p(\omega) = -\varepsilon \tag{3}$$

erhält, wo ε die Dielektrizitätskonstante des angrenzenden Isolators (Metalloxid) ist. Aus (3) und (2) folgt dann in diesem Falle

$$\omega = \omega_r = \omega_p/(1+\varepsilon)^{1/2}. \tag{4}$$

Tatsächlich geht (4) für ein angrenzendes Vakuum ($\varepsilon = 1$) in die Formel von Ritchie über. Weiter wird noch das Problem von der Streuung eines einfallenden Elektrons unter gleichzeitiger Anregung eines solchen Oberflächenquants ausführlicher besprochen. Das Problem des nicht senkrechten Einfalls und das von zwei aneinander angrenzenden Plasmen wird ebenfalls erörtert und es wird gezeigt, dass wenn die Absorption sehr stark ist, ω_r ganz unbeobachtbar werden kann.

Im letzten Abschnitt wird noch das Problem der nicht unendlich dicken Isolatorschicht, die dann weiter an ein Vakuum angrenzt, besprochen. In diesem Falle folgt statt (4)

$$\omega_r = \omega_p\left[\frac{\varepsilon + \tanh k\tau}{2\varepsilon + (1+\varepsilon^2)\tanh k\tau}\right]^{1/2}, \tag{5}$$

wo τ die Dicke der Oxidschicht bedeutet und $k = 2\pi/\lambda$ ist. *T. Neugebauer* (Budapest)

9020:

Kitao, Kazuo. Energy loss and radiation of a gyrating charged particle in a magnetic field. Non-ionized medium. Progr. Theoret. Phys. **23** (1960), 759–775.

The author studies the energy loss of a gyrating charged particle in a non-ionized medium with a uniform magnetic field. In general, this loss consists of one part due to close collisions and another due to distant collisions in the form of polarization loss and loss due to Cerenkov radiation, and, if the particle is gyrating, an additional loss due to cyclotron or synchrotron radiation. Because the motion is circular in this problem, it is convenient to resolve the potential of the electromagnetic field into harmonic components by means of a Fourier series expansion. The general formula of the total energy loss per unit time is derived from the work of the reaction force of the medium on the particle. For a non-relativistic particle it is shown that this total energy loss is split into the collision loss and the loss due to cyclotron radiations. The ratio of the latter to the former is less than $(\omega_0/\omega_p)^2$, where ω_0 is the cyclotron frequency and $\omega_p^2 = 4\pi n_e e^2/m_e$, where n_e and m_e are the density and mass of electrons in the medium. The asymptotic formulas of the Bessel functions with large order and large argument are used in the relativistic case to obtain the polarization loss, which is found to depend on the external magnetic field. Also the losses due to the Cerenkov and synchrotron radiations are computed and the spectral and angular distributions of these two radiations are discussed. In an appendix another treatment of the same problem by the method of Poynting vector is presented. *R. S. B. Ong* (Leiden)

9021:

Rosen, Philip. Generation of the third harmonic by an electromagnetic signal in a plasma. Phys. Fluids **4** (1961), 341–345.

The generation by an oscillating electric field in a gas discharge of a third harmonic in the current density is studied by means of a perturbation solution of the Boltzmann equation. By assuming that in each order of the main perturbation the higher harmonics represent perturbations on the fundamental it is possible to develop differential equations for the components of the electron distribution function. These equations are then integrated over velocity space to give the components of the current density. The integrals are approximated by the method of steepest descent and the results given in the form of power series expansions. *R. D. Kodis* (Providence, R.I.)

9022:

Kruskal, Martin. Landau damping. La théorie des gaz neutres et ionisés (Grenoble, 1959), pp. 285–292. Hermann, Paris; Wiley, New York; 1960.

This paper is a short review of the theory of Landau damping, and compares two methods of treating the linearized Boltzmann equation to solve the plasma oscillation problem. One of the methods gives a result that seems physically unreasonable. The author then gives a summary of some attempts by himself and others to solve the complete, non-linear equation, the results of which seem to explain the disturbing factor in the treatment of the linearized problem.

The two methods of treating the linearized problem are the Laplace transform method of Landau (initial value problem), and Van Kampen's method of normal modes. The latter method gives rise to waves which are undamped, and which can have infinities in their amplitudes. The non-damping can be reconciled with Landau's result, because the normal modes are a complete set, so that a linear combination of them can damp in time due to phase mixing. However, the singularities of the normal modes seem to contradict the assumption that they are small of the first order. An explanation of this result is given on the basis of the non-linear theory which characterizes the amount of non-linearity of the solution by an amplitude factor, A, and the solution is examined in the limit $A \to 0$. It is found that if f_1 represents the perturbed part of the distribution function, f_1 is singular like $1/\sqrt{A}$, so that Af_1 remains small. *R. Gerwin* (Seattle, Wash.)

9023:

Theimer, O.; Taylor, L. S. The dependence of the plasma conductivity on frequency and collision time. Ann. Physics **11** (1960), 377–392.

The collision frequency is determined for a fully ionized plasma by a detailed examination of the microscopic motion of electrons. The microscopic motion of charged particles is then averaged over the different physical parameters to obtain an expression for the collision frequency which is used in a phenomonological description of plasma bulk properties.

The derivation is straightforward for relaxation problems; however the section which analyzes the a.c. collision frequency is not so clear and is difficult to follow.

The result of the analysis is of interest in that a complex collision frequency is obtained which can offer corrections of a few percent at microwave frequencies.

R. Hall (Seattle, Wash.)

9024:

Kadomcev, B. B.; Rokotyan, V. E. Stability of plasma in the field of a magnetic dipole. Dokl. Akad. Nauk SSSR **133** (1960), 68–70 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 747–749.

9025:

Vandakurov, Yu. V. Stationary state of a thin circular plasma pinch of finite conductivity. Ž. Tehn. Fiz. **30** (1960), 1134–1136 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 1069–1071.

9026:

Golant, V. E.; Žilinskiĭ, A. P. Propagation of electromagnetic waves through waveguides filled with plasma. Ž. Tehn. Fiz. **30** (1960), 15–24 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 12–21.

Authors' summary: "In this paper we consider the propagation of electromagnetic waves through a waveguide filled with a medium of varying conductivity. The analysis is carried out by means of perturbation theory. The phase shift and attenuation introduced by the plasma are calculated to first and second order."

9027:

Hurley, James. Interaction of a streaming plasma with the magnetic field of a line current. Phys. Fluids **4** (1961), 109–111.

Author's summary: "The interaction between the magnetic field of a line current and a rare plasma streaming toward it is investigated. It is assumed that the field of the line current is confined to a cavity with the plasma particles specularly reflected in a thin layer of the cavity wall. The shape of the cavity wall and the field in the cavity are calculated."

9028:

Tonks, Lewi. Diffusion through a finite plasma in a uniform magnetic field. Phys. Fluids **3** (1960), 758–764.

Author's summary: "The anisotropic diffusion of ions and electrons in a nonmultiplying partially ionized plasma lying in a strong uniform magnetic field has been analyzed to zero and first order in the parameter Ω^{-1}, where Ω^2 is the ratio of the reduction of transverse electron mobility to the reduction of transverse ion mobility due to the magnetic field. This obviates the need for an earlier erroneous assumption. Potential distribution and electron and ion current flows have been formulated for two-dimensional flow, using a lowest-harmonic charged-particle distribution in a rectangular space and various wall-current conditions. The encroachment of sheaths inward from the walls has not been considered on the basis that, in principle, the charged-particle concentration could always be assumed to be large enough to keep this complication small."

9029:

Cytovič, V. N. Elastic reflection of plasma and coils with current. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 6, 142–145. (Russian)

9030:

Vandakurov, Yu. V. Oscillations of a thin circular pinch in a magnetic field. Ž. Tehn. Fiz. **30** (1960), 711–722 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 667–677.

Author's summary: "The axially symmetric oscillations of a thin circular pinch with zero real resistance are considered in the magnetohydrodynamic approximation. The oscillation frequencies are determined for a pinch located inside an ideally conducting chamber and for a plasma loop with no chamber. In the second case we consider a pinch whose cross section is not strictly circular; hence,

there is a possibility of varying the transverse magnetic containing field. It is shown that the presence of a radial component in the transverse field in the plane which contains the axial line of the pinch leads to a deterioration of stability."

9031:

Cairó, Lorenzo. Méthode de perturbation pour la propagation des ondes électromagnétiques dans un plasma remplissant partiellement un guide d'ondes circulaire. C. R. Acad. Sci. Paris **250** (1960), 4129–4131.

Author's summary: "On considère un guide circulaire, le plasma étant disposé à son intérieur dans un tube concentrique. On trouve une relation qui relie la valeur de la constante de propagation aux diamètres des deux tubes concentriques. On se borne au mode fondamental du guide vide."

9032:

Frank-Kameneckiĭ, D. A. Natural oscillations of a bound plasma. Ž. Èksper. Teoret. Fiz. **39** (1960), 669–679 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 469–475.

Author's summary: "The general properties of cylindrical waves in a cold plasma are examined. The results are applied to low-frequency natural oscillations of a plasma cylinder surrounded by conductive walls. The conditions of magneto-acoustic resonances that ensure effective penetration of the oscillations inside the plasma have been found. The nature of the resonance phenomena depends on the linear density of the electrons. Approximate formulas are given for the natural oscillations of a long plasma cylinder. It is shown that purely radial oscillations are not feasible in a region close to the geometric mean of the electron and ion cyclotron frequencies, for even slight deviations from it sharply change the resonance frequency."

9033:

Polovin, R. V. Flow of a plasma into vacuum in the presence of a magnetic field. Ž. Èksper. Teoret. Fiz. **39** (1960), 657–661 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 461–463.

Author's summary: "Hydromagnetic waves excited by disintegration of the boundary between the plasma and vacuum are investigated. The boundary velocity (escape velocity) is determined. The amplitude of the electromagnetic wave radiated into the vacuum during the disintegration of the discontinuity is determined."

9034:

Aleksin, V. F.; Yašin, V. I. Investigation of the stability of a plasma by a generalized energy principle. Ž. Èksper. Teoret. Fiz. **39** (1960), 822–826 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 572–574.

Authors' summary: "Stability conditions are derived for a plasma possessing an anisotropic particle velocity distribution and located in a cylindrically symmetric magnetic field. Cases of longitudinal and azimuthal magnetic fields are considered."

9035:

Gerjuoy, E.; Rosenbluth, M. N. Pinch with rotating plasma. Phys. Fluids **4** (1961), 112–122.

Authors' summary: "The magnetohydrodynamic stability of an infinitely conducting incompressible plasma, in the shape of an infinitely long circular cylinder within an external conductor, compressed by external B_θ and B_z against internal B_z, is investigated in the circumstance that the plasma has an angular velocity about the z-axis of the cylinder. Two different cases are examined in detail: (i) arbitrarily large constant angular velocity, in which case the differential equation for the perturbed total (magnetic plus hydrodynamic) pressure inside the plasma is solved exactly in terms of Bessel functions; (ii) nonconstant but small angular velocity, in which event perturbation theory can be employed. For both these cases the rotating plasma is less stable than the nonrotating plasma." *E. A. Jackson* (Princeton, N.J.)

9036:

Ecker, G. Enhanced interaction in the positive column. Phys. Fluids **4** (1961), 127–138.

The author considers the influence of enhanced interactions on the characteristics of a positive column in a longitudinal magnetic field. The enhanced interactions (assumed to be produced by some underlying microinstability, such as a drift instability) are accounted for by effective parameters in the Boltzmann transport equations. The main effects of this interaction, aside from the increased diffusion across the magnetic field, is the alteration of the energy exchange between electrons and ions, which changes the relationship between the longitudinal electric field and the electron temperature, and a pronounced influence on the radial electric field.

E. A. Jackson (Princeton, N.J.)

9037:

Katz, Sylvan. On a moving boundary problem. J. Mathematical Phys. **2** (1961), 149–150.

Author's summary: "An exact analytic solution is obtained for a uniformly expanding, neutral, infinitely conducting plasma sphere in an external uniform and constant magnetic field."

E. A. Jackson (Princeton, N.J.)

9038:

Nekrasov, F. M. Nonlinear theory of oscillations in an electron-ion plasma. Ž. Tehn. Fiz. **30** (1960), 774–780 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 727–733.

Author's summary: "The stationary oscillations of an electron-ion plasma are considered in the nonlinear approximation on the basis of the kinetic equation; the collision integral is neglected.

"It is shown that the oscillations of charge density in the plasma, where there is no relative directed motion of the electron and ion components, are primarily due to the electrons. The ion component makes an important contribution to the oscillations only when there is relative motion. The relation between the oscillation period and amplitude is determined and the maximum fields for which stationary plasma waves are possible are found."

9039:
Kovižnyh, L. M. Stability of longitudinal oscillations of an electron-ion plasma in an external electric field. Ž. Tehn. Fiz. **30** (1960), 1186–1192 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 1122–1128.

Author's summary: "We consider instabilities of the longitudinal oscillations of an electron-ion plasma in an external electric field for adiabatic variation of the parameters. The stability criteria and expressions for the growth of instabilities are derived."

9040:
Golant, V. E. Diffusion of charged particles across a magnetic field in a three-component plasma. Ž. Tehn. Fiz. **30** (1960), 881–892 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 831–841.

Author's summary: "The motion of charged particles under the action of the concentration and temperature gradients in a plasma consisting of electrons, ions, and neutral atoms is considered. Expressions are obtained which determine the flow of electrons and ions across the magnetic field."

9041:
Frank-Kamenecki̇ĭ, D. A. Magnetic sound in three-component plasma. Ž. Tehn. Fiz. **30** (1960), 893–898 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 842–846.

Author's summary: "Under the simplest approximations formulas are deduced describing the influence of neutral particles on magnetosonic plasma vibrations. At high frequencies the neutral gas is entirely free from, and at low frequencies entirely entrained by, the plasma vibrations. In the case of incomplete entrainment, recharging collisions lead to additional decay. The influence of the collisions on the penetration into the plasma of a variable field has opposite characters (causing a skin effect or spatial decay) depending on whether a strong static magnetic field is present or not."

9042:
Rudakov, L. I. A magnetic trap with rotating plugs. Ž. Tehn. Fiz. **30** (1960), 907–912 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 853–857.

Author's summary: "If electromagnetic oscillations of large amplitude are excited in a plasma filling a trap with static magnetic plugs, the plasma particles in the region of the plugs will be acted on by an additional force which will prevent the escape of particles from the trap. It is shown that by this method the cone of escape of the particles can be closed."

9043:
Timofeev, A. V. Build up of ion acoustic vibrations in an anisotropic plasma. Ž. Éksper. Teoret. Fiz. **39** (1960), 397–399 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 281–282.

9044:
Eichenbaum, A. L.; Hernqvist, K. G. Space charge instabilities in synthesized plasmas. J. Appl. Phys. **32** (1961), 16–21.

9045:
Young, W. H. A variational approach to correlation in an electron gas. Philos. Mag. (8) **6** (1961), 371–377.

Author's summary: "A method is proposed for calculating correlation energies and pair functions associated with an electron gas at any density. Upper bounds to the exact energy, lying however at least as low as the corresponding plane wave results, are guaranteed, and exact solutions are obtained in the high- and low-density limits. The technique used is that of allowing a periodic electron density and a proof emerges of the recent conjecture of Overhauser that there exist one-particle states energetically more favourable than the usual plane wave orbitals. It is concluded that in the ground state a single-particle approximation implies static spin density waves of wavelength proportional to the usual separation electron parameter r_s, at least for r_s sufficiently large."

9046:
Clavier, P. A. Propagation in a plasma. J. Appl. Phys. **32** (1961), 578–582.

Author's summary: "The boundary conditions for a radio signal incident normally on a layer of plasma are discussed. It is shown that 10 different modes must in general propagate in the layer, instead of the usually assumed six. The 10 modes are obtained when the Langevin form of the force is used in Boltzmann's equation."

9047:
Fano, Robert M.; Chu, Lan Jen; Adler, Richard B. ★Electromagnetic fields, energy, and forces. John Wiley & Sons, Inc., New York-London, 1960. xv+520 pp. $12.00.

This is a textbook designed for junior students in electrical engineering. As such, it represents a refreshing departure from traditional emphasis of subject matter and presents new developments and ideas as well as classical results. The traditional emphasis on solution of special boundary value problems of electromagnetic theory is abandoned and in its place we find: (1) a commendable effort to relate circuit theory to the field viewpoint; (2) a searching inquiry into the concepts of electromagnetic energy conversion and force in magnetizable and polarizable media in motion. The ideas presented with regard to the latter problem are new and interesting, and demand attention. The advanced student of electrodynamics probably will find the four-dimensional formulation of the field laws and of the Poynting theorems of electrodynamics and the comparisons among the authors' (E-H) scheme, the Amperian (E_a-B_a) scheme, and Minkowski's (D-B) scheme the most useful and interesting aspect of this book. As is clearly stated on page 486, the (E-H) and the traditional (E_a-B_a) formulation of the Poynting theorems and field equations are mathematically equivalent but inspire different definitions for the rate at which electromagnetic energy and momentum is converted into other forms of energy and momentum. As has been pointed out by C. Truesdell and the reviewer [#8778, § 284], definitions of electromagnetic force and rate of electromagnetic energy conversion remain mere definitions in the absence of further equations linking them to the motion, deformation, and thermodynamic state of the

medium. The authors imply that the quantity they choose to define as the electromagnetic force, for example, is to be equated to the mass density times the acceleration of the medium. But this certainly cannot be intended to hold generally, for example, in a magnetizable, piezoelectric deformable solid. Thus there remain open many questions concerning the means by which the new Poynting identities of the (E-H) scheme may be usefully employed in the construction of determinate special theories of specific types of material media.

R. A. Toupin (Washington, D.C.)

9048:

Moon, Parry; Spencer, Domina Eberle. ★Foundations of electrodynamics. The Van Nostrand Series in Electronics and Communications. D. Van Nostrand Co., Inc., Princeton, N.J.-Toronto-London-New York, 1960. vii+314 pp. $9.75.

This book presents an introduction to the basic principles and applications of electromagnetic field theory and is written with the needs of the engineer in mind. A novel feature of the book occurs in the development of the basic electrodynamic principles. This development is contained in the second and third chapters, the first chapter being devoted to introducing the notions of vector field theory. The development of the theory is purely macroscopic and it is assumed that a medium may be characterised by the macroscopic parameters ε the permittivity, μ the permeability and $\mathscr{R}$ the resistivity. The authors then build up the whole electrodynamic field theory from the fundamental concept of charge. This development requires the introduction of five postulates: (i) The validity of the Newtonian concepts of length, mass, time and force; (ii) Independence of charge on motion; (iii) Conservation of charge; (iv) The electromagnetic potentials are assumed to propagate in any medium at a velocity $(\varepsilon\mu)^{-1/2}$ with respect to the receiver; (v) The force per unit charge is given by $\mathbf{E}+\mathbf{V}\times\mathbf{B}$. The authors show that the above postulates together with appropriate definitions of the field components and the scalar and vector potentials yield Maxwell's field equations.

Chapters 4 to 10 are devoted to the application of Maxwell's equations to various engineering problems. The chapter headings are: (a) Charges with no relative motion; (b) Charges in uniform motion; (c) Accelerated charges; (d) Skin effect; (e) Waves; (f) Waveguides; (g) Antennas. In these chapters some of the simpler mathematical boundary value problems of electromagnetic field theory are formulated in a precise manner and the reader is introduced to some of the techniques available for their solution. Most of the boundary value problems are for plane, spherical and cylindrical boundaries and they are solved by the separation of variables technique. The appropriate mathematical solutions of Helmholtz's and Laplace's equations are presented in an appendix. Throughout these chapters considerable emphasis is placed on the detailed physical interpretation of the solutions of the field equations. The text also contains a large number of graphs to illustrate the actual numerical behaviour of the quantities of physical interest.

The two remaining chapters treat moving coordinate systems and the electrodynamics of moving media. In chapter 11 five postulates are formulated to deal with the transformation of events from one coordinate system to another one moving with uniform velocity relative to the first. An infinite number of transformations are shown to obey all the postulates and the Lorentz and Galilean transforms emerge as two particular cases of the general transformation. In the final chapter the transformations of Chapter 11 are employed to determine the transformation equations for the various electromagnetic quantities. In this chapter the elementary ideas of tensor analysis are required. The particular role of the Lorentz transformation is also examined in this chapter and also the validity of the original fundamental postulates for the case of a moving medium.

It should perhaps be pointed out that, apart from chapters 2 and 3, the book is virtually independent of the postulational formulation of electrodynamics.

W. E. Williams (Liverpool)

9049:

Karp, Samuel N. Two dimensional Green's function for a right angled wedge under an impedance boundary condition. Comm. Pure Appl. Math. **13** (1960), 203–216.

A formal solution for the inhomogeneous, two-dimensional wave equation is carried out in the region exterior to a right angled wedge of infinite extent. The boundary conditions on the wedge surfaces are mixed. One surface is taken to be a perfect reflector; the other is a partial absorber (impedance surface).

Following an idea of Stoker [Quart. Appl. Math. **5** (1947), 1–54; MR **9**, 163], the solution is found by introducing an auxiliary wave function which is related to the original through the operator $(\partial/\partial x-\lambda)$. The auxiliary function satisfies the same scalar wave equation with a modified inhomogeneous term. In addition, the transformed boundary condition on both wedge surfaces turns out to be of simple Dirichlet type. Since the equation is separable and the boundary condition simple, the solution of this related problem can be expressed as a Fourier series in the angular variables. The coefficients are Bessel functions of order $\frac{1}{2}m$.

Unfortunately, the determination of the original wave function requires an integration, the constant of integration being fixed by the original boundary conditions. This leads to a solution which is a rather complicated expression involving derivatives, integrals, products and ratios of a number of different infinite series. Although the convergence of these series is not discussed, it is clear that it would be extremely slow for a source more than a few wavelengths from the corner of the wedge. Indeed, when this distance is only 2λ, it seems that at least fifteen terms of each series would be required to calculate the far zone field.

The above difficulty probably explains why the author has left his solution in the form of a substitutional recipe whose basic ingredient is a set of Fourier series. Whether any of the physical implications of the solution can be extracted from the formalism is a question that is not considered in this paper. *R. D. Kodis* (Providence, R.I.)

9050:

Eckart, Gottfried. Wellenoptische Behandlung der Strahlung eines magnetischen Dipols in einem eben geschichteten Medium nach der Methode von Epstein. Bayer. Akad. Wiss. Math.-Nat. Kl. Abh. **102** (1960), 85 pp.

Bekannterweise können abnormal grosse Reichweiten

bei Ultrakurzwellen (die von der Ionosphäre mehr kaum beeinflusst werden) infolge von drei Ursachen auftreten: Es können atmosphärische Wellenleiter entstehen, an atmosphärischen Inhomogenitäten können Reflexionen auftreten und die Schichtung der ruhenden Atmosphäre könnte ebenfalls ein bemerkbares Streufeld verursachen. Die strenge Behandlung dieses letzteren problematischen Effektes, über den bis jetzt nur Überschlagsrechnungen vorliegen, ist das Ziel der vorliegenden Arbeit.

Für die wellenoptische Behandlung der Reflexionen an inhomogenen Schichten hat P. S. Epstein [Proc. Nat. Acad. Sci. U.S.A. **16** (1930), 627–637] eine Methode angegeben, welche dann von ihm und von K. Rawer [Ann. Physik (5) **35** (1939), 385–416] auf atmosphärische Probleme angewandt wurde. Der Grundgedanke dieser Methode ist, dass man nicht die Wellengleichung

$$\Delta U + k_0^2 \varepsilon(z) U = 0, \tag{1}$$

wo U eine Feldgrösse und $\varepsilon(z)$ die Dielektrizitätskonstante bedeutet (ausserdem ist $k_0 = 2\pi/\lambda_0$) zu lösen versucht, sondern die Gauss-sche hypergeometrische Differentialgleichung

$$\frac{d^2u}{dv^2} + \frac{\gamma - (\alpha+\beta+1)v}{v(1-v)} \frac{du}{dv} - \frac{\alpha\beta}{v(1-v)} u = 0 \tag{2}$$

(deren Lösungen ja bekannt sind) in die Form (1) zu transformieren trachtet. Nach Rawer wird $v = v(z)$ und $u = U \cdot r(z)$ gesetzt. Tatsächlich gelingt es dann (2) in die Form

$$\frac{d^2U}{dz^2} + g(z) U = 0 \tag{3}$$

zu bringen, wo $g(z)$ mit Hilfe von v, mit Hilfe der Ableitungen dieser Funktion und den Konstanten α, β und γ ausgedrückt wird. Weiter wird $v(z) = -e^{\kappa z}$ gesetzt und es wird gezeigt, dass man auf diesem Wege $\varepsilon(z) = 1 + \delta/(1+e^{\kappa z})$ erhält und diesen Ausdruck kann man dann durch entsprechende Wahl der Konstanten δ and κ den tatsächlich auftretenden Verhältnissen leicht anpassen.

Zuerst wird die Ausbreitung von ebenen Wellen in diesem "Epsteinmedium" berechnet. Die auftretenden Wellentypen, die Reflexions- und Übertragungskoeffizienten, die partielle Reflexion, die Totalreflexion und im Zusammenhang damit die dabei auftretenden stehenden Wellen und Interferenzstreifen, der Grenzübergang zu einem homogenen Medium ($\delta \to 0$) und zu einem unstetigen Medium ($\kappa \to \infty$) werden berechnet.

Danach wird das Problem des magnetischen Dipols betrachtet, den man sich als realisiert durch eine horizontal liegende Drahtschleife zu denken hat. Die Strahlung eines solchen Dipols im homogenen Medium wird nach Lamb und Sommerfeld aus Zylinderwellen in Form eines Integrals aufgebaut. Analog zu dem Hertzschen Vektor Π des elektrischen Dipols wird auch hier ein Hertzscher Vektor Ψ eingeführt und es wird wieder die besprochene Methode zur Lösung der entstehenden Differentialgleichung benutzt. Die auftretenden 12 Lösungsintegrale werden explizit angegeben, die Grenzübergänge $\delta \to 0$ und $\kappa \to \infty$ werden wieder besprochen, die Pole der Integranden werden berechnet und die Sattelpunktapproximationen der Integrale werden tabelliert.

Im ersten Teil des Anhanges behandelt der Verfasser das selbe Problem nach der geometrischen Optik (Strahlenoptik) und leitet eine Näherungsformel für das Eikonal her. Im zweiten Teil werden einige Eigenschaften der Γ-Funktionen, durch welche die Lösungen der hypergeometrischen Differentialgleichung miteinander in Zusammenhang gebracht werden, besprochen und die Sommerfeldschen Integrale werden explizit angegeben.

T. Neugebauer (Budapest)

9051:

Gaponov, A. V. Instability of a system of excited oscillators with respect to electromagnetic perturbations. Ž. Èksper. Teoret. Fiz. **39** (1960), 326–331 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 232–236.

Author's summary: "Possible mechanisms of autophase selection of excited oscillators in a radiation field leading to instability of the system with respect to electromagnetic disturbances are considered. It is shown that from the quantum point of view the instability of such systems may be due to unequal spacing in the (anharmonic) oscillator spectrum or to recoil during the emission of a photon."

9052:

Poincelot, Paul. Sur la condition de Lorentz. C. R. Acad. Sci. Paris **251** (1960), 1986–1988.

The author points out that one can write a generalized Lorentz condition for classical electromagnetism,

$$\operatorname{div} \mathbf{A} + \mu\varepsilon + \mu\sigma V = 0,$$

where $\mathbf{A}$ and V and vector and scalar potentials (which is well known) and that this condition is not invariant under a Lorentz transformation whereas the special case, $\operatorname{div} \mathbf{A} + \mu_0\varepsilon_0(\partial V/\partial t) = 0$ is.

M. Kline (New York)

9053:

Stier, F. Bestimmung der Energie eines ebenen Magnetfeldes durch Abbildung desselben auf ein elektrisches Stromfeld. Arch. Elektrotech. **45** (1960), 343–346.

The author considers a curl-free magnetic field and seeks a representation of this magnetic field by an electric current field. He first considers plane fields and by way of example refers to a transformer-window. He then proceeds to calculate the energy content of the magnetic field and finds a corresponding expression using the representing electric field. Finally these considerations are extended to non-curl-free magnetic fields and an expression for the energy is also found in this case.

M. J. O. Strutt (Zürich)

9054:

Hauser, Walter; Brown, Lowell. The size effect of ferrite spheres. Quart. J. Mech. Appl. Math. **13** (1960), 257–271.

Approximate solutions are obtained for the electromagnetic field in a small ferrite sphere in a cavity. The integral equation for the fields in terms of the dyadic Green's function is used. This integral equation is combined with the one for the unperturbed cavity modes (in the absence of the sphere). The resultant equation is studied when the cavity Green's function is approximated by the free space Green's function and applied to a

sphere placed near the center of the cavity; and when it is approximated by a Green's function for one plane reflecting boundary and applied to a ferrite sphere near a cavity wall. The equations are considerably simplified upon dropping terms of the order $(ka)^3$ or higher, where k is the free-space propagation constant and a is the sphere radius. In this simpler form, the equations are shown to follow from a variational principle. While the approximations are justified for simple cavity geometries, it does not seem that these same approximations would lead to good results for ferrite spheres in unusual field configurations.

The frequency perturbation of the cavity is first determined for an assumed uniform field inside the ferrite sphere (quasistationary approximation) and the effect of a nearby wall is determined. The result checks well with cited experiments. A better approximation to the field within the ferrite is then employed that yields corrections to order $(ka)^2$ to the previously obtained formulae.

H. A. Haus (Cambridge, Mass.)

9055:

Êfros, A. L. Potential distribution near a cylindrical probe. Ž. Tehn. Fiz. **30** (1960), 1024–1029 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 954–959.

A circular cylinder rests with its flat top on a plane boundary of a semi-infinite conductor (or semiconductor). The potential in the cylinder is given by a Fourier-Bessel expansion. The solution in the semispace is given in terms of oblate spheroidal coordinates by an expansion in spheroidal harmonics. The coefficients are related by the boundary conditions at the interface. The first approximation, giving a uniform field strength at infinity, is calculated from the said boundary conditions and series expansions. Taking higher terms of the expansions into account, a second approximation is calculated and partly tabulated and compared with the first approximation.

M. J. O. Strutt (Zürich)

9056:

Obermair, G.; Schwink, Ch. Über Magnetisierung und Feld stabförmiger Körper. Z. Physik **160** (1960), 268–276.

9057:

Gélard, Jacques. Recherche d'équations aux différences finies pour des points situés à la limite d'une distribution superficielle de charges. Cas des systèmes de révolution. C. R. Acad. Sci. Paris **251** (1960), 1871–1873.

Il s'agit d'un problème d'électrostatique, concernant la résolution approximative de l'équation de Laplace, à laquelle satisfait le potentiel. L'auteur considère particulièrement des distributions superficielles de charges sur des surfaces de révolution de plusieurs formes, au voisinage de discontinuités. L'exposé s'appuie sur une méthode développée par M. Emile Durand dans *Electrostatique et magnétostatique*, Masson, Paris, 1953; C. R. Acad. Sci. Paris **244** (1957), 2355–2358; **249** (1959), 1864–1866 [MR **16**, 99; **19**, 463; **22** #495]. Le potentiel étant développé, à l'approximation du quatrième ordre, en fonction d'une certaine distance, l'auteur forme des équations aux différences finies dont il déduit la densité superficielle de charge à l'approximation du second ordre.

P. M. Poincelot (Issy-les-Moulineaux)

9058:

Barsukov, K. A. Radiation of electromagnetic waves from a point source in a gyrotropic medium with separation boundary. Radiotehn. i Èlektron. **4** (1959), 1759–1764 (Russian); translated as Radio Engrg. and Electronics **4**, no. 11, 1–9.

Author's summary: "The problem of the field of a point source of electromagnetic waves in a semi-infinite gyrotropic medium with plane boundary of separation from an isotropic medium is considered."

9059:

Kaganov, V. I. The application of Chaplygin's method for the solution of some non-linear problems of radio engineering. Radiotehn. i Èlektron. **4** (1959), 1990–1992 (Russian); translated as Radio Engrg. and Electronics **4**, no. 12, 54–58.

Author's summary: "The advantages of using Chaplygin's method for the solution of the dynamic equations of systems, containing non-linear inertia-free elements are shown. This method enables us to find a solution quickly and to evaluate the error. As an example we examine the transient process in an automatic frequency control system."

9060:

Benenson, L. S. The phase velocity of waves in an anisotropic artificial metal-dielectric for an arbitrary direction of propagation. Radiotehn. i. Èlektron. **4** (1959), 1806–1815 (Russian); translated as Radio Engrg. and Electronics **4**, no. 11, 73–87.

Author's summary: "The characteristic equation for finding the phase velocity of waves in an artificial anisotropic metal-dielectric for an arbitrary direction of propagation is obtained by the method of induced e.m.f. Practical formulae are found for the case of rod grids and graphs of the phase constant are plotted."

9061:

Schmitt, Hans J.; Wu, Tai Tsun. Electromagnetic reflection from sound waves. J. Acoust. Soc. Amer. **32** (1960), 1660–1667.

The reflection coefficient is calculated for an electromagnetic wave normally incident on a liquid half-space in which there is an acoustic standing wave. The reflected wave is obtained from an approximate solution of the Mathieu equation and shows the effect of a time dependent Bragg reflection from the standing wave as well as the motion of the interface. The time dependence of the standing wave produces sum and difference frequencies and the strength of these sideband terms is plotted against the ratio of electromagnetic to acoustic wavelength.

E. T. Kornhauser (Providence, R.I.)

9062:

Amatuni, A. C.; Korhmazyan, N. A. Transition radiation in the case of a diffuse boundary between two media. Ž. Èksper. Teoret. Fiz. **39** (1960), 1011–1019 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 703–708.

Authors' summary: "We consider the transition radiation of a charged relativistic particle in the case of a

diffuse boundary between two media. General formulas are obtained for the main part of the radiation, and a detailed study is made of the case when the dimension of the diffusion zone z_0 is much smaller than that of the zone where the transition radiation is produced in either medium, R_0 or R_m, and also the cases when $z_0 \gg R_0 > R_m$ and $R_m \ll z_0 \ll R_0$."

9063:

Kielich, S. Molecular theory of light scattering by multi-component systems. Acta Phys. Polon. **19** (1960), 573–597.

Author's summary: "A molecular theory of light scattering by multi-component systems of optically anisotropic molecules is proposed. General expressions, accounting for the degree of depolarization D of the light scattered, for Rayleigh's ratio S, and for the turbidity h of the medium are derived. These expressions contain the molecular factors F_{is} and F_{anis} of isotropic and anisotropic light scattering, respectively, which are discussed with special reference to multi-component systems of: (i) anisotropic molecules with constant polarizability; (ii) dipolar molecules with hyperpolarizability; and (iii) non-dipolar molecules with quadrupole moments. Formulas are derived relating anisotropic light scattering and the optico-optical birefringence of the medium, and others relating isotropic light scattering to the divergence between the molecular refraction of the substance in the condensed and gaseous states. Applications to one-component systems are given."

9064:

Hurd, R. A. An electromagnetic diffraction problem involving unidirectionally conducting surfaces. Canad. J. Phys. **38** (1960), 1229–1244.

The author treats the scattering by a certain plane surface of an obliquely incident plane electromagnetic wave. This surface is the union of two half planes, each of which conducts in one direction only. The Fourier transform techniques used are similar to those employed in a previous paper by the same author [same J. **38** (1960), 168–175; MR **22** #1320].

The crux of the problem is the decomposition of a function into two factors, one of which is regular in an upper half of the complex plane, while the other factor is regular in a lower half plane. This decomposition is performed, and the asymptotic behaviour of the factors is determined. A discontinuity in some tangential field components across the boundary between the half planes is noted and discussed. Expressions are obtained for the far field. In addition to waves which are not exponentially attenuated in the direction normal to the plane, the existence of surface waves is demonstrated.

R. F. Millar (New York)

9065:

Garibyan, G. M. Radiation of a particle moving across the interface of two media with account of multiple scattering. Ž. Èksper. Teoret. Fiz. **39** (1960), 332–336 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 237–239.

Author's summary: "The spectral intensity distribution of the electromagnetic radiation emitted by a charged particle moving across the interface between two media is derived, taking the effect of multiple scattering into account."

9066:

Russek, Arnold. Effect of target gas temperature on the scattering cross section. Phys. Rev. (2) **120** (1960), 1536–1542.

Author's summary: "If a beam of particles is scattered by a gas or plasma, the differential scattering cross section that is observed experimentally may, in some cases, be altered a discernible amount by the random thermal motion of the target particles. In order to explore the feasibility of using this effect as a means of measuring high temperatures, or to correct for the temperature of the target in the event that the desired cross section must be measured at high temperatures, this work presents a theoretical study of the temperature dependence of the cross section. A general expression is obtained for the observed differential scattering cross section in the laboratory frame in terms of the differential cross section in the center-of-mass frame for the general case of arbitrary initial motion of the target. Detailed results for the temperature dependence are given for hard-sphere scattering (which is also applicable to low-energy neutron scattering) and for Coulomb scattering, in the approximation in which the projectiles are light and rapidly moving, compared to the targets. For hard-sphere scattering the case of equal projectile and target mass is also considered."

9067:

Buldyrev, V. S.; Molotkov, I. A. Investigation of exact solutions of time-dependent diffraction problems in the neighborhood of slide-off fronts. Dokl. Akad. Nauk SSSR **134** (1960), 1051–1054 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 979–983.

The authors study the properties of a time-dependent scalar wave field due to a Heaviside pulse in the region of the geometric shadow of a fixed circular cylinder or sphere. The medium external to the body is characterized by a constant velocity of propagation c_1 while the internal medium is characterized by c_2. Various asymptotic representations of the field are developed and explicit formulae are given for the cylinder in the cases $c_2 > c_1$ and $c_2 < c_1$. The methods used by the authors seem unduly complicated to the reviewer and strongly dependent upon the fact that circular or spherical boundaries are used. It is doubtful if this approach could be used successfully for more complicated boundary shapes (e.g., smooth convex cylinder). *R. Barakat* (Lexington, Mass.)

9068:

Barabanenkov, Yu. N. Coulomb scattering of charges in a strong magnetic field. Ž. Èksper. Teoret. Fiz. **38** (1960), 1512–1514 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1091–1092.

Author's summary: "Coulomb scattering of charges in a strong magnetic field, when the impact parameter is much greater than the Larmor radius, is considered. The solution of the problem indicates that the collisions cannot change the plasma electric (ion) distribution function, which is

symmetric with respect to the velocity component transverse to the magnetic field." *T. Regge* (Turin)

9069:

Govorun, N. N. Uniqueness of the solution to Fredholm's integral equations of the first kind in antenna theory. Dokl. Akad. Nauk SSSR **132** (1960), 91–94 (Russian); translated as Soviet Physics. Dokl. **5**, 509–512.

Author's summary: "Fredholm's integral equations of the first kind were obtained in same Dokl. **126** (1959), 49–52 [MR **22** #508] for the electric current density in an antenna which has rotational symmetry and surface impedance. Considered in that article was the problem of the uniqueness of the solution of the integral equations for the case of a perfectly conducting antenna."

G. Sinclair (Toronto)

9070:

Kontorovič, M. I. The application of the Laplace transform to the theory of rectangular cavity resonators and waveguides. Radiotehn. i Ėlektron. **4** (1959), 1475–1479 (Russian); translated as Radio Engrg. and Electronics **4**, no. 9, 93–101.

The field distribution in a rectangular cavity is expressed as a triple contour integral in terms of current excitation by means of Laplace transform. Residue evaluation would lead to the usual mode expansions. Application to slit excitation and to guides is indicated.

H. G. Baerwald (Albuquerque, N.M.)

9071:

Gercenšteĭn, M. E.; Vasil'ev, V. B. Waveguide with random nonhomogeneities and Brownian motion on the Lobachevsky plane. Teor. Veroyatnost. i Primenen. **4** (1959), 424–432. (Russian. English summary)

Authors' summary: "It has been shown that the probability density for the continuous random process of the resultant of independent values which are 'summed up' according to the fractional-linear law satisfies the diffusion equation of the Lobachevsky plane. Green's function of the diffusion equation, which apparently is a new distribution, has been found."

9072:

Yaškin, A. Ya. Π- and T-waveguides uniformly bent in the H-plane. Radiotehn. i Ėlektron. **4** (1959), 1831–1837 (Russian); translated as Radio Engrg. and Electronics **4**, no. 11, 111–120.

Author's summary: "A system of equations is derived by the method of field matching for the calculation of the propagation constant and critical wavelength in Π- and T-waveguides uniformly bent in the H-plane. The effectiveness of the system obtained is illustrated by the calculation of the critical wave for a curved Π-waveguide."

9073:

Tartakovskiĭ, L. B. On the theory of double-curvature reflectors. Radiotehn. i. Ėlektron. **4** (1959), 1821–1830 (Russian); translated as Radio Engrg. and Electronics **4**, no. 11, 95–110.

Author's summary: "The properties of the envelope of a single-parametric family of confocal paraboloids are studied and simple formulae are proposed for the design of reflector antennas with fan-shaped beams. The differences between the vertical patterns of a double-curvature reflector and a cylindrical reflector with the same curve in the central section are analysed. The position of the equivalent linear antenna is found for which it is possible to apply the aperture method to correct the curve of the central section. A specific method for this correction based on the application of the phase diagrams of linear antenna theory is illustrated."

9074:

Kazancev, Yu. N.; Kaznačeev, Yu. I.; Meriakri, V. V. Investigation of helical waveguides. Radiotehn. i. Ėlektron. **4** (1959), 1816–1820 (Russian); translated as Radio Engrg. and Electronics **4**, no. 11, 88–94.

Authors' summary: "An investigation of propagation of various wave types in helical waveguides both with dielectric and metal envelopes has been carried out. For the waveguide with dielectric envelope the dependence of the H_{01}-wave and H_{11}- and E_{11}-wave attenuation on the helix parameters was found. The attenuation of the H_{01}-wave was studied in straight and curved helical waveguides with metal envelope. The relationships found permit determining the optimum parameters of straight waveguides, rejection filters for undesired wave types, and bends for the H_{01}-wave."

9075:

Silin, R. A. Properties of a waveguide matrix with $2(P+1)$ terminals. Radiotehn. i Ėlektron. **4** (1959), 1628–1631 (Russian); translated as Radio Engrg. and Electronics **4**, no. 10, 92–98.

Author's summary: "The properties of waveguide matrices with $2(P+1)$-terminals are found as a function of their reversibility, horizontal and circular symmetry, the absence of loss and the absence of reactance. The scattering properties of the waveguide matrix are found as a function of the circular symmetry of the $2(P+1)$-terminals."

9076:

Armand, N. A. Surface-wave excitation at the open end of a coaxial line. Radiotehn. i Ėlektron. **4** (1959), 1609–1616 (Russian); translated as Radio Engrg. and Electronics **4**, no. 10, 60–72.

Author's summary: "The problem of surface-wave excitation by a coaxial line is solved in the article by the method of integral equations. Expressions are obtained for the surface-wave field, radiation field and reflected field. Quantitative results of the calculation of reflection coefficient and efficiency of excitation of surface-waves are presented."

9077:

Gutman, A. L. Resonant regions in waveguides with continuously variable cross-section. Radiotehn. i Ėlektron. **4** (1959), 2020–2030 (Russian); translated as Radio Engrg. and Electronics **4**, no. 12, 97–113.

Author's summary: "A method is proposed for calculating resonant wavelengths for resonant regions formed

in waveguides with continuously variable cross-sections. The method is applied to the calculation of so-called helical waveguides. Calculated and experimental results are compared."

9078:

Bulgakov, B. M.; Šestopalov, V. P.; Šiškin, L. A.; Yakimenko, I. P. Slow waves in a spiral waveguide with a plasma. Ž. Tehn. Fiz. **30** (1960), 840–850 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 791–801.

Authors' summary: "The dispersion equation is derived and studied for a system consisting of a spiral wound on a dielectric tube inside which there is a plasma, for the case in which there is a constant magnetic field applied along the axis of the system."

9079:

Caprioli, Luigi; Ronchi, Laura; Scheggi, Annamaria; Toraldo di Francia, Giuliano. Metodo per la sintesi di diagrammi di radiazione al finito. Alta Frequenza **29** (1960), 653–666.

9080:

Miranker, W. L. Periodic solutions of the wave equation with a nonlinear interface condition. IBM J. Res. Develop. **5** (1961), 2–24.

Author's summary: "In this paper we consider the problem of the voltage oscillations in a transmission line when a diode represented as a nonlinear capacitance is placed in shunt in that line. In particular we consider the response of this line to periodic driving voltages and study the periodic responses. This physical situation is shown to lead to the mathematical model of the wave equation in the voltage in a domain with an internal boundary (the interface), at which the voltage is required to satisfy a nonlinear jump condition. By an application of Gauss' theorem, the problem is reduced to a nonlinear difference-differential equation. In the case that the generator driving the line is matched to it, this family of equations reduces to a family of nonlinear differential equations. The paper is concerned with a study of the periodic solutions of these two classes of equations."

V. M. Papadopoulos (Melbourne)

9081:

Bertein, François. Sur quelques aspects du calcul matriciel des circuits non linéaires à oscillation locale. J. Phys. Radium **21** (1960), supplément, 137 A–148 A. (English summary)

Author's summary: "General principles are surveyed about matrix calculus applied to the above mentioned circuits. This method leads occasionally to the description of currents not by their various componenta, but by amplitude and phase modulations. Applications are presented concerning chiefly problems of oscillator stability."

9082:

Milić, Mirko M. Electrical networks which exhibit ideal-generator characteristics toward a branch. Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. No. 41 (1960), 8 pp. (Serbo-Croatian. English summary)

Author's summary: "The first part of the paper is concerned with the conditions of obtaining branch-voltage and current in a linear network, that do not depend on parameters which characterize that branch. By means of theorem I, this problem is proved to consist of finding the natural complex frequencies of the network with that branch removed or short-circuited. Under these circumstances the over-all network behaviour toward that branch is like that of an ideal-current/voltage generator.

"In the second part of the paper, theorem II gives the conditions under which the voltage or current of a branch is independent of another branch-impedance. These conditions, being general, are valid for reciprocal as well for non-reciprocal linear networks. However, since these conditions involve zeros of the transfer impedances and admittances, they may not be related to any natural complex frequencies of the given network.

"Two examples illustrate the application of these theorems."

9083:

Utkin, G. M. The conditions of stability of simultaneous oscillations in multiple-tuned oscillatory systems. Radiotehn. i Èlektron. **4** (1959), 1993–2003 (Russian); translated as Radio Engrg. and Electronics **4**, no. 12, 59–73.

Author's summary: "Simple conditions are obtained for the stability of simultaneous oscillations in symmetrical and partially symmetrical multiple-tuned oscillating systems. The stability of simultaneous oscillations in a system of coupled symmetrical oscillators with anode current characteristics approximated by polynomials of third and fifth degrees is investigated. The quasi-synchronous case for three oscillators is studied, the frequency of one of which is equal to the sum of the frequencies of the two others."

9084:

Yavič, L. R. Structure of the matrix and properties of reactive four-terminal networks. Radiotehn. i Èlektron. **4** (1959), 1796–1800 (Russian); translated as Radio Engrg. and Electronics **4**, no. 11, 57–64.

Author's summary: "The general case of irreversible reactive four-terminal networks is considered. It is proved that employing an ideal power transformer it is possible to extract from it a reversible four-terminal network having known properties."

9085:

Pazdzerskiĭ, V. A. On the theory of non-stationary processes in electrical circuits containing a semiconductor diode. Izv. Akad. Nauk UzSSR. Ser. Fiz.-Mat. **1958**, no. 2, 25–38. (Russian. Uzbek summary)

9086:

van Kampen, N. G. Non-linear thermal fluctuations in a diode. Physica **26** (1960), 585–604.

This is a remarkable paper on the subject of thermal noise in a nonlinear diode. The diode is connected in parallel with a condenser C, and is in contact with a heat

reservoir at temperature T. The analysis starts with Richardson's formula and develops from it the differential equation in time for the probability $P(N, t)$ of finding N electrons at the time t on the cathode of the diode. Transit time effects are neglected. The individual electron emissions from the diode electrodes are assumed uncorrelated. The time independent equilibrium distribution of N is determined first and is found to be Gaussian. While no exact solution is obtained for the time dependent probability $P(N, t)$, approximate solutions are obtained by means of expansions in terms of e^2/kTC (e—electron charge, k—Boltzmann's constant). The power spectra of the current and voltage fluctuations are determined to first order in e^2/kTC. These power spectra show the effect of the nonlinearity. In particular, the current power spectrum $S_I(f)$ as a function of the frequency f is

$$S_I(f) = \frac{4kT}{R_0}\left[1-\left(1+\frac{2e^2}{kTC}\right)(2\pi R_0(f))^{-2}+\cdots\right],$$

where R_0 is the diode resistance in the linear region. The Fokker-Planck equation is shown to lead to correct results to first order in e^2/kTC. It is further shown that the fluctuation-dissipation theorem does not apply to the nonlinear case. Finally a set of eigenvalue solutions is presented for the differential equation of $P(N, t)$. This set is found not to be complete, however, and therefore a rigorous solution is not obtained by this method.

H. A. Haus (Cambridge, Mass.)

9087:

Landauer, R. Shock waves in nonlinear transmission lines and their effect on parametric amplification. IBM J. Res. Develop. 4 (1960), 391–401.

The transmission line or telegraphist's equations without leakage or resistance parameters are studied in the case that the capacitance parameter is a function of the potential difference between the lines. The propagation of a periodic signal in the presence of this nonlinear distributed capacitance is studied by solving the basic differential equations to determine the nature of the resulting traveling waves. It is found that the signal is deformed during its propagation and that electromagnetic shock waves are generated. The analysis indicates that the shock wave will form in a distance which is short for any parametric amplification purposes. The growth of the shock and its decay are analyzed on the basis that the capacitance variations are small in comparison to the total capacitance. The analysis was motivated by the possibility of constructing transmission lines with a ferroelectric material. {In the opinion of the reviewer this paper is a notable contribution to the theory of nonlinear transmission lines.} *L. A. Pipes* (Los Angeles, Calif.)

9088:

Bolinder, E. Folke. General method of analyzing bilateral two-port networks from three arbitrary impedance or reflection coefficient measurements. Proc. National Electronics Conference, Chicago, 1958 (Vol. 14), pp. 363–376. National Electronics Conference, Inc., Chicago, Ill., 1959.

This paper presents an elegant method based on modern geometry for the determination of the equivalent T or the equivalent Pi lumped system that represents a given microwave transmission system at a given fixed frequency. The method is based on plotting the given load impedances and input impedances in the complex Z-plane and then projecting stereographically these impedances on the unit Riemann sphere. A generalization of the Pascal theorem by Klein is used to give the problem of determining the impedances of the T or Pi network an elegant geometrical interpretation. {The basic problem can, of course, be solved simply without recourse to this geometrical interpretation but, in the opinion of the reviewer, the method presented is useful in that it reveals some simple geometrical relations connecting the basic parameters of the system and therefore a better insight of the general problem is attained by this method.}

L. A. Pipes (Los Angeles, Calif.)

9089:

Bolinder, E. Folke. Geometric-analytic theory of transition in electrical engineering. Proc. IRE 47 (1959), 1124–1129.

When the elements of an electric circuit are varied, then the critical points of the impedance representation of the circuit move in the complex plane representing the impedance of the circuit. In the usual case, two points are considered. When a circuit element is altered these two points approach each other, finally they coincide and then they separate again. This movement represents the transition of the circuit from one state to another. In this paper, the transition between two states of a circuit are studied by the use of the complex plane and the Riemann sphere imbedded in a three-dimensional space.

The movement of the corresponding poles of the admittance function in the complex frequency plane and that of saddle-points in the same plane are studied by the same method. It is shown that the use of modern geometry yields an elegant and systematic description of the trajectories of the moving points involved.

L. A. Pipes (Los Angeles, Calif.)

CLASSICAL THERMODYNAMICS, HEAT TRANSFER

See also 8735, 8736, 8780, 8965.

9090:

Muki, Rokuro; Sternberg, Eli. Steady-state heat conduction in a circular cone. Z. Angew. Math. Phys. 11 (1960), 306–315. (German summary)

Let $T(r, \theta)$ denote steady-state temperatures in an unbounded conical medium $r>0$, $0\leqq\theta<\theta_0$, where r and θ are spherical polar coordinates and $0<\theta_0<\pi$. Write $p=\cos\theta$ and $p_0=\cos\theta_0$. The boundary value problem consists of Laplace's equation $r^2T_{rr}+2rT_r+[(1-p^2)T_p]_p=0$, the boundary condition $T(r, p_0)=T_0$ when $0<r<a$, $T(r, p_0)=0$ when $r>a$, and the order conditions $T=o(r^{-1})$ and $T_r=o(r^{-2})$ as $r\to\infty$, $T=O(1)$ and $T_r=O(r^{-1})$ as $r\to 0$. The authors solve this problem with the aid of the Mellin transformation $\int_0^\infty T(r, p)r^{s-1}dr$ to obtain both an integral representation and a series representation for $T(r, p)$. The representations involve Legendre functions $P_s(p)$. Numerical values of temperatures on the axis of the cone are displayed for the two cases $\theta_0=\pi/6$ and $\theta_0=\pi/2$.

R. V. Churchill (Ann Arbor, Mich.)

9091:
Chen, Shih-Yuan. One-dimensional heat conduction with arbitrary heating rate. J. Aerospace Sci. **28** (1961), 336–337.

9092:
Vlasov, V. P.; Markin, S. A. Solution to the non-stationary heat conduction problem for a rod with two masses attached to the ends. Analysis of the differential thermopile. Ž. Tehn. Fiz. **30** (1960), 1128–1133 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 1062–1068.

9093:
Crane, Lawrence, J. Thermal convection from a horizontal wire. Z. Angew. Math. Phys. **10** (1959), 453–460. (German summary)

By a combination of similarity methods and series expansions, velocity and temperature profiles are found for the steady flow, generated in uniform fluid by diffusion of heat, at a large distance above a heated wire placed horizontally. The solution is valid even when the density differences in the flow are large. *D. C. Pack* (Glasgow)

9094:
Eckhaus, Wiktor. Theory of flame-front stability. J. Fluid Mech. **10** (1961), 80–100.

The paper presents a theoretical study of whether a "crinkled" laminar flame will become planar or more crinkled. The author concludes that the theory leads to results which can be interpreted as being in qualitative agreement with experimentally observed behaviour.

{The reviewer found the model too crude, and the simplifying assumptions too numerous, for the conclusions to be convincing. The results appear not to be in accordance with exact theoretical studies of curved laminar flames.} *D. B. Spalding* (London)

9095:
Eschenroeder, Alan Q. Combustion in the boundary layer on a porous surface. J. Aerospace Sci. **27** (1960), 901–906.

Es wird die Lage der Diffusionsflamme in der laminaren Grenzschicht ohne Druckgradient untersucht, wenn ein brennbares Gemisch gleichmässig verteilt durch die poröse Wand zugeführt wird. Bei der Behandlung des Problems werden folgende Annahmen getroffen: (1) Die durch die Verbrennung hervorgerufene Energieänderung ist gross gegenüber der Dissipationsenergie. (2) Die kinetische Energie der thermischen Molekularbewegung ist gross gegenüber der kinetischen Energie der mittleren Strömungsgeschwindigkeit. (3) Der Energietransport durch Strahlung wird vernachlässigt. (4) Die Diffusion infolge von Druck- und Temperaturgradienten ist klein gegenüber der durch Konzentrationsgradienten.

Methoden zur Behandlung komplexer Reaktionsprobleme werden beschrieben. *L. Speidel* (Mülheim)

9096:
Davies, R. O. The kinetics of isomeric reactions. Proc. Roy. Soc. London. Ser. A **257** (1960), 541–551.

Author's summary: "A careful consideration of the kinetics of simple reacting systems near equilibrium shows that the observable relaxation times of the system involve not only transition probabilities (or rate constants) but also the dependence of these quantities on concentrations. By using a new method to describe the kinetics of 2- and 3-state processes (which introduces a time-dependent distribution function for the reaction co-ordinates) it is shown that, under certain conditions, the concentration-dependent terms can be evaluated in terms of the solution properties of the system."

H. L. Frisch (Murray Hill, N.J.)

9097:
Bartholomay, Anthony F. Molecular set theory: a mathematical representation for chemical reaction mechanisms. Bull. Math. Biophys. **22** (1960), 285–307.

QUANTUM MECHANICS

See also 8746, 8762, 8763, 8764, 8765, 8766, 8769, 8936, 9239, 9245, 9273.

9098:
Sillitto, R. M. ★Non-relativistic quantum mechanics: An introduction. Quadrangle Books, Chicago, Ill., 1960. x + 230 pp. $7.50.

This book is intended as an introduction to non-relativistic quantum mechanics destined especially for the British undergraduate honors student majoring in physics. The material is presented in a deductive manner, beginning with a chapter on general principles and developing numerous applications in subsequent chapters. It is a skilful compilation from some of the best sources avoiding most redundances and covering a great deal of material within 230 pages. The following topics are treated: The uncertainty principle and the commutation relations, the harmonic oscillator, angular momentum and spin, central field problems, perturbation theory and the variation method, many-electron atoms and the hydrogen molecule, quantum statistics, the classical limit, Ehrenfest's theorem and WKB-approximation, time-dependent perturbations and collision theory. Two chapters on radiation problems conclude the book.

Every chapter except the first and last is supplied with problems which illustrate and elaborate certains points of the text. In conformity with the prevailing tendencies in textbooks on this level the author uses the MKS system of units.

The deductive method of presentation used in this book has a great advantage over the inductive or historical one. The entire theory derives from a few general principles which are stated at the very beginning and which in turn receive their support from a few crucial empirical facts. The various examples are selected so as to exemplify the general formalism and they give a good illustration of the great range of applications which are covered by the fundamental principles. At the same time it is possible to show how wave mechanics and matrix mechanics emerge from the same principles and, throughout the book, they are treated on an equal footing.

The author has tried to write with the minimum of mathematical sophistication. The frequent use of phrases

such as: "It can be shown . . ." indicates that this minimum is just a trifle too low for an adequate presentation of the subject matter. The deficiency is especially felt in the first chapter where the author has unfortunately repeated much of the celebrated nonsense which has been written around the use of the δ-function. All this contributes to the feeling that quantum mechanics is presented here more as a technique rather than as a logically consistent physical theory, a technique which with the help of this book, can easily be learned and which produces quick and useful results. For the great majority of students this pragmatic point of view will suffice. There will almost certainly remain a minority with an inquisitive mind who will raise questions which are not answered in this book. No doubt the author had this minority in mind when he added a section at the end entitled "Background reading and further study", where one finds a selection of references to and comments about other presentations of quantum mechanics with some notable omissions (for instance Pauli's handbook article is not mentioned).

The two references to the original literature are not very representative and serve no useful purpose.

J. M. Jauch (Geneva)

9099:

Yourgrau, Wolfgang; Mandelstam, Stanley. ★Variational principles in dynamics and quantum theory. 2nd ed. Pitman Publishing Corp., New York-Toronto-London, 1960. xi+180 pp. $5.75.

The first edition [I. Pitman, London, 1955] of this book received an enthusiastic review in MR **16**, 873. The reviewer (A. H. Taub) there noted one omission, that the authors did not discuss the rôle of variational principles in modern field theories and quantum mechanics. This omission is remedied, at least partially, by the present edition, which contains an extra chapter on the formulations of quantum mechanics evolved by Feynman and Schwinger. The authors take as their starting point Feynman's integral formulation, of which they give a most elegant presentation, and derive from it the differential formulation due to Schwinger. {A minor point which troubles the reviewer is that their expression for the weight factors in the Feynman integral does not appear to agree with that of Morette [Phys. Rev. (2) **81** (1951), 848–852; MR **12**, 780].}

The other main addition to the book is an appendix on variational principles and chemical reactions. And there are two postscripts to the chapters on the old quantum theory and wave mechanics, based on some comments which Schrödinger made on the first edition. Altogether these alterations have improved an already excellent book.

P. W. Higgs (Edinburgh)

9100:

Dicke, Robert H.; Wittke, James P. ★Introduction to quantum mechanics. Addison-Wesley Publishing Co., Inc., Reading, Mass.-London, 1960. xi+369 pp. $8.75.

The scope of this book is non-relativistic quantum mechanics. It gives a clear and readable account of its subject keeping a good balance between abstract principle and its application. In fact it is a worthwhile addition to an already vast literature. The concluding chapter gives a much fuller account of quantum statistical mechanics than is usually to be found in texts of this kind.

J. C. Polkinghorne (Cambridge, England)

9101:

Schönberg, Mario. Quantum mechanics and geometry. II. An. Acad. Brasil. Ci. **30** (1958), 1–20.

This is part two of five. The notation is the same as in the review of part one [Nuovo Cimento (10) **6** (1957), supplemento, 356–380; An. Acad. Brasil Ci. **29** (1957), 473–499; MR **20** #2190, 2191]. The author uses the metric g_{jk} to define new operators d_j and q^j via

$$2^{1/2}d_j = \{I_j\} - g_{jk}\{I^k\}, \quad 2^{1/2}q^j = \{I^j\} + g^{jk}\{I_k\}.$$

In the case $g_{jk}=\delta_{jk}$, q^j and $p_j=-id_j$ are self-adjoint and can represent coordinates and momenta of a particle. He then constructs a calculus of generalized functions of the p's (and an analogous one of the q's) writing

$$F(p) \sim \sum_r C_{r_1\cdots r_n} \prod_{j=1}^{n} h_{r_j}(p_j),$$

with $h_r(u)=(\pi^{1/2}r!2^r)^{-1/2}\exp(-\frac{1}{2}u^2)H_r(u)$ and H_r the Hermite polynomial $H_r(u)=(-1)^r\exp(u^2)D^r\exp(-u^2)$. The $F(p)$ correspond to ordinary functions if the series converges, but also are defined if not, e.g.,

$$\prod_{j=1}^{n}\delta(p_j-a_j 1L_n) = \sum_r \prod_{j=1}^{n} h_{r_j}(a_j)h_{r_j}(p_j).$$

Fourier transformation and differentiation can be defined for all $F(q)$ because they correspond to algebraic operations on the h_r. A further development shows that L_n (an extension of the L_n defined in the review of part one obtained by adjoining a projection operator $\{P\}$ satisfying $\{I_j\}\{P\}=0$, $\{P\}\{I^j\}=0$) is the analysis of the functions $F(q)$ of n commuting non-numerical variables q^j, the $G(p)$ corresponding to functions of the derivatives $\partial/\partial q^j$, viz., $G(p)F(q)\{P\}=(G(-i(\partial/\partial q))F(q))\{P\}$. In part one an affine calculus of points was constructed using L_n. Here an analogous metric calculus of points is constructed. It is shown that one can pass from one to the other by an automorphism of L_n. There is further detailed discussion of the geometrical significance of this calculus too complicated to summarize.

A. S. Wightman (Princeton, N.J.)

9102:

Klauder, John R. The action option and a Feynman quantization of spinor fields in terms of ordinary c-numbers. Ann. Physics **11** (1960), 123–168.

According to Feynman, the transition amplitude of a quantum mechanical system can be written as an integral over all intervening histories of $\exp iI$ where I is the classical action corresponding to the particular system and history. In the application of this idea to spinor fields it has been customary to introduce anti-commuting "classical" fields. [See, for example, P. T. Matthews and A. Salam, Nuovo Cimento (10) **2** (1955), 120–134; MR **17**, 693.] The present author remarks that to make the integral well defined one has to extend the definition of I from "continuous" to "unruly" histories and that there are several possibilities for such an extension which agree for "continuous" histories. By an appropriate choice of extension he obtains a quantization of spinor fields. {In the reviewer's opinion, the mathematical arguments offered in support of this intriguing idea are almost entirely heuristic.} The cases of non-relativistic field with Bose and Fermi statistics and relativistic spinor fields are discussed in detail. A principal tool used by the author is what he calls expansion in an over-complete family of states. The

essential idea of this can be seen in the case of a system of one degree of freedom described by the canonical variables q and p with $[q, p]=i$. If one introduces $a=(1/\sqrt{2})(q+ip)$, $a^*=(1/\sqrt{2})(q-ip)$ one can diagonalize a as follows. (See a forthcoming paper by V. Bargmann in Comm. Pure Appl. Math.) Consider the set H, of entire functions in one complex variable, z, for which the scalar product

$$(f_1, f_2) = \pi^{-1} \int \exp(-|z|^2)\overline{f_1(z)} f_2(z)\, dx\, dy$$

is defined. Realize the operator a as $\partial/\partial z$. Then a^* will be multiplication by z, and is therefore diagonal. Other operators can be written as integral operators whose kernels are functions of two complex variables. The overcomplete family of states is the set of eigenfunctions of a, namely e^{bz}, where b is the eigenvalue. H may be very useful for calculations. Compare M. Schönberg [see review #9101] for a somewhat similar calculus.

A. S. Wightman (Princeton, N.J.)

9103:

Kar, K. C. On the perturbation theory with time-dependent wave functions. Indian J. Theoret. Phys. **7** (1959), 65–72.

Author's summary: "The well-known perturbation formula used in radiation problems has been derived from an entirely wave statistical point of view."

9104:

Kar, K. C. Note on Dirac method of linearising the relativistic Hamiltonian. Indian J. Theoret. Phys. **7** (1959), 53–55.

Author's summary: "It is pointed out that conditions for linearising the relativistic Hamiltonian by Dirac method are ignored in its subsequent use. It is also shown that all the Dirac results can be obtained if unit vectors are introduced instead of unnecessarily complicated matrices."

9105:

Zel'dovič, Ya. B. On the theory of unstable states. Ž. Èksper. Teoret. Fiz. **39** (1960), 776–780 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 542–545.

Author's summary: "A perturbation theory is developed and an expression is given for the amplitude (corresponding to a given initial state) of a state that decays exponentially with time. A final expression is obtained which plays the role of the norm of such a state."

9106:

Hōshi, Reiun; Wakasa, Akira. Dirac particles with anomalous magnetic moment in arbitrary external electromagnetic fields. Sci. Rep. Kanazawa Univ. **7**, 1–4 (1960).

It is well-known that, as a consequence of the two-component neutrino theory, and the universal fermion interaction, the electrons emitted in β-decay, and the muons emitted in the π-μ decay are longitudinally polarised. K. M. Case and others have discussed the effects of external electric and magnetic fields on polarised electron beams in the region of relativistic energies of the particles. The authors in this paper have treated the same problem, taking into consideration further the anomalous magnetic moments of the Dirac particles, and indicating the corrections that arise. For this purpose, they introduce phenomenologically an additional term in the Hamiltonian operator representing the anomalous magnetic moment, and carry out the polarisation analysis. The results obtained are that (i) in the case of an external pure magnetic field, a longitudinally polarised beam will be depolarised, and it is even possible to change a state of longitudinal polarisation to one of transverse polarisation, and (ii) in the case of a pure electric field, the particles are depolarised. The authors further remark that, from the degree of depolarisation, one can estimate the order of magnitude of the anomalous magnetic moment. It is to be hoped that the authors will undertake a further investigation to substantiate this remark quantitatively.

B. S. Madhavarao (Poona)

9107:

Saxl, I. The quantum theory of multipole radiation in a dielectric. Czechoslovak J. Phys. **10** (1960), 650–658. (Russian summary)

Author's summary: "The potentials of an electromagnetic field of multipoles in a dielectric, which is realized by a dielectric sphere having a perfectly conducting surface, are derived. The diagonal values of the energy for the z component of the angular momentum and the square of the angular momentum of the field are determined and also the ratio between the z component of the angular momentum and the energy and the ratio between the square of the angular momentum and the square of the energy. It is shown that the total angular momentum can be divided in the usual way into orbital and spin parts but that these parts cannot be interpreted as the orbital and spin angular momentum because their eigenvalues cannot be the eigenvalues of any operator of infinitesimal rotation. In the commutation rules of a multipole field the vector character of the field is to a certain extent suppressed and the spin of the photon in a state with a certain value of the energy, parity z component of the angular momentum and the square of the angular momentum is not defined."

9108:

Yamazaki, Kazuo. A new variational method for calculating excited energy levels and its application to the anharmonic oscillator. Nuclear Phys. **24** (1961), 313–317.

Author's summary: "A simple variational method for the calculation of the excited levels of a quantal system is developed and as an illustration it is applied to the anharmonic oscillator. For this case it agrees in the weak coupling limit with the perturbation calculation and for the strong coupling limit it also agrees with the exact values within a 1% error."

9109:

Kurşunoğlu, Behram. Complex orthogonal and anti-orthogonal representation of Lorentz group. J. Mathematical Phys. **2** (1961), 22–32.

Essentially an account of certain finite-dimensional representations of the homogeneous Lorentz group, with emphasis on the reversal operations. The analytical form of these representations is thought to be suggestive in

connection with elementary particle classification, and especially with regard to the notion of charge.

I. E. Segal (Cambridge, Mass.)

9110:

McIntosh, Harold V. Symmetry-adapted functions belonging to the symmetric groups. J. Mathematical Phys. **1** (1960), 453–460.

Author's summary: "Young's factorization of idempotents belonging to the symmetric groups is given a necessary and sufficient characterization, by means of a lemma due to Burrow. The use of these idempotents is contrasted with Yamanouchi's representation, and finally the equivalence of Löwdin's path diagram method to the group-theoretical treatment of the angular momentum states arising from the coupling of an assemblage of spin $\frac{1}{2}$ particles is demonstrated." *H. K. Farahat* (Sheffield)

9111:

Kar, K. C.; Paria, B. N. Note on the wavestatistical derivation of Klein-Nishina formula for Compton scattering. Indian J. Theoret. Phys. **7** (1959), 57–60.

Authors' summary: "The wave-statistical theory of Klein-Nishina formula and the mechanisms of the different processes involved have been critically discussed."

9112:

Agranovič, Z. S.; Marčenko, V. A. Reconstruction of the potential energy from the dispersion matrix. Amer. Math. Soc. Transl. (2) **16** (1960), 355–357.

Translation of Uspehi Mat. Nauk **12** (1957), no. 1 (73), 143–145 [MR **19**, 278].

9113:

Livšic, M. S. The method of non-selfadjoint operators in dispersion theory. Amer. Math. Soc. Transl. (2) **16** (1960), 427–434.

Translation of Uspehi Mat. Nauk **12** (1957), no. 1 (73), 212–218 [MR **19**, 216].

9114:

Gribov, V. N. Asymptotic behaviour of the scattering amplitude at high energies. Nuclear Phys. **22** (1961), 249–261.

The Mandelstam representation is used to explore the properties of the scattering amplitude at high energies, corresponding to small momentum transfer and high angular momenta. It is then shown that if this amplitude is of the nature of diffraction scattering, the unitarity condition will not be satisfied. Furthermore, unitarity and analyticity together require that the total cross section decrease at high energies faster than $1/\ln E$. The arguments are partly of a heuristic nature and are made only for spin-less particles. *F. Rohrlich* (Iowa City, Iowa)

9115:

Zav'yalov, O. I. "Nonphysical" thresholds in perturbation theory. Dokl. Akad. Nauk SSSR **133** (1960), 64–67 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 743–746.

Sufficient conditions are given for the presence of anomalous thresholds in the dispersion relations of quantum field theory. These conditions are shown to be valid for any diagram and in any order of perturbation theory. Two auxiliary theorems are proven.

F. Rohrlich (Iowa City, Iowa)

9116:

Englefield, M. J.; Margolis, B. Electromagnetic scattering of hyperons. Nuovo Cimento (10) **19** (1961), 274–276. (Italian summary)

Authors' summary: "The interaction between the magnetic moment of a neutral hyperon and the charge of a nucleus causes large polarizations in small-angle scattering. The feasibility of using the effect to measure the magnetic moment is considered."

9117:

Minguzzi, A. π-$\mathcal{N}$ forward dispersion relations with electromagnetic corrections. Nuovo Cimento (10) **19** (1961), 981–986. (Italian summary)

Author's summary: "Meson-nucleon dispersion relations with e.m. correction to e^2 order are proved, and a conjecture about the unphysical range absorptive part is discussed together with a comment about the Chew-Low extrapolation procedure."

9118:

Barut, A. O.; Ruei, K. H. Extra solutions of the dispersion relations and resonance scattering. Nuclear Phys. **21** (1960), 300–309.

It is remarked that the system of dispersion relations, together with unitarity, gives rise to integral equations with many solutions. For potential scattering it is shown that only one corresponds to a given Hamiltonian, namely, the unique solution with a power-series expansion in the coupling constant. The extra solutions correspond to situations in which arbitrary unstable states are included in the spectrum. Conversely the theory with the inclusion of arbitrary unstable states gives rise to resonances and satisfies the dispersion relations. Since potential scattering cannot give rise to any resonances, they must correspond to the extra solutions. So it would seem that there is no method for predicting resonant behaviour: the existence of unstable states must be put into the theory.

The treatment of the paper follows the formal scattering theory in, say, Dirac's *The principles of quantum mechanics* [3d ed., Clarendon, Oxford, 1947; MR **9**, 319]. The conclusions of the paper are not proved by completely rigorous methods. *R. F. Streater* (Princeton, N.J.)

9119:

Bowcock, J.; Martin, A. A Mandelstam representation in potential scattering. Nuovo Cimento (10) **14** (1959), 516–526. (Italian summary)

The authors discuss the analytic properties of the scattering amplitude corresponding to the scattering by a central potential which consists of a mixture of ordinary and exchange potential each of which is a linear superposition of Yukawa potentials. It is shown that the nth Born approximation is analytic in the complex $\cos\theta$ plane (θ = angle of scattering) but having a cut along the

positive real axis extending from a point depending on n to ∞. The convergence of the Born series for a pure Yukawa potential is proved inside an ellipse in the $\cos\theta$ plane with foci at ± 1. *H. Feshbach* (Cambridge, Mass.)

9120:

Wanders, G. The analytic properties of scattering amplitudes in sixth-order perturbation theory. Nuovo Cimento (10) **18** (1960), 580–594. (Italian summary)

The author discusses the analytic properties of sixth order graphs in perturbation theory. These are of interest because they include the first case of a graph not having a proper dual diagram. It is asserted that all these contributions satisfy the Mandelstam representation for the case of nucleon-nucleon scattering.

J. C. Polkinghorne (Cambridge, England)

9121:

Malenka, B. J.; Valk, H. S. High-energy potential scattering with short-range forces. Phys. Rev. (2) **122** (1961), 931–933.

Authors' summary: "An attempt is made to separate out long- and short-range effects for high-energy elastic scattering. Within the context of a high-energy approximation, expressions for the scattering amplitudes are obtained for the cases $kR \gg ka \gg 1$ and $kR \gg 1 > ka$, where R and a denote the long and short ranges, respectively. For the latter case, the entire short-range effect is included in a phenomenological S-wave term while the long-range contributions are written explicitly."

9122:

Hara, Osamu. Collective correlation between vacuum nucleons and the S-wave pion-nucleon scattering. Progr. Theoret. Phys. **24** (1960), 495–511.

Introducing Bogoliubov's method in the theory of superconductivity, the author calculates the phase shift which corresponds to the total isotopic spin of 3/2 in the scattering of S-wave pion by nucleon. Although some rough approximations are used, an overall agreement with experiment is obtained. *N. Kumasawa* (Tokyo)

9123:

Halfin, L. A. The influence of inelastic processes on elastic scattering in the vicinity of thresholds for inelastic reaction modes. Ž. Èksper. Teoret. Fiz. **39** (1960), 1020–1022 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 709–710.

Author's summary: "It is shown that the specific effects that occur in the vicinity of thresholds for inelastic reactions can be treated in the general case by using dispersion relations."

9124:

Cutkosky, R. E. Singularities and discontinuities of Feynman amplitudes. J. Mathematical Phys. **1** (1960), 429–433.

This paper solves the important problem of obtaining a concise expression in perturbation theory for the discontinuity of the scattering amplitude on encircling one of its singularities. The rule is simply that those lines which appear in the corresponding dual diagram are to have their propagators replaced by mass-shell delta functions. This is shown to be a natural generalisation of the prescription provided by unitarity in the physical regions. However in its application rather subtle problems relating to the choice of branch for the multivalued functions involved have to be solved.

J. C. Polkinghorne (Cambridge, England)

9125:

Fowler, T. K.; Watson, K. M. The quasi-elastic scattering of fast particles by atomic nuclei. Nuclear Phys. **13** (1959), 549–575.

The scattering and reactions induced by fast particles striking a nucleus are considered in the high-energy limit in which the impulse and multiple scattering approximations are valid. Both elastic and inelastic scattering cross-sections are obtained in terms of form factors as well as the appropriate Fourier transforms of correlation functions for space symmetric and anti-symmetric states. A multiple scattering correction including terms involving three-particle correlation functions are derived. It is assumed that these correlation functions may be decomposed into pair-correlation functions. These results are applied to the analysis of the scattering of high energy electrons, protons and pions from nuclei. Evidence for the existence of a repulsion in the nucleon-nucleon interaction at small energies is adduced.

H. Feshbach (Cambridge, Mass.)

9126:

Blankenbecler, R.; Goldberger, M. L.; Khuri, N. N.; Treiman, S. B. Mandelstam representation for potential scattering. Ann. Physics **10** (1960), 62–93.

This paper contains most of the elements of a proof of the Mandelstam or double dispersion representation of the Schrödinger scattering amplitude for scattering by a class of potentials which can be represented as a linear superposition of Yukawa interactions. Starting from the dispersion relation in the energy variable (s) which holds for an even wider class of potentials, the authors are led to investigate the analyticity properties in the momentum transfer variable (t) of the imaginary part of the amplitude. With the help of the Fredholm solution of the Schrödinger equation, analyticity in the finite t plane, exclusive of the cut associated with the potential, is established. Assuming no essential singularity at infinity, the double representation follows. The important question of the actual number of subtractions needed is unresolved, except for the determination that if bound states are present, the absence of a suitable number of subtractions is inconsistent with the unitarity requirement. The extension of the double representation to exchange potentials is carried out. The unitarity condition is applied in the absence of bound states and it is shown how the spectral function entering the dispersion integral can be determined as the limit of a convergent sequence of polynomials in the strength of the potential. In this case the Mandelstam representation plus unitarity uniquely determine the scattering amplitude. In the presence of a bound state, e.g., a single s-state, it is shown that the determination of the scattering amplitude requires in addition the solution of a mapping problem whose complexity depends on the number of subtractions. As an alternative approach to the problem of determining the

scattering amplitude, dispersion relations for the partial wave amplitudes, which follow from the Mandelstam representation, are established. The (N/D) method of solving these is rehearsed. In both this method and the previous mapping problem it is made plausible that bound state energies and effective coupling strengths may be determined by analytic continuation of the solutions in the potential coupling strength, indicating once more that one has a complete dynamical scheme equivalent to the Schrödinger equation. *A. Klein* (Philadelphia, Pa.)

9127:

Moroz, L. G.; Fedorov, F. I. The scattering matrix with the Pauli interaction taken into account. Ž. Èksper. Teoret. Fiz. **39** (1960), 293–303 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 209–215.

The free electromagnetic field can be described by a 10-component object ψ satisfying the linear homogeneous equation $(\gamma_\rho\nabla_\rho+\gamma_0)\psi=0$, where the γ-matrices are 10×10 and satisfy the Kemmer-Duffin algebra. The first four components of ψ are the potential four-vector A_μ and the last six are the linearly independent components of the electromagnetic field tensor $F_{\mu\nu}$. When this formulation is used in conjunction with the usual Dirac equation for the electron field φ to construct a quantum electrodynamics, it lends itself in a natural way to an interaction Lagrangian consisting of the usual term $-ie\bar\varphi\beta_\mu A_\mu\varphi$ (β_μ = Dirac matrices) and a "Pauli interaction" $\frac{1}{4}iel\bar\varphi F_{\mu\nu}\beta_\mu\beta_\nu\varphi$ (l = constant). Using the Coulomb gauge the corresponding Hamiltonian is separated into photon and Coulomb part. The latter includes also the Coulomb contributions of the Pauli interaction. The usual scattering matrix formulation in the interaction picture is then constructed. The evaluation of S-matrix elements and transition probabilities in perturbation expansion is discussed.

F. Rohrlich (Iowa City, Iowa)

9128:

Passatore, G. On the longitudinal depolarization of beams of spin 1/2 particles by multiple Coulomb scattering. Nuovo Cimento (10) **18** (1960), 532–544. (Italian summary)

Author's summary: "The average longitudinal depolarization of an electron beam by multiple Coulomb scattering has been evaluated. The treatment is based on the iteration, extended to a whole succession of scatterings, of the matrix which describes the variation of the polarization state for a single scattering, and on the average of the iterated matrix on all possible scattering successions. One gets a very simple formula for the average longitudinal polarization degree of the whole outgoing beam, and the final expressions are evaluated by using the second Born approximation. Numerical values of the depolarization for Al and Au targets and for several values of energy and thickness are given; these values in all cases considered are never greater than few percent."

9129:

Efremov, A. V.; Meščeryakov, V. A.; Širkov, D. V. Pion-nucleon scattering at low energies. I. Ž. Èksper. Teoret. Fiz. **39** (1960), 438–449 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 308–315.

Authors' summary: "Integral equations for πN-scattering partial waves in the low-energy region are derived on the basis of the Mandelstam representation. The kernels of the equations contain the lowest $\pi\pi$-scattering phases."

9130:

Titc, T. [Tietz, T.]. Electron scattering in the Thomas-Fermi model. Ž. Èksper. Teoret. Fiz. **39** (1960), 1347–1348 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 940.

9131:

Kukin, V. D.; Frenkin, A. R. "Ghost" states and the condition of crossing symmetry. Dokl. Akad. Nauk SSSR **133** (1960), 49–51 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 698–700.

From the introduction: "The suggestion has been made [S. Mandelstam, Phys. Rev. (2) **112** (1958), 1344–1360; MR **20** #5057] that the problem of 'ghost' states arises because of the neglect of the condition of crossing symmetry. This suggestion is based on results obtained for the Lee model [K. W. Ford, ibid. **105** (1957), 320–327; MR **18**, 626], which does not possess crossing symmetry, and for the charged scalar meson theory with fixed source [L. Castillejo, R. H. Dalitz and F. J. Dyson, ibid. **101** (1956), 453–458], which possesses crossing symmetry. We here construct a model with a fixed source which has an exact solution and which, although it possesses crossing symmetry, at the same time can have 'ghost' states for sufficiently large values of the renormalized coupling constant g^2."

9132:

Kolkunov, V. A.; Okun', L. B.; Rudik, A. P.; Sudakov, V. V. Location of the nearest singularities of the $\pi\pi$-scattering amplitude. Ž. Èksper. Teoret. Fiz. **39** (1960), 340–344 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 242–244.

9133:

Kaempffer, F. A. Representation of space inversion, time reversal, and particle conjugation in quantum field theory. Canad. J. Phys. **39** (1961), 22–34.

In this paper the author proposes explicit representations for the operators of parity (Π), charge conjugation (Γ) and time reversal (Θ) acting on a field, in terms of the creation and annihilation operators (C.A.O.) of the same field in momentum space. This procedure is suggested by analogy with the case of the mechanical operators (such as momentum, energy, angular momentum) which can be expressed as simple bilinear combinations of the C.A.O.

As a starting point one recalls the transformation laws of the C.A.O. of a field under Π, Γ and Θ. Bilinear expressions of the C.A.O. are then constructed, which act on the C.A.O. themselves in the same way as Π, Γ and Θ. These expressions, however, look much more complicated than the corresponding ones valid for mechanical operators. In particular, if the C.A.O. considered belong to the wave number $\mathbf{k}$, the combination of them which represents the parity operator is different for $k_3>0$ and $k_3<0$. A unique formula can still be written, by introducing the

singular step function $\theta(\mathbf{k})$, which is equal to 1 if $k_3>0$ and 0 if $k_3<0$. In fact one obtains finally

$$\Pi = \exp\{\tfrac{1}{2}i\Pi[\Omega_1\theta(\mathbf{k})+\Omega_2\theta(-\mathbf{k})]\},$$

where Ω_1, Ω_2 are given bilinear combinations of the C.A.O. Similar formulae hold also for the operators Θ and Γ. The computations are worked out for spin 0 and $\frac{1}{2}$ fields; but the argument can easily be extended to fields of higher spin. *E. Ferrari* (Geneva)

9134:

Ter-Martirosyan, K. A. Equations for the Mandelstam representation spectral functions. Ž. Èksper. Teoret. Fiz. **39** (1960), 827–840 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 575–583.

Author's summary: "On the basis of unitarity, a closed system of equations is derived for the Mandelstam-representation spectral functions, fully symmetric with respect to the three channels of the four-particle vertex. The question of consistency of equations obtained by applying unitarity in different channels is clarified. If the integral representation is written down with subtractions, one obtains a set of coupled equations for the one-variable and two-variable spectral functions. Consistent iteration of these equations corresponds to taking into account the contribution (or part of the contribution) from a number of Feynman diagrams consisting of two parts connected by two lines. This set of equations reduces to a Chew-Mandelstam type equation if the terms containing the two-variable spectral functions are neglected."

9135:

Braun, M. A. Normalization constants of state vectors in field theory. Ž. Èksper. Teoret. Fiz. **39** (1960), 737–740 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 514–516.

Author's summary: "By means of weak-convergence methods it is shown that the normalization constant introduced by Van Hove [Physica **21** (1955), 901–923; **22** (1956), 343–354; MR **21** #3218, 3219] and De Witt [Phys. Rev. (2) **100** (1955), 905–911; MR **17**, 810] for an n-particle state in field theory is equal to the product of the so-called vacuum constant and a factor Z^n, where Z is the constant of wave-function renormalization in the usual formalism of field theory."

9136:

Grossmann, A. Algebraic characterization of the TCP operation. J. Mathematical Phys. **1** (1960), 424–428.

The author considers the proper orthochronous Lorentz group and defines an operation as being something acting on Lorentz covariant quantities in such a way as to preserve Lorentz covariant relations. It is shown that an operation is either a Lorentz transformation or the product of a Lorentz transformation with the TCP operation.

J. C. Polkinghorne (Cambridge, England)

9137:

Brodskiĭ, A. M. Effect of weak interactions on the electromagnetic properties of fermions. Ž. Èksper. Teoret. Fiz. **39** (1960), 322–325 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 229–231.

Author's summary: "Self-mass effects and the singularities of behavior of fermions in a magnetic field, brought about by the universal Fermi interaction, are considered in the first order in the constant G_F. It is shown that parity nonconservation in weak interactions does not induce parity nonconservation in the field mass. An expression describing the effect of weak interactions on the electromagnetic properties of the fermions is found. The expression is proportional to the external current and is $\sim 10^{-2}$ times the similar expression obtained by taking into account the vacuum terms in electrodynamics."

9138:

Šahbazyan, V. A. On the infrared catastrophe in scalar quantum electrodynamics. Ž. Èksper. Teoret. Fiz. **39** (1960), 484–490 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 340–343.

Author's summary: "The asymptotic behavior of the four-vertex function in the infrared region is studied in the quantum electrodynamics of particles with zero spin. A procedure for removing the infrared divergences and summing the probabilities is discussed for the process of scattering of charged mesons by charged mesons with the emission of an arbitrary number of soft quanta."

9139:

Vanyašin, V. S. Second-order equations for spinor fields. Ž. Èksper. Teoret. Fiz. **39** (1960), 337–339 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 240–241.

Author's summary: "Second order equations for fermions are investigated on the basis of the Lagrangian formalism. It is shown that an additional degree of freedom of the particles is due to two orientations of an imaginary electric dipole moment, which, however, may have a real radiation correction."

9140:

Barbašov, B. M.; Efimov, G. V. A method of solution of field-theoretical problems involving a static nucleon. Ž. Èksper. Teoret. Fiz. **39** (1960), 450–460 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 316–323.

Authors' summary: "A new method of solving field-theoretical problems involving a static nucleon is proposed. The formalism is not connected with the magnitude of the coupling constant and is based on the matrix methods for solving linear differential equations developed by Lappo-Danilevskiĭ. The solution is obtained in the form of a series for which the concrete form of the nth term is known. The S matrix has been derived for a 'charged' scalar theory with a static source. The renormalization constants are calculated by the method proposed. In this model, the transition to a point interaction does not lead to the appearance of logarithmic singularities in the renormalized charge g_r."

9141:

Sutula, V. D. On causality in nonlocal field theory. Dokl. Akad. Nauk SSSR **133** (1960), 77–80 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 757–760.

From the introduction: "In connection with the increased possibilities of experimental technique, the question of the limit of applicability of existing theory has become timely. A natural generalization of the ordinary field theory is nonlocal field theory. A number of papers have been devoted to this subject [C. Bloch, Danske Vid. Selsk. Mat.-Fys. Medd. **27** (1952), no. 8; M. Chrétien and R. E. Peierls, Nuovo Cimento (9) **10** (1953), 668–676; MR **15**, 81]. Later, however, it turned out that attempts to combine the demands of relativistic invariance, of macrocausality, and unitariness encounters considerable difficulties [E. C. G. Stueckelberg and G. Wanders, Helv. Phys. Acta **27** (1954), 667–682; MR **16**, 1185; C. Møller, Proc. Internat. Conf. Theoret. Phys. (Kyoto and Tokyo, 1953), pp. 13–17; discussion, 17–23; Science Council of Japan, Tokyo, 1954; MR **16**, 656]. We shall show that this demand may be combined in the second order of perturbation theory if the class of form factors is somewhat narrowed in comparison with that introduced by Chrétien and Peierls [op. cit.]. Moreover, the macrocausality of the model of field theory considered by Bloch [op. cit.] will be established."

9142:

Borneas, M. The Lagrange function in a general problem. Nuovo Cimento (10) **16** (1960), 806–810. (Italian summary)

Author's summary: "A general formalism, based upon the use of a Lagrange function with n-order derivatives with respect to σ independent variables, is presented."

John G. Taylor (Baltimore, Md.)

9143:

Scarfone, L. M. Renormalization in a combined Lee-Machida field theory. Nuovo Cimento (10) **19** (1961), 377–381. (Italian summary)

Author's summary: "The renormalization of a combined Lee-Machida field theory is re-examined. It is shown that the energy of a new V-ghost found in an earlier investigation is sensitive to the renormalization procedure."

9144:

Ida, Masakuni. Space-time description of collision and decay processes. Progr. Theoret. Phys. **24** (1960), 1135–1148.

An attempt to express the scattering amplitude on the basis of Lorentz invariants and existence of vacuum state. It is for the characteristic feature that the author considers observation function (four-dimensional wave packet) which acts as a rôle of weight of the field and imposes the stable condition on the mass spectrum so that it may be applied to unstable particle too. The Bethe-Salpeter equation for stable composite particles and physical states of unstable particles are also discussed. *N. Kumasawa* (Tokyo)

9145:

Ishida, Shin. Pion-nucleon scattering and pion-pion interaction. Progr. Theoret. Phys. **24** (1960), 1262–1274.

A calculation of the s-wave phase shifts of pion-nucleon scattering is carried out taking into account also the pion-pion interaction. The interaction Lagrangian contains in addition to the pion-nucleon coupling constant G, three adjustable parameters, namely the range of the pion-nucleon interaction, Λ_σ, the range of the pion-pion interaction, Λ_π, and a constant, $V(0)$, which gives the strength of the pion-pion interaction and is related to the coupling constant λ of the elementary interaction $\lambda\phi^4$.

A reasonable choice of the above parameters ($\Lambda_\sigma \simeq 5\mu$, $\Lambda_\pi \simeq 3.5\mu$, $(2\pi)^{-1}(G^2/2M + V(0)) \simeq (0.4 \sim 0)\mu^{-1}$) gives good agreement with the experiment. Some earlier results of other authors are summarised and discussed.

An evaluation of λ from $V(0)$ is made by means of a simple perturbative calculation. It is found $\lambda/4\pi \simeq 0.4$: this value seems to be consistent with the value obtained from the analysis of the $(\pi \to 2\pi)$ process at 1.4 GeV.

Finally, the small p-wave phase shifts are evaluated: the results obtained are in qualitative agreement with the results of the phase shift analysis by Lomon and Chiu.

E. Ferrari (Geneva)

9146:

Caianiello, E. R.; Okubo, S. An analysis of infrared divergences. Nuovo Cimento (10) **17** (1960), 355–364. (Italian summary)

In a series of papers Caianiello has reformulated renormalizable field theory in terms of certain algebraic expressions like pfaffians, hafnians, etc. [Nuovo Cimento (10) **14** (1959), supplemento, 177–191; MR **22** #550, where further references are given]. The present paper is an application of this formulation to the infra-red divergences occurring in electron-electron scattering (forward scattering being explicitly excluded). Their techniques permit the authors to factor out of the scattering matrix element the infrared divergent contributions independent of perturbative methods. The results agree with those known from other approaches to the problem.

F. Rohrlich (Iowa City, Iowa)

9147:

Fowler, M.; Landshoff, P. V.; Lardner, R. W. Some properties of single loop diagrams in perturbation theory. Nuovo Cimento (10) **17** (1960), 956–963. (Italian summary)

This paper applies the method of J. Tarski [J. Mathematical Phys. **1** (1960), 149–163; MR **22** #4468] to the analysis of loop diagrams in which internal stability conditions are relaxed. It is pointed out that there cannot be cut-plane analyticity in two or more variables for production processes.

{There is a misprint in fig. 1b, which should be rotated clockwise through 90°.}

J. C. Polkinghorne (Cambridge, England)

9148:

Sen, P. Field metrics. II. A correction. Nuovo Cimento (10) **18** (1960), 613.

In a previous paper [Nuovo Cimento (10) **15** (1960), 513–518; MR **22** #3545] the author has suggested that the field metrics and commutators are correlated. He now points out that it is necessary to postulate that the τ space is also four-dimensional. *G. L. Clark* (London)

9149:

Baraff, Gene A.; Borowitz, Sidney. Green's function method for quantum corrections to the Thomas-Fermi model of the atom. Phys. Rev. (2) **121** (1961), 1704–1713.

The Thomas-Fermi equation for an atom and the quantum corrections thereto are derived from a many-body Green's function approach, which demonstrates that the uniform density assumption of the T-F derivation is equivalent to the assumption that the atom is in its ground state. A formal expansion in powers of $\hbar$ for the quantum corrections to the T-F equation is derived in the mixed position-momentum representation of the two-particle Green's functions defined by the average position and the Fourier transform of the space-time separation. Corrections to second order are calculated in Hartree-Fock approximation, and the equation of Kompaneets and Pavlovskii [Soviet Physics. JETP **4** (1957), 328–336; MR **18**, 971] is rederived.

J. R. Klauder (Murray Hill, N.J.)

9150:

Porter, Charles E.; Rosenzweig, Norbert. Statistical properties of atomic and nuclear spectra. Ann. Acad. Sci. Fenn. Ser. A VI No. 44 (1960), 66 pp.

Consider the energy levels of an atomic or nuclear spectrum which refer to the same parity and total angular momentum. The nearest neighbor spacings S have an average D. One considers the fractional frequency of spacings per unit interval, P, as a function of $x \equiv S/D$. In the nuclear case, let y denote the square root of the ratio of the neutron width to its average; one can then also consider P as a function of y. The empirical curves presented by the authors in this and another paper [Rosenzweig and Porter, Phys. Rev. (2) **120** (1960), 1698–1714] fit well with $P(x) = (\pi/2)x \exp(-\pi x^2/4)$ conjectured by E. P. Wigner. One has $P(0) = 0$, i.e., "level repulsion". P as a function of y is a normal distribution, $(2/\pi)^{1/2} \exp(-y^2/2)$. If either parity or angular momentum is not used as criterion of the level selection, the distribution $P(x)$ approaches an exponential. Since the levels are the eigenvalues of real symmetric matrices one considers an ensemble of them and attempts to explain the above results by suitable statistical hypotheses for their matrix elements. Heuristic arguments are presented to the effect that statistical independence and rotational invariance are sufficient for all matrix elements to be normally distributed (zero mean for off-diagonal elements), and for the dispersion of the diagonal elements to be twice that of the off-diagonal ones. Various consequences of these statistical assumptions are given and Monte Carlo machine calculations are made to confirm the arguments.

F. Rohrlich (Iowa City, Iowa)

9151:

Sinanoğlu, Oktay. Relation of perturbation theory to variation method. J. Chem. Phys. **34** (1961), 1237–1240.

Author's summary: "The various order wave functions of the Rayleigh-Schrödinger perturbation theory can be obtained directly by solving certain differential equations or by minimizing equivalent variational expressions. These expressions are related to the ordinary variation method. The perturbation series is shown to result in a unique way from the minimization of larger and larger portions of $\langle \psi, H\psi \rangle / \langle \psi, \psi \rangle$. In addition to several orders of perturbation, each step gives the exact remainders and upper limits to the energy. The approach suggests several 'variation-perturbation' schemes."

9152:

Burgess, A.; Seaton, M. J. A general formula for the calculation of atomic photo-ionization cross sections. Monthly Not. Roy. Astr. Soc. **120** (1960), 121–151.

A general formula for the photo-ionization cross sections of atomic systems is obtained by using the Coulomb approximation to describe the initial and final states, and numerical tables are presented covering most cases of interest. Several examples are considered.

A. Dalgarno (Belfast)

9153:

Vaĭnshteĭn, L. A.; Sobel'man, I. I. Deduction of the radial equations of the theory of collisions between electrons and atoms. Ž. Èksper. Teoret. Fiz. **39** (1960), 767–775 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 536–541.

Authors' summary: "Radial equations are derived for the excitation of an arbitrary level of a many-electron atom, with allowance for the nonorthogonality of the wave functions of the external and optical electrons. The well-known ambiguity which appears when approximate atomic wave functions are used is discussed. An approximate form of the equations is proposed, in which terms that contain simultaneously nonorthogonality integrals and higher multiple interactions are neglected. In this approximation the ambiguity disappears if semi-empirical wave functions are employed for the optical electron."

9154:

Krutov, V. A.; Gorškov, V. G. Higher Born approximations in pair conversion. Ž. Èksper. Teoret. Fiz. **39** (1960), 591–599 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 417–421.

Authors' summary: "Higher Born approximations with respect to the nuclear Coulomb field are considered in pair creation processes. The integrals of the first Born approximation can be computed exactly and lead to a simple analytic result."

9155:

Troubetzkoy, E. S. Statistical theory of gamma-ray spectra following nuclear reactions. Phys. Rev. (2) **122** (1961), 212–217.

Author's summary: "A theory predicting γ-ray spectra following intermediate or high-energy nuclear reactions is derived on the basis of the statistical theory. The equations take a particularly simple form if the assumption is made that all radiative transitions are of the electric dipole type. The theory is applied to two specific reactions: inelastic neutron scattering and thermal neutron capture. Numerical calculations of spectra arising from thermal neutron capture by two gadolinium isotopes are shown to compare well with experiments."

9156:

Goldhammer, Paul. Spin-orbit splitting in nuclei due to tensor interaction. Phys. Rev. (2) **122** (1961), 207–211.

Author's summary: "The effect of the tensor force in nuclei with closed shells plus one nucleon has been investigated using second-order perturbation theory. It is found that one can explicitly exhibit the spin-orbit

splitting due to the tensor force using some simple identities. The spin-orbit splitting in He^5 is computed, and found to be 3.4 Mev compared with an experimental value of 2.6 Mev."

9157:

Jones, P. B. Convergence of the cluster expansion for the optical model potential. Nuclear Phys. **23** (1961), 349–352.

Author's summary: "The convergence of the cluster expansion for the optical model potential is examined, at low energies, by comparison of the series with the potential given by a closed form. The problem of antisymmetrization is not considered."

9158:

Baranger, Elizabeth; Lee, Chong Wan. Spurious states arising from the centre-of-mass motion of a nucleus. Nuclear Phys. **22** (1961), 157–163.

Authors' summary: "A simple method is given for generating the spurious states which arise from the incorrect treatment of the centre-of-mass motion by nuclear shell model calculations. It is not necessary to know the spurious states in order to find their weight in a particular shell model state. It is also shown which *jj* configurations have no spurious states. Some examples are given."

9159:

Young, James E. The optical model and deuteron stripping. Nuclear Phys. **23** (1961), 40–52.

Author's summary: "A development is given which stresses the underlying similarity between the deuteron stripping theory on the one hand and direct interaction theory of inelastic nucleon scattering on the other. The assumption here is that both formulations are given in distorted Born approximation. Some implications for the form of the stripping theory, e.g., construction of the distorting potentials, are discussed. The entire theory, as given, for the transition matrix element is first-order in the two-body scattering operators. Probable corrections to this description, such as those arising from multiple scattering, are discussed."

9160:

Nagata, Sinobu. Nuclear deformation and nuclear force. I. Progr. Theoret. Phys. **25** (1961), 35–50.

Using the wave function by Nilson and the central part of the reaction matrix by Brueckner, a variational calculation is performed with respect to the deformation parameter δ and the inter-nucleon distance r_0 for Mg^{24}. The result gives a reasonable value for δ and r_0, though this success should be taken as the qualitative one. A more realistic force is applied in another paper.

T. Sasakawa (Cambridge, Mass.)

9161:

Ikeda, Kiyomi; Nagata, Sinobu; Takada, Kenjiro. Nuclear deformation and nuclear force. II. Progr. Theoret. Phys. **25** (1961), 51–63.

The previous calculation [see #9160] is extended to include the tensor force and the spin-orbit force. The result shows that one can expect an energy minimum for a reasonable set of the deformation parameter and the inter-nucleon distance. Also it is shown that the spin-orbit force gives a serious effect for the splitting and the tensor force is not as great an effect for the splitting and it gives a minor effect for the deformation.

T. Sasakawa (Cambridge, Mass.)

9162:

Flores Maldonado, Victor. Effect of a repulsive core on a statistical model of heavy nuclei. Rev. Mexicana Fis. **9** (1960), 163–179. (Spanish. English summary)

Author's summary: "In earlier works [J. A. McLennan, Jr. and G. P. De Vault, unpublished], the binding energies of heavy nuclei have been computed theoretically by using a statistical model and a Yukawa interaction. The values obtained were very high compared with the experimental values. The present work uses the same model but a repulsive core is added to the Yukawa interaction. It is found that the repulsive core reduces the binding energy to a large extent, greatly improving the values obtained earlier. Nevertheless, the present results are not satisfactory yet. Some ways that would improve these results are indicated here."

9163:

Gorškov, V. G. Account of the nuclear Coulomb field in the interaction between electrons and an electromagnetic field. Ž. Èksper. Teoret. Fiz. **39** (1960), 1411–1421 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 983–989.

Author's summary: "Convenient expressions for practical calculations are derived for the matrix elements of the interaction between electrons and an electromagnetic field near the nucleus. The Coulomb field is taken into account with an accuracy up to terms proportional to $(\alpha Z)^2$."

9164:

Kammuri, Tetsuo. Compound nucleus processes in the reactions between complex nuclei. Progr. Theoret. Phys. **25** (1961), 235–246.

Author's summary: "A modification is made for the statistical theory of nuclear reactions so as to take account of the angular momenta of the nuclear states relevant in the reactions. Since the average values of the angular momenta of the compound nuclei formed by different projectiles at the same excitation energy will be different, various aspects of the decay process will also depend on the incident particles. For our purpose it is essential to take the spin dependence of the nuclear level density not as $(2I+1)$, but $(2I+1)\exp[-I(I+1)/2cT]$. In this paper, the energy spectra of emitted particles are treated. The average kinetic energy of evaporating neutrons from a high-spin nucleus becomes higher than twice the nuclear temperature T. It is shown that the anomalies found in the branching ratios or excitation functions in the heavy ion reactions can be explained by the angular momentum effect. The degree of these anomalies depends sensitively on the nuclear moment of inertia $c\hbar^2$."

9165:

Sakamoto, Yoshiyuki. On the charge exchange collision in nucleon-deuteron. Progr. Theoret. Phys. **25** (1961), 259–276.

9166:

Mitra, A. N.; Pandya, S. P. Nuclear spectroscopy with non-local potentials. Nuclear Phys. **20** (1960), 455–463.

Authors' summary: "A non-local separable potential derived earlier [A. N. Mitra and V. L. Narasimhan, Nuclear Phys. **14** (1959/60), 407–428; MR **22** #6519] has been used to calculate the $T=0$ energy levels of the shell model configuration $(d_{3/2})^2$. The results are satisfactory, and the advantages of such a realistic and well-behaved potential over the other potentials, local but invoking singularities, hard cores, etc., for calculating nuclear energy levels are pointed out."

9167:

Uehara, Masayuki. The hyperon and nucleon interactions. I. Progr. Theoret. Phys. **24** (1960), 629–647.

From the author's summary: "Two- and three-body Y-N potentials including the rescattering corrections are calculated by using Miyazawa's method."

9168:

Solov'ev, V. G. A superfluid model of the nucleus. Dokl. Akad. Nauk SSSR **133** (1960), 325–328 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 778–781.

The author, following the methods of Bogolyubov, has calculated the ground-state energy, on the basis of a deformed potential. He has further assumed an adiabatic approximation. The suggestion is made that an improvement of the Nilsson level scheme, such that single particle states of odd nuclei can be determined by a superfluid model, will enable complex calculations for a number of properties of deformed nuclei.

N. S. Wall (Cambridge, Mass.)

9169:

Baldin, A. M.; Semenko, S. F. On the theory of the optical anisotropy of atomic nuclei. Ž. Èksper. Teoret. Fiz. **39** (1960), 434–437 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 306–307.

9170:

Phillips, G. C.; Griffy, T. A.; Biedenharn, L. C. The significance of the generalized density of states function for nuclear spectra. Nuclear Phys. **21** (1960), 327–339.

The question of three-(or more)-body decay of a compound system is analyzed on the basis of a sequence of two-body interactions. In the case that the earlier decaying bodies can be localized in space and decays through a channel for which the scattering phase shifts are known the decay spectrum is shown to be determined by the density of states given by the rate of change of the phase shifts and an equivalent hard sphere phase shift. This analysis shows how anomalies in the spectral yield of the emitted particles may be interpreted, not in terms of states of the residual system, but correspond to the fact that the final state may be localized.

N. S. Wall (Cambridge, Mass.)

9171:

Agodi, A.; Eberle, E. An unified theory of direct reactions. Nuovo Cimento (10) **18** (1960), 718–743. (Italian summary)

Feshbach and others have derived a unified theory of nuclear reactions from a starting point of well-defined compound states. The present authors start with a description of direct reactions and analyze the result of a projection operator connecting just the two channels of the real incident and outgoing states.

N. S. Wall (Cambridge, Mass.)

9172:

Simons, Lennart. Remarks on Coulomb excitation. Soc. Sci. Fenn. Comment. Phys.-Math. **21** (1958), no. 2, 14 pp.

Values of the reduced transition probability $B(E2)$ for Coulomb excitation obtained from different theories are compared with each other and with experiment. This comparison favors the most recent wave-mechanical theory, but uncertainties in the experimental quantities are too large to permit a decisive test of the theories.

A. C. Hurley (Melbourne)

9173:

Longuet-Higgins, H. C.; Salem, L. The forces between polyatomic molecules. I. Long-range forces. Proc. Roy. Soc. London. Ser. A **259** (1960/61), 433–441.

Authors' summary: "It is known that the electrostatic long-range forces between two molecules are 'locally additive' in the sense that the first-order interaction energy between molecules bearing charges or permanent multipole moments may be written as a double integral over pairs of points one in each molecule. In this paper we examine the second-order interaction energy, which comprises polarization and dispersion terms, and find that the corresponding forces are locally additive or not according as the electron correlation within each molecule does or does not die away rapidly with distance."

A. C. Hurley (Melbourne)

9174:

O-ohata, Kiyosi. The polarity of the diatomic molecule. Progr. Theoret. Phys. **25** (1961), 215–228.

Author's summary: "The relation between the ionicity of the bond and the electronegativity difference is discussed by the use of the semilocalized orbital method. It is found that the ionicity of the bond depends not only on the electronegativity difference but also on the overlap integral between the atomic valence orbitals. According to the magnitude of the overlap integral the diatomic molecules are divided into the two groups which show the different features with respect to the dependence of the ionicity of the bond on the electronegativity difference. The S-shaped curve given by Townes and Dailey from the analysis of the results of the measurements of eQq seems to be the composite of the above two characteristic groups."

A. C. Hurley (Melbourne)

9175:

Livšic, B. L. Perturbation theory for a simple structure operator. Dokl. Akad. Nauk SSSR **133** (1960), 800–803 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 827–830.

A discussion of the modifications of quantum-mechanical perturbation theory which are required for dealing with non-Hermitian operators of simple structure.

A. C. Hurley (Melbourne)

9176:

Adamov, M. N. Integral representation of the dispersion formula for a hydrogen atom in the ground state. Dokl. Akad. Nauk SSSR **133** (1960), 315–317 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 768–770.

The ordinary dispersion formula for the hydrogen atom is transformed to give an integral representation of the optical polarizability which is convenient for calculations. The transformation is carried out by resolving the basic differential equation by means of a Laplace transformation and subjecting the resultant expression to successive integration by parts. *A. C. Hurley* (Melbourne)

9177:

Gioumousis, George; Curtiss, C. F. Molecular collisions. II. Diatomic molecules. J. Mathematical Phys. **2** (1961), 96–104.

[For part I see J. Chem. Phys. **29** (1958), 996–1001; MR **21** #597.] The irreducible representations of the three-dimensional relation group are used to analyze the equations for the inelastic scattering of diatomic molecules. This leads to a set of coupled integral equations and an expansion of the cross-section in spherical harmonics.

A. C. Hurley (Melbourne)

9178:

Cohen, M.; Coulson, C. A. Single-centre expansions for the hydrogen molecular ion. With an appendix by L. Fox. Proc. Cambridge Philos. Soc. **57** (1961), 96–106.

Expansion of the electronic wave function for the hydrogen molecular ion about the centre of change leads to an infinite set of coupled differential equations. For the ground state it is shown that the higher members of this set are relatively unimportant, and the equations obtained by truncating the set after one, two, three and four members are solved numerically. The resulting eigenfunction is shown to be quite accurate except in the neighborhood of the singularities at the nuclei. The desirability of approximate wave functions which reflect the actual singularities of a system, and only these, is stressed. *A. C. Hurley* (Melbourne)

9179:

Mulliken, Robert S. The interaction of differently excited like atoms at large distances. Phys. Rev. (2) **120** (1960), 1674–1684.

The theory of the long-range first-order interaction energies of two like atoms in differently excited states is presented. It is pointed out that in many cases the first order dispersion forces are not the most important even at large internuclear distances. *A. Dalgarno* (Belfast)

9180:

O-ohata, Kiyosi. The binding energy of the σ bond. I. General theory. J. Phys. Soc. Japan **15** (1960), 1048–1056.

This paper is the first in a series of four papers dealing with the calculation of binding energies of σ bonds. In this paper the general theory is elucidated with particular emphasis given to a discussion of the non-orthogonality of the orbitals. In dealing with this problem, overlap integrals are divided into two classes according to their magnitude. The overlap integral between paired orbitals (S) in saturated molecules may be nearly equal to unity, whereas that between unpaired orbitals (T) is generally small. Thus, S overlap integrals are fully taken into account, while the contribution from non-bonded orbitals is terminated with second order terms.

Under the additional assumption that all ionic terms are negligible, a detailed expression for the binding energy of the σ bond is developed. The expression thus obtained is then compared with corresponding results based on electron pair bond theory. Differences resulting from a consideration of the non-orthogonality between the valence orbitals are discussed.

R. E. Kagarise (Washington, D.C.)

9181:

O-ohata, Kiyosi. The binding energy of the σ bond. II. Application to the diamond crystal. J. Phys. Soc. Japan **15** (1960), 1258–1263.

The second paper of the series applies the results of I [#9180] to a calculation of the *C-C* bond energy in the diamond crystal. Atomic orbitals of the Slater type are first used to calculate the total energy of the ground state of the carbon atom and the energies of the excited states relative to the ground state. Agreement between calculated and observed value is satisfactory. The atomic orbitals are next used to calculate the ionization potential and electron affinity for the valence state $V_4(sp)^3$ of the carbon atom. Agreement between the calculated values and those estimated by Mulliken are satisfactory in the case of the ionization potential, but unsatisfactory for the electron affinity.

The binding energy of the *C-C* bond was calculated using the results of I and the assumption that the core potential of an atom can be replaced by the nuclear potential with an effective nuclear charge. The value thus obtained was -3.54 eV. In making this calculation only those interactions between nearest and second neighbors were considered. The calculation is further refined by considering the effect of extending $1s$ orbitals, which contributes 0.28 eV per bond to the binding energy. The resultant binding energy of the *C-C* bond in diamond is thus -3.26 eV. *R. E. Kagarise* (Washington, D.C.)

9182:

O-ohata, Kiyoshi. The binding energy of the σ bond. III. Discussion on the ionic structures. J. Phys. Soc. Japan **15** (1960), 1449–1455.

In this paper, the author considers the ionic structures of σ bonds through the use of the semi-localized orbital method first proposed by Mueller and Eyring. This approach is chosen because it permits the description of heteropolar bonds, takes into consideration the correlational interaction between bonding pairs and yields a simple formula for the total bond energy. Such a formula is developed along lines similar to those used in paper I [#9180], but uses semi-localized orbitals rather than the paired orbitals utilized in Heitler-London approximation.

From the expression of the binding energy of the bond

A-B, one can obtain the difference between the electronegativities of atoms A and B on the Mulliken scale. By making certain assumptions concerning the extra ionic resonance energy as defined by Pauling a similar expression can be obtained for the Pauling electronegativities. Thus a theoretical relation between the Pauling and Mulliken electronegativity scales is developed which is excellent agreement with the empirical relationship.

Finally the theoretical results are used to re-calculate the binding energy of the C-C bond in diamond and a value of -3.64 eV calculated, which is in good agreement with the observed value of -3.716 eV.

R. E. Kagarise (Washington, D.C.)

9183:

O-ohata, Kiyoshi. The binding energy of the σ bond. IV. Discussion on the higher order terms. J. Phys. Soc. Japan **15** (1960), 1632–1637.

In the final paper of the series, the author considers the role of higher order terms. A bond orbital method is used to describe the electronic state of the saturated molecule. Bond orbitals are orthogonalized by Loewdin's method which involves a power series expansion of the overlap integral. The condition for convergence is barely satisfied in the present case which accounts for the significant contribution of the third order terms to the binding energy. Finally, it is shown that the additivity rule in the binding energies of saturated hydrocarbons is valid even if one considers the contributions of higher order terms.

R. E. Kagarise (Washington, D.C.)

9184:

Kvasnica, J. A possible estimate of the elementary length in electromagnetic interactions. Czechoslovak J. Phys. **10** (1960), 625–627. (Russian summary)

Author's summary: "The difference of 0.2 MHz between the theoretical and experimental values of the Lamb shift of the hydrogen terms $2s_{1/2}$ and $2p_{1/2}$ is used to estimate the elementary length λ in an electromagnetic interaction. The calculated difference of the hydrogen terms in the Bopp-Podolsky potential field $\varphi(r)=(e/4\pi r)(1-e^{-r/\lambda})$ is compared with the given difference $\Delta\simeq 0.2$ MHz. From this we then get for the elementary length $\lambda\simeq 0.1\times 10^{-13}$ cm. The distribution of the electric charge in the proton $\rho(r)=(-e/4\pi r\lambda^2)e^{-r/\lambda}$ leads to the Bopp-Podolsky potential if the Coulomb interaction is assumed to be valid in the whole space."

9185:

Boys, S. F. The integral formulae for the variational solution of the molecular many-electron wave equation in terms of Gaussian functions with direct electronic correlation. Proc. Roy. Soc. London. Ser. A **258** (1960), 402–411.

The author has shown previously [same Proc. **200** (1950), 542–554] that all molecular integrals may be evaluated in closed form if primary functions of the form $P(x_A, y_A, z_A)\times\exp(-ar_A^2)$ are employed. Here x_A, y_A, z_A are cartesian coordinates relative to centre A, $r_A^2=x_A^2+y_A^2+z_A^2$, P is any polynomial, and any number of centres may be used. Here it is shown that a similar reduction is possible for the more general class of functions

$$P(x_{iA}, y_{iA}, z_{iA})\exp\Big(-\sum_i a_i r_{iA}^2-\sum_{ij} c_{ij} r_{ij}^2\Big),$$

where r_{ij} is the distance between electrons i and j. These functions provide a new tool for the detailed calculation of wave functions of several nuclei and electrons. They may also provide a convenient method for introducing correlation terms into wave functions for large molecules.

A. C. Hurley (Melbourne)

9186:

McLellan, A. G. Eigenfunctions for integer and half-odd integer values of J symmetrized according to the icosahedral group and the group C_{3v}. J. Chem. Phys. **34** (1961), 1350–1359.

Author's summary: "Eigenfunctions of J^2 for $J=0, \frac{1}{2}, 1, \cdots, 8$, have been obtained and tabulated which are symmetrized according to the icosahedral group and the group C_{3v}, in terms of eigenfunctions of J^2 and J_z, referred to a fivefold axis of rotation as the z axis. The functions are symmetrized according to C_{3v} in such a way that the symmetry operation of a $2\pi/3$ rotation is represented by a diagonal matrix. The results have been applied to calculate the anisotropic magnetic splitting factors for the doublet states arising from the above symmetry."

9187:

★Interazioni deboli. Rendiconti della Scuola Internazionale di Fisica "Enrico Fermi". Corso XI, 29 giugno–11 luglio 1959 (Direttore: L. A. Radicati). Nicola Zanichelli, Bologna, 1960. xv+406 pp. (1 plate) L. 4500.

Papers of mathematical interest in these Proceedings will be reviewed separately.

9188:

Ikeda, Mineo; Miyachi, Yoshihiko; Ogawa, Shuzo. Symmetry in Sakata's model and weak interactions. I. Progr. Theoret. Phys. **24** (1960), 569–587.

In the Sakata model all the "elementary" particles are supposed to be composed of three basic particles P, N, Λ. Neglecting the mass permutation symmetry between these three particles is expressed by assuming invariance under $U(3)$, the unitary group of degree three. The nine infinitesimal generators of $U(3)$ can be identified with the baryon number and hypercharge number, the three components of isotopic spin, and four operators changing the hypercharge by ± 1 and the third component of isospin by $\pm\frac{1}{2}$. There are therefore two linearly independent operators, O_1 and O_2, that preserve the electric charge and change strangeness by $+1$. In the notation of this paper, $O_1=N_{23}$ and $O_2=N_{13}I_-=I_-N_{13}+N_{23}$.

The authors confine themselves to configurations built up of four or less basic particles, i.e., the particles are considered to be Λ, $\mathcal{N}=N$, P, $\Sigma=\Lambda\pi$, $\Xi=\Lambda\bar{K}$, $K=\mathcal{N}\bar{\Lambda}$, $\bar{K}=\bar{\Lambda}\mathcal{N}$, $\pi=\bar{\mathcal{N}}\mathcal{N}$ (isovector), $\pi^{0'}=\bar{\mathcal{N}}\mathcal{N}$ (isosinglet), surrounded by at most one pion. They find that the weak interaction operator O_1 reproduces, for these configurations, the $\Delta I=\frac{1}{2}$ rule in hyperon and $K_{2\pi}$, K_{e3} decays. Assuming a purely vector interaction, they also consider the effect on the K_{e3} electron spectrum of an isoscalar particle $\pi^{0'}$ somewhat heavier than the conventional isovector π^0, and the relation of K_{e3} decay to $\Lambda\,\beta$ decay. They are aware that the unitary symmetry in weak interactions would suggest comparable rates for the β decay of Λ and N, in contradiction with experiment.

S. Bludman (Berkeley, Calif.)

9189:

Lomont, J. S.; Moses, H. E. Dirac-like wave equations for particles of nonzero rest mass, and their quantization. Phys. Rev. (2) **118** (1960), 337–348.

The authors consider wave equations of the form

$$(\gamma^\mu\nabla_\mu+m)\psi = 0, \tag{1}$$

where $[\gamma_\mu, \gamma_\nu]_+=2g_{\mu\nu}$. Under an infinitesimal Lorentz transformation $\delta x^\lambda=\omega^{\lambda\mu}x_\mu$ the wave function ψ transforms as follows

$$\delta\psi = -\tfrac{1}{2}\omega^{\mu\nu}K_{\mu\nu}\psi,$$

where $K_{\mu\nu}=-K_{\nu\mu}$;

$$[K_{\kappa\lambda}, K_{\mu\nu}]_- = g_{\kappa\mu}K_{\lambda\nu}+g_{\lambda\nu}K_{\kappa\mu}-g_{\nu\kappa}K_{\lambda\mu}-g_{\lambda\mu}K_{\kappa\nu} \tag{2}$$

and

$$[\gamma_\lambda, K_{\mu\nu}]_- = g_{\lambda\nu}\gamma_\mu-g_{\lambda\mu}\gamma_\nu.$$

Equation (1) is then covariant and the problem reduces to one of finding suitable irreducible representations of the γ's and K's. This is accomplished by introducing

$$M_{\mu\nu} \equiv K_{\mu\nu}+\tfrac{1}{4}[\gamma_\mu, \gamma_\nu]_-.$$

M satisfies the commutation relation (2) and commutes with γ_λ. Once the consequent representations are obtained the behavior of K and γ under time reversal and space inversion is discussed. In general the latter requires a doubling of the number of components of ψ. The electromagnetic field is introduced and the behavior of ψ under charge conjugation. Quantization of the free field is introduced formally with the aid of creation and destruction operators in momentum space. Quantization in the presence of interaction is not discussed.

H. Feshbach (Cambridge, Mass.)

9190:

Brodski, A. M.; Ivanenko, D. On the influence of weak interactions on electromagnetic properties of fermions. Nuovo Cimento (10) **16** (1960), 556–559. (Italian summary)

Making use of renormalized form of universal Fermi-interaction Lagrangian

$$\mathscr{L} = \frac{G}{\sqrt{2}}\{[\bar{p}\gamma_\mu(1+\lambda\gamma_5)n, \bar{n}\gamma_\mu(1+\lambda\gamma_5)p]_+ + \zeta[\bar{p}\gamma_\mu\gamma_5 n, \bar{n}\gamma_\mu\gamma_5 p]_+\},$$

the authors report that in the case of the presence of variable external electromagnetic field the equation of the particle is added by a kinematic magnetic moment term which is divergent logarithmically.

N. Kumasawa (Tokyo)

9191:

Gürsey, F. On the symmetries of strong and weak interactions. Nuovo Cimento (10) **16** (1960), 230–240. (Italian summary)

Author's summary: "A model of strong interactions is proposed which admits a group $(G_4\times H)$ in the limit of a suitably defined doublet approximation, G_4 being a 4-dimensional extension of the isotopic spin group and H the hypercharge gauge group. Weak interactions having phenomenological chirality invariance properties or obeying the $\Delta I=\frac{1}{2}$ rule are shown to be invariant under an unitary subgroup of the group $(G_4\times H)$."

S. A. Bludman (Berkeley, Calif.)

9192:

Vavilov, B. T.; Grigor'ev, V. I. Multiple processes and the relations between different transition matrices. Ž. Èksper. Teoret. Fiz. **39** (1960), 794–799 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 554–557.

Author's summary: "An infinite chain of equations relating the matrices of different transitions is constructed. After mass renormalization the equations are applied to multiple boson production in two-fermion collisions."

9193:

Komar, A. "Minimal property" of electromagnetic interaction for spin $\frac{3}{2}$ particles. Nuclear Phys. **22** (1961), 101–103.

Author's summary: "The possibility of the extended electromagnetic interaction for spin $\frac{3}{2}$ particles with terms in the Lagrangian depending directly on electromagnetic field tensor $F_{\mu\nu}$ is considered. It is shown that inclusion of such terms leads in all variants to a non-consistent theory, thus making the minimal electromagnetic interaction the only one allowed."

9194:

Gražin, A. F.; Nikitin, Yu. P. On the nucleon-nucleon potential. Ž. Èksper. Teoret. Fiz. **39** (1960), 713–719 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 498–502.

Authors' summary: "A convenient method is proposed for setting up a potential in the form of the series $U(x)=\sum U^{(n)}(x)$, where $U^{(n)}\sim e^{-nx}$ for $x\to\infty$, starting from the relativistic meson-theoretical scattering amplitude expressed as an expansion in the number of exchange mesons (x is the distance in the units $1/\mu$, where μ is the meson mass). In the case of peripheral nucleon interaction this method yields a two-meson potential with a broad locality region $p^2/m^2\ll 1$ (nonrelativistic region). The two-meson potential consists essentially of attractive tensor and central forces which depend weakly on the isotopic state."

9195:

Maksimenko, V. M.; Rozental', I. L. Covariant statistical theories of multiple particle production. Ž. Èksper. Teoret. Fiz. **39** (1960), 754–756 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 527–528.

Authors' summary: "Possible covariant theories of multiple particle production are analyzed under the condition that the matrix element can be factorized. The multiplicity of the secondary particles is computed by assuming that the matrix element is a power function of the energy of the particles involved in the process."

9196:

Širokov, M. I. A relativistic general theory of reactions. Ž. Èksper. Teoret. Fiz. **39** (1960), 633–638 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 445–448.

Author's summary: "The formulas of a relativistic general theory of reactions for the cross-section and the polarization in terms of phase shifts may assume a

different form for different definitions of the relativistic spin operator. However, in the rest-system of the particle the spin operators coincide. This allows one to express the general theory in a form which is the same for all equivalent definitions of the spin."

9197:

Eder, G. The two meson approximation for the real nucleon. Nuovo Cimento (10) **18** (1960), 430–442. (Italian summary)

The main part of this paper is a repetition of a part of F. R. Halpern, L. Sartori, K. Nishimura and R. Spitzer's paper in Ann. Physics **7** (1959), 154–173 [MR **21** #2500]. *H. Wakita* (Hiroshima)

9198:

Maki, Ziro; Ohnuki, Yoshio. The non-leptonic decays of hyperons. Progr. Theoret. Phys. **25** (1961), 353–360.

Authors' summary: "The non-leptonic decays of hyperons are investigated based on the Sakata model and the V-A theory of weak interactions. The Λ-decay and the Σ-decay are treated in a unified way. In the latter case, a possibility of interpreting the experimental evidences is given provided that the structure of Σ-hyperon satisfies some suitable conditions."

9199:

Cassels, J. M. Low energy pion physics. Nuovo Cimento (10) **14** (1959), supplemento, 259–277.

9200:

Teodorovič, È. V. Pion-pion interaction and the electromagnetic structure of the nucleon in the static theory. Ž. Èksper. Teoret. Fiz. **39** (1960), 476–479 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 334–336.

Author's summary: "The mean-square radius of the pion charge distribution is estimated on the basis of the relationship between this radius and the pion-pion scattering resonance energy derived from the dispersion relations. The mean square radius of nucleon charge is calculated within the framework of the Chew-Low-Wick static theory by taking the electromagnetic structure of the meson into account, and good agreement with experimental data is obtained."

9201:

Downs, B. W. Hypernuclei and the Λ-nucleon interaction. Lectures in theoretical physics (Boulder, Colo., 1959), pp. 210–239. Interscience, New York, 1960.

9202:

Sugimoto, Yoshiko. On the electromagnetic mass differences among the sigma hyperons. Progr. Theoret. Phys. **25** (1961), 189–200.

Author's summary: "The problem of the mass difference of the triplet sigma is discussed in terms of an electromagnetic self-energy by taking account of the virtual process $\Sigma^0 \to \Lambda^0 + \gamma \to \Sigma^0$ and the process $\Sigma^0 \to \Sigma^0 + \gamma \to \Sigma^0$ for the self-mass of the neutral sigma on an equal footing. The electromagnetic form factor is introduced to serve as a cutoff factor. With the e^2-approximation, it can be seen that the process $\Sigma^0 \to \Lambda^0 + \gamma \to \Sigma^0$ plays an important role in studying the mass splitting of Σ's along this line."

9203:

Ihara, Chiaki. Electromagnetic structure of the nucleon and the composite model for pion. Progr. Theoret. Phys. **25** (1961), 229–234.

Author's summary: "The electromagnetic structure of the nucleon is investigated based on the composite model for the pion. The mean-square radius of charge distribution of the nucleon is calculated by the second-order perturbation and the same coupling constants are used with those in the calculation of the pion-nucleon interaction and the anomalous magnetic moment of the nucleon on this model. The results are qualitatively in agreement with experiment and it is possible to explain all these phenomena consistently from the standpoint of the composite model."

9204:

Širokov, M. I.; Okonov, È. O. The spatial and charge parities and two-meson annihilation of the proton-antiproton system. Ž. Èksper. Teoret. Fiz. **39** (1960), 285–292 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 204–208.

9205:

Takabayasi, Takehiko. Theory of relativistic rotators and elementary particles. I. Progr. Theoret. Phys. **23** (1960), 915–941.

Isospin is identified with the space part of the internal angular momentum generated by a spinning material sphere, suggested as a relativistic model for particles. The admission of accelerated orbital external motions, on top of the internal rotation, allows the author to define his internal momentum as a 6-component entity, decomposing into Pauli spin and a precessional fluctuation of the orbital momentum. Various detailed models arise from possible choices of a Lagrangian. A weakness of the theory, pointed out by Yukawa, is that it entails a definite correlation between allowed values of spin and isospin, which does not agree with the facts. The treatment is not field-theoretical at this stage. *Y. Neeman* (London)

9206:

Hamaguchi, M. On the two fluid model in multiple production of mesons. Nuovo Cimento (10) **17** (1960), 35–44. (Italian summary)

This is the third member of a series of papers in which the author investigates the hydrodynamical aspect of the multiple meson production [Nuovo Cimento (10) **4** (1956), 1242–1261; **5** (1957), 1622–1635; MR **18**, 856; **19**, 924].

The two-fluid model is now employed to calculate the inelasticity of the meson production due to the nucleon-nucleon collision. Assuming that the core part of the incident nucleon and the cloud part surrounding the core behave as a perfect fluid and a viscous fluid, respectively, the author sets up the equation: (1) $\partial_\nu T_{\mu\nu} = 0$, where $T_{\mu\nu}$ is the energy-momentum tensor of the whole system. Besides the condition that the Reynolds number is larger

than unity, the approximations used in solving equation (1) are introduced rather intuitively in order to simplify mathematical expressions. This fact makes it very difficult to justify the numerically calculated inelasticity given in table I. *Y. Takahashi* (Dublin)

9207:

Konisi, Gaku; Yamamoto, Kunio. A quantitative investigation on the "compositeness" of a particle. Progr. Theoret. Phys. **24** (1960), 817–824.

The concept of a "composite" system is a quite clear one in non-relativistic quantum mechanics. In relativistic quantum field theory, however, when a virtual dissociation of a particle of mass M into two particles of masses m, m' is possible, it is not clear to what extent the particle M can be considered as a "composite system" of m, m' with binding energy $B=M-m-m'$. In this paper a quantitative criterion for establishing the degree of "compositeness" of a particle is tried by analogy with the non-relativistic case. One starts from an identity which holds in non-relativistic quantum mechanics for a 2-particle system, and which connects the two individual probability densities of the component particles. This identity can be reinterpreted in terms of relativistic quantum field theory, but now it can be considered as a condition for the 2-particle system to be "composite". This extension to the relativistic case is only possible under certain conditions, which allow the non-relativistic "probability density" of a particle to be well represented by the corresponding field-theoretical "particle density". One of these conditions is that $M^2>m^2+m'^2$. Some particular examples are then worked out in perturbation theory. For these the explicit calculation of a parameter is possible, whose magnitude increases with the discrepancy in the reinterpreted form of the basic identity. This parameter is called the "acompositeness", since it vanishes when the identity is satisfied. It turns out to be proportional to the square root of the binding energy B. *E. Ferrari* (Geneva)

9208:

Ueda, Tamotsu; Sawamura, Makoto. The electric polarizability of the neutron. Progr. Theoret. Phys. **24** (1960), 519–528.

From the authors' summary: "Electric polarizability of the neutron is calculated with the relativistic meson theory in the lowest order of perturbation method."

9209:

Halfin, L. A. Relationship between the decay law and the first-order moment of energy distribution. Dokl. Akad. Nauk SSSR **132** (1960), 1051–1054 (Russian); translated as Soviet Physics. Dokl. **5**, 515–518.

The author discusses the problem of the determination of the energy distribution $\tilde{\omega}(E)$ in a physical system from a given decay law:

$$p_1(t) \equiv M_1(t)e^{iN_1(t)} = \int_{-\infty}^{\infty} \tilde{\omega}(E)e^{iEt}\,dt.$$

Defining the first moment of the distribution as

$$\tilde{\omega}^{(1)} = \int_{-\infty}^{\infty} E\tilde{\omega}(E)\,dE < \infty,$$

he derives a relation between $\tilde{\omega}^{(1)}$ and the number of zeros of $p_1(t)$ in the upper half-plane $\mathrm{Im}\,(t)>0$. It follows that the moment is increased by the presence of complex zeros of $p_1(t)$. The lower bound corresponding to the absence of zeros provides a possible determination of the effective mass of unstable particles in accordance with a theory put forward by P. T. Matthews and A. Salam [Phys. Rev. (2) **112** (1958), 283–287; MR **20** #4427]. Moreover, it is shown that, for a distribution with the least first moment, $\tilde{\omega}(E)$ is uniquely determined by the law of decay, that is, by the modulus $M_1(t)$ of $p_1(t)$.

A. H. Klotz (Newcastle-upon-Tyne)

9210:

Edwards, S. F.; Naqvi, M. A. Integrable many body wave functions for a system of Bose particles. Proc. Phys. Soc. **77** (1961), 433–446.

Authors' summary: "It is shown that, by means of a parametric representation, a variational calculation for a system of Bose particles can be carried through using explicit trial wave functions which describe a state with a fixed number of particles and include pair correlations. For the ground state the results of previous work are obtained as the first term in a steepest-descent calculation. An effective Schrödinger equation is obtained for the relative motion of two particles. The application of the method to excited states is discussed; in particular for the case of a vortex line in the system it is shown that the method leads to an analysis which is similar to that for the ground state and could be carried through in the same way." *F. Rohrlich* (Iowa City, Iowa)

9211:

Mentkovskiĭ, Yu. L. On perturbation theory for large quantum systems. Ž. Èksper. Teoret. Fiz. **39** (1960), 317–321 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 225–228.

A variation of the Van Hove-Hugenholtz perturbation theory expansion is proposed which involves simpler energy denominators. *D. ter Haar* (Oxford)

9212:

Bloch, Claude. On the theory of imperfect Fermi gases. Physica **26** (1960), supplement, S 62–S 74.

A survey is given of recent work on expansions of the grand partition function using diagram techniques.

D. ter Haar (Oxford)

9213:

Mohling, F.; Morita, M. Temperature dependence of the low-momentum excitations in a Bose gas of hard spheres. Phys. Rev. (2) **120** (1960), 681–688.

The quasi-particle energy for a Bose gas of hard spheres in the low density limit was determined by Lee and Yang at $T<Tc$. The leading correction for this quasi-particle energy at low but finite densities has been given by several authors at $T=0$. The present work gives the leading correction for $T\neq 0$. The approximate Hamiltonian given by Wu based on Lee and Yang's pseudopotential method is adopted. Further, certain terms of this Hamiltonian is neglected, though this procedure is justified only at $T=0$. Thus, the real part of the correction term is shown to be small compared to the leading term ω_0 given by Lee and

Yang for all temperatures lower than Tc. The imaginary part of the correction term shows that the quasi-particle life time τ is small for temperatures slightly below Tc and in this case $\tau\omega_0$ approaches zero in the low-momentum limit. *R. Kubo* (Tokyo)

9214:

Mehta, M. L. On the singularities of a generalized Brueckner t-matrix. Nuclear Phys. **12** (1959), 333–341.

It is known that in certain examples (separable potentials, with the total momentum of all interacting pairs set equal to zero) the Brueckner t-matrix has singularities below the Fermi surface. Chisholm and Squires [see #9215] have proposed a modification of Brueckner's definition of the t-matrix which is symmetrical with respect to particles and holes. In this paper it is shown that this modified definition does not remove the singularities; similar singularities, symmetrical about the Fermi surface, are still present. *J. L. Gammel* (Los Alamos, N.M.)

9215:

Chisholm, J. S. R.; Squires, E. J. On the inclusion of hole-hole scattering in the Brueckner t-matrix. Nuclear Phys. **13** (1959), 156–163.

A t-matrix for a many-fermion system is defined which allows for hole-hole scattering as well as particle-particle scattering, although the creation or destruction of pairs of particles and holes is not allowed for. The potential is assumed to be separable, and integral equations are found for the new t-matrix. The object of the paper is to show how the singularity of the t-matrix that occurs for an attractive potential is altered in this new definition. The authors study the integral equations, but their conclusions are not borne out by an examination of the explicit solution of the equation, and their results are incorrect. While Brueckner's t-matrix goes to infinity at one value of the momentum, their t-matrix oscillates infinitely as the volume of the system is increased, for all values of the momentum. It is claimed that the work sheds some light on the relation between the singularity of the t-matrix and the energy gap of a system of fermions.

D. J. Thouless (Birmingham)

9216:

Squires, E. J. Brueckner theory and the shell model effective potential. Nuclear Phys. **13** (1959), 224–234.

The idea of effective potential in several situations is discussed. The t-matrix used to calculate the ground state energy of a closed shell nucleus does not give the energy of a single particle outside a closed shell directly, but corrections that contribute to the rearrangement energy must be made. The effective potential may be different again if there are two or more particles outside a closed shell, and so proposals to determine an effective potential from experimental data run into difficulties.

D. J. Thouless (Birmingham)

9217:

Kröner, E.; Hofelich, F. Zur Behandlung des quantenmechanischen Vielteilchenproblems mit Hilfe von Mehrteilchenfunktionen. Z. Physik. **160** (1960), 297–309.

Authors' summary: "Die Boppsche Zweiteilchenbehandlung des quantenmechanischen Vielteilchenproblems wird zu einer Mehrteilchenmethode verallgemeinert. Probleme mit N Teilchen lassen sich näherungsweise auf solche mit L Teilchen zurückführen ($2 \leq L < N$). Die Näherung wird im allgemeinen um so besser, je größer das Verhältnis L/N ist. Als Beispiel wird gezeigt, daß die Zustände des N-Elektronenatoms näherungsweise aus denen des $(N-1)$-Elektronenatoms folgen. Entnimmt man die letzteren Zustände dem Experiment, so kann man die Methode testen. Ausreichende Tabellenwerte findet man bis zum Phosphor (15 Elektronen). Der z.B. danach halbtheoretisch berechnete Grundzustand des P-Atoms stimmt auf 1‰ mit dem experimentellen Wert überein. Dies kann als Beweis für die Brauchbarkeit der Methode, wie auch als experimentelle Bestätigung der (unrelativistischen) Mehrelektronen-Schrödinger-Gleichung angesehen werden. Die Anwendung der Methode auf Molekül- und Kristallprobleme, eventuell auch auf Kernprobleme, erscheint möglich."

9218:

Lipkin, Harry J. Collective motion in many-particle systems. I. The violation of conservation laws. Ann. Physics **9** (1960), 272–291.

It is pointed out that the trial wave-functions used for various many particle systems necessarily violate some conservation law, because of the incomplete reflection of the correlation between particles. Conversely, one may find the appropriate approximate wave-function by looking for "a suitable conservation law to violate". Nuclear shell model, plasma oscillation in the electron-gas nonspherical nuclei, superconductivity are treated briefly as illustrative examples. An appendix presents a concise re-derivation of some of the random phase approximation results and some ideas for its extension are given. *G. Kalman* (Haifa)

9219:

Sawicki, J. Notes on the theory of vibrational states of systems of fermions. I. Nuclear Phys. **23** (1961), 285–295.

Author's summary: "The system of nonlinear equations for the components of the single-particle density matrix operator is discussed, for both infinite and finite systems of fermions interacting through a two-body "residual" interaction. The Hartree-Fock factorization and linearization of that system of equations for the density fluctuations can be performed. Hartree-Fock exchange terms and, particularly, deviations from the random-phase approximation, are discussed. The importance of these deviations is illustrated by the example of the quadrupole vibrations of a closed spherical nuclear subshell. An apparent ambiguity in the linearization procedure is discussed and the importance of the nonlinearity of equations is stressed."

9220:

Amus'ya, M. Ya. On the ground states of atomic nuclei. I. Ž. Èksper. Teoret. Fiz. **39** (1960), 639–650 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 449–456.

Author's summary: "A variational principle is formulated and applied to a many-particle system with two-particle interaction. The wave function is chosen in a form which permits exact account to be taken of pair correlation. Equations are obtained for one- and two-particle

functions in the first approximation in the correlation. It is shown that these can easily be extended to the case of strong correlations and three-particle interactions. The results are applied to the case of the so-called nuclear matter. The equations obtained are compared with those of Brueckner."

9221:

Proceedings of the International Congress on Many-Particle Problems, Utrecht, June 13–18, 1960. Organized by the Nederlandse Natuurkundige Vereniging. Supplement to Physica **26** (1960).

A collection of 25 papers, with discussion. Those of theoretical interest will be reviewed individually in these pages.

9222:

Vonsovskiĭ, S. V.; Svirskiĭ, M. S. On the superconductivity of a ferromagnetic with a weak exchange interaction. Ž. Eksper. Teoret. Fiz. **39** (1960), 384–392 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 272–277.

The influence of *s-d* exchange on superconducting properties is discussed. It is shown that if this exchange becomes too strong, superconductivity disappears. The exchange also reduces the jump in the specific heat and it reduces the value of the critical field as well as introducing a linear term in the temperature in the expression for the critical field. *D. ter Haar* (Oxford)

RELATIVITY

See also 8453, 8728, 9010, 9205.

9223:

Rindler, W. ★Special relativity. University Mathematical Texts. Oliver and Boyd, Edinburgh-London; Interscience Publishers Inc., New York; 1960. x+186 pp. $2.25.

The Oliver and Boyd series have been known for a long time to provide useful texts on pure and applied mathematical topics at a level suitable for honours students. The book under review aims to present special relativity at such a level. It is characterised by very copious sets of examples of varying degrees of difficulty, and for that, at least, is very welcome. It makes no attempt to go very deeply into the experimental, historical or philosophical foundations of the subject, for these topics are dealt with more properly in physics texts, and are not usually lingered over in a course for mathematical physicists. (After all, even in the usual lecture presentations of classical mechanics, the philosophical subtleties are best left until students have some idea what the subject is about.)

The presentation and coverage is orthodox throughout, although some quite recent references are given. (It was interesting to note a problem referring to Terrell's recent work on the appearance of rapidly moving objects.) Full use is made of tensor techniques which are adequately presented in an appendix.

The standard of printing is high, and no misprints of any consequence were noted. Altogether this is a very useful little book.

Contents: I. Special relativity; II. Relativistic kinematics; III. Relativistic optics; IV. Space-Time; V. Relativistic mechanics of mass points; VI. Relativistic electrodynamics in vacuo; VII. Waves; VIII. Relativistic mechanics of continuous matter; Appendix (Tensors for special relativity); Bibliography; Index.

C. A. Hurst (Edinburgh)

9224:

Halbwachs, Francis; Hillion, Pierre; Vigier, Jean-Pierre. Théorie mathématique des angles d'Euler dans l'espace-temps. Ann. Inst. H. Poincaré **16**, 115–143 (1959).

In this paper, the authors investigate in detail the use of Euler angles in a four-dimensional space-time of special relativity described by J. L. Synge [*Relativity: The special theory*, North-Holland, Amsterdam, 1956; MR **17**, 1013]. Following the doctoral thesis of C. Van Winter (Groningen, 1957) they consider rotations in a three-dimensional, complex, euclidean space, with complex Euler angles: $\varphi^{\pm}=\varphi_1 \pm i\varphi_2$, $\theta^{\pm}=\theta_1 \pm i\theta_2$, $\psi^{\pm}=\psi_1 \pm i\psi_2$. It is shown that any Lorentz transformation can be represented as a product of six complex, but mutually conjugate in pairs, three-dimensional rotations about skew-symmetric tensors formed from pairs of null vectors in this space, which define invariant hyperplanes therein. In fact, every pair of such complex conjugate rotations yields a Lorentz transformation. The invariant planes replace the axes of rotation of three-dimensional kinematics, and the invariant tensors of rotation are seen to be self-dual. The authors show also how to employ infinitesimal Euler angles in calculating the differential transformations of the proper, future-preserving Lorentz group.

A. H. Klotz (Newcastle-upon-Tyne)

9225:

Halbwachs, Francis. Rotation instantanée et angle d'Euler dans l'espace-temps. Ann. Inst. H. Poincaré **16**, 145–160 (1959).

This paper continues the study of the relativistic generalisation of Euler angles [see preceding review]. By writing derivatives of matrices of transformation in the form of matrix products, the author arrives at formulae with a natural extension in the corresponding expressions of special relativity. It is shown, in particular, that there exists a complete analogy between the theory of a real, classical spinning top and two tops whose motion is given by complex conjugate Euler angles (loc. cit.) characterising the instantaneous rotation of a relativistic top.

A. H. Klotz (Newcastle-upon-Tyne)

9226:

Wolfson, C. Jake. Newtonian and Galilean reference frames (of the special theory of relativity) defined and compared by elementary mathematics. Proc. Iowa Acad. Sci. **67** (1960), 394–398.

The author considers the various definitions of Newtonian-Galilean frames of reference, and discusses the linearity of the Lorentz transformation on the basis of a definition of a Galilean inertial reference system which presupposes linearity. *G. J. Whitrow* (London)

9227:

Bakamjian, B. Relativistic particle dynamics. Phys. Rev. (2) **121** (1961), 1849–1851.

Following Dirac [Rev. Mod. Phys. **21** (1949), 392–399; MR **11**, 409], the author distinguishes the "instant" formulation of relativistic mechanics (common time) from the "point form". In the instant form he performs a contact transformation which brings it about that interaction terms enter only the generators of translations, whereas the four-dimensional angular momentum (i.e., the generators of homogeneous Lorentz transformations) are free of interaction terms between the constituent particles of the mechanical system. The contact transformation performed converts the three components of the total linear momentum into the velocity of the centroid, and the coordinates of the centroid into the first moment of the mass distribution. {Note by reviewer: Though this transformation appears to bring about the four-dimensional symmetry as regards dependence on the total rest-mass, which incorporates the internal potential and kinetic energy of the system, it also destroys the four-vector character of the energy-momentum vector.}

P. G. Bergmann (Syracuse, N.Y.)

9228:

Hlavatý, V. Proper time, apparent time, and formal time in the twin paradox. J. Math. Mech. **9** (1960), 733–744.

The author considers the purely geometrical aspect of the clock paradox. The apparent time $\{P_i\}_{P_j}$ of observer P_i as observed by observer P_j, where $i, j = 1, 2$ and $i = j$, or $i \neq j$, obeys the equation $\{P_i\}_{P_i} = \{P_j\}_{P_i} \cosh \omega$, where the relative velocity $V = \tanh \omega$. The author also introduces the formal time $[E]_i$ of an event E, which he defines as the proper time of the segment O_iE, where O_i is the origin of the inertial frame of reference in which P_i is at rest. If P_i changes his inertial frame of reference, and $E_j{}^i$ denotes the event on the world-line of P_i which is simultaneous with E_j (an event on the world-line of P_j) from the point of view of P_i, then $[E_i{}^j]_j/[E_i]_j = [E_j{}^i]_i/[E_j]_i = \cosh \Omega(t_j)$, where t_j is the time-coordinate of $E_i = E_j{}^i$ in the inertial frame of reference of P_j, and Ω is the appropriate value of ω.

From purely geometrical data alone, P_i cannot decide whether he changes his inertial frame of reference or not. Each observer regards the apparent time of the other as smaller than his own by the same factor $1/\cosh \omega$. But clocks which show formal time will synchronize when the observers meet again. The author stresses that he is considering only geometrical invariants without attempting to identify them with either physical or biological concepts.

G. J. Whitrow (London)

9229:

Figueras, Henri. Discontinuités des invariants différentiels des trajectoires des particules chargées. C. R. Acad. Sci. Paris **250** (1960), 3567–3569.

Résumé de l'auteur: "Dans cette Note, nous appliquons les résultats déjà obtenus [mêmes C. R. **250** (1960), 2143–2145; MR **22** #7807] pour les invariants différentiels de la trajectoire d'une particule chargée à l'étude des discontinuités de ces mêmes invariants lorsque la particule traverse une surface S de discontinuité des dérivées du champ électromagnétique."

J. Charles-Renaudie (Montpellier)

9230:

Synge, J. L. ★Relativity: The general theory. Series in Physics. North-Holland Publishing Co., Amsterdam; Interscience Publishers, Inc., New York; 1960. xv+505 pp. $16.50.

General relativity is mathematically the most beautiful and satisfactory fundamental theory; and yet, at the same time, it is more divorced from experiment than any other such theory. The mathematical structure is extremely conducive to the pursuit of a wealth of models, ideas geometrical, topological, and cosmological. However, it remains pure speculation from the physicists' point of view, since there is hardly ever observational corroboration. This situation was particularly in evidence from the time after the establishment of the theory until recently. As a consequence, almost no attention was paid to the questions of observability and observables, physical meaning in an operational sense, and fundamental experiments—with three famous exceptions.

This reviewer therefore enthusiastically welcomes this long-awaited second volume of the author's work on relativity. [The first volume was entitled *Relativity: The special theory*, North-Holland, Amsterdam, 1956; MR **17**, 1013.] Perhaps its two strongest points are the heavy emphasis on the geometrical aspects of the formalism, following Minkowski's spirit, and, especially, the serious concern of this "chronometry". The latter is the author's name for his study of the foundations of the theory (assumptions and concepts) in terms of basic measurements. Here at least a serious attempt is made at the reconciliation of mathematics and physical meaning, which is a lot more than can be said of many other works on this subject. His chapter on "Chronometry in Riemannian space-time" is preceded by one on Riemann spaces in general (four-dimensional and with signature plus two) and one on the "world-function" or distance function which to a large extent is based on the author's work (originally introduced by H. S. Ruse), and which occupies a central position in the whole structure of the book. It is essentially the square of the length of the geodesic between two points. In these three introductory chapters the reader is given a very good mathematical background for the things to come and at the same time is told how it is related to measurements and observations which permit him to adapt this powerful and extensive mathematical tool to his physical intuition. Fermi coordinates, Fermi transport, Fermi-Walker transport, Born rigidity, and the systematic measurement of gravitational fields, are samples of the items discussed.

The next three chapters, The material continuum, Some properties of Einstein fields, Integral conservation laws and equations of motion, deal with general relativistic kinematics and dynamics but contain much in addition to the standard material and, in particular, contain an extensive discussion of the Cauchy problem. This is followed by three chapters concerned with solutions of the field equations, spherically and axially symmetric solutions, special universes (including those of Gödel), and gravitational waves. The latter are defined in the spirit of the Bondi-Pirani-Robinson school. The final two chapters deal with electromagnetism and geometrical optics. The electromagnetism field is considered as being in curved space-time. Unified theories are not mentioned. In the last chapter, optical considerations are extended to various basic astronomical observations and the author turns once more to questions of chronometry.

Two appendices cover notation (including a long list of all the symbols used) and numerical values (in seconds) of many quantities. A very remarkable part of the book is a bibliography of general relativity which covers over 63 pages and is the most complete up-to-date collection of references known to this reviewer. It includes many papers not discussed or even mentioned in the text.

Inevitably, a book of this sort must omit many topics. Which of these one misses is partly a matter of taste. This reviewer would have especially enjoyed the following: The principle of equivalence is mentioned only in the preface as either false or trivial, playing only the role of "midwife at the birth of general relativity". {This does not seem to be a fair statement, but whatever the case may be, it is evident for example from the well-known works by Møller and Fock that the confusion on this principle is so great that its discussion would have been very worthwhile.} The Petrov classification is not mentioned nor is the work of Wheeler and his school, or even that of Rainich from which it started. Some of these papers may be of too recent origin for their inclusion, but they are listed in the bibliography.

On the whole, the book is refreshing, because it is in many ways unorthodox. Here is an author who is not afraid to deviate from the beaten path as long as he can show that he has a better way. Furthermore, the clear and lucid presentation, amply supplemented by space-time diagrams, and the very fine informal and often humorous style of the author of *Kandleman's Krim: A realistics fantasy* [Jonathan Cape, London, 1957; MR **21** #1] greatly contributes to the success of this book and makes it delightful reading. The publishers did a fine job of printing. In all, Professor Synge has written a two-volume work on relativity which will no doubt become one of the standard texts in the field. *F. Rohrlich*(Iowa City, Iowa)

9231:

Synge, J. L. Some properties of a world-function. Colloque sur la théorie de la relativité 1959, pp. 25–35. Centre Belge Rech. Math., 1960.

In a Riemannian space (metric possibly indefinite), let $S(PQ)=\int_P^Q ds$, the integral being taken along a geodesic, supposed unique, and let $\Omega(PQ)=\frac{1}{2}\varepsilon S^2$, where $\varepsilon=\pm 1$ and $\varepsilon S\geqq 0$. This "characteristic function" Ω has applications in the theory of differential equations [Hadamard, *Lectures on Cauchy's problem in linear partial differential equations*, Dover, New York, 1953; MR **14**, 474] and in Riemannian geometry [cf. Schouten, *Ricci-calculus*, Springer, Berlin, 1954; MR **16**, 521; p. 382]. Here some simple properties of Ω and its first few derivatives are developed for the Riemannian space-time of general relativity, in which application the author calls Ω the "world-function". [Many further details can now be found in his book, #9230 above.]

F. A. E. Pirani (London)

9232:

Kohler, Max. Zur Rotations- und Deformationsgeschwindigkeit in der Relativitätstheorie. Z. Physik **156** (1959), 248–255.

This paper contains a neat analysis of spin and rate of strain in a material body defined by a matter tensor **T** in general relativity. Let **s** be the future-directed time-like proper unit vector of **T** and define $p_{ik}=s_{k|i}+s_i s_{k|h}s^h$. The spin and rate of strain tensors are the skew and symmetric parts of the tensor **p**. It is shown that the spin tensor is identically zero if and only if the world-lines tangential to **s** form a normal congruence.

An almost identical analysis was given, independently, by Synge [#9230; Ch. IV, § 3].

W. Noll (Pittsburgh, Pa.)

9233:

Uhlmann, A. Zur Lokalisierung der Energie in der allgemeinen Relativitätstheorie. Acta Phys. Polon. **19** (1960), 133–138.

Aufbauend auf Ergebnissen von Møller [Ann. Physics **4** (1958), 347–371; MR **20** #732] wird untersucht, in welchem Maße ein Energiebegriff, der sich auf eine affine Tensordichte τ_i^k mit $\tau_{i,k}^k=0$ stützt, eine Lokalisierung gestattet.

D. W. Sciama (Ithaca, N.Y.)

9234:

Arnowitt, R.; Deser, S.; Misner, C. W. Consistency of the canonical reduction of general relativity. J. Mathematical Phys. **1** (1960), 434–439.

Authors' summary: "The question of consistency of the canonical reduction of general relativity (obtained by eliminating constraints and also imposing coordinate conditions in the action or generator) is examined. It is shown that the equations of motion obtained from this 'reduced' formalism agree with the original Einstein equations. Agreement is also established for the generators of space-time translations. In order to establish consistency it is necessary to discard certain well-defined divergence terms in the original Lagrangian. These would otherwise appear as nondivergences in the reduced Lagrangian incorrectly altering the equations."

[The authors described their canonical reduction in Phys. Rev. (2) **117** (1960), 1595–1602; MR **22** #4505b.]

P. W. Higgs (Edinburgh)

9235:

Good, Myron L. K_2^0 and the equivalence principle. Phys. Rev. (2) **121** (1961), 311–313.

Author's summary: "It is shown that the existence of the long-lived neutral K meson, and the absence of its decay into two pions, establishes that the gravitational masses of the K^0 and $\overline{K}^0$ are equal to a few parts in 10^{-10} of the K inertial mass. This is of interest since the $\overline{K}^0$ is the antiparticle of the K^0, and is not identical with the K^0. The gravitational mass of such a nonidentical antiparticle has never been directly measured.

"Also, the $\overline{K}^0$ has opposite strangeness to the K^0. Thus the argument rules out any linear dependence of the gravitational mass on the strangeness quantum number, a point on which all previous experiments say nothing.

"These observations are in accord with, and serve as a confirmation of, the equivalence principle of Einstein."

P. W. Higgs (Edinburgh)

9236:

Yem, Pham Xuan. Interactions entre graviton et électron en théorie fonctionnelle. J. Phys. Radium **22** (1961), 165–168. (English summary)

Author's summary: "The interactions between an electron and a graviton are expressed by the coupling terms in the equations of those two types of particles. The action of the electron in the gravitational equations is represented

in terms of its energy-momentum tensor. Conversely, the potentials of the gravitational field associated with the graviton will react upon the movement of the electron."

9237:
Libois, M. P. Relativité générale et tridimensionalité. Colloque sur la théorie de la relativité 1959, pp. 93–106. Centre Belge Rech. Math., 1960.

Consideration d'espaces temps de la Relativité générale possédant des familles de sections d'espace privilégiées.

Y. Bruhat (Paris)

9238:
Lenoir, Marcel. Sur le mouvement d'une particule en théorie du champ gravitationnel de spin. C. R. Acad. Sci. Paris **251** (1960), 2303–2304.

9239:
Bechert, Karl; Lindner, Jochen. Modell einer Coulombschen Ladung in der nichtlinearen Feldtheorie. Ann. Physik (7) **6** (1960), 361–369.

If the energy-tensor has the form of the electromagnetic energy-tensor added to that for dust, the electromagnetic field satisfying the covariant of Maxwell's equations, there is a static spherically symmetric solution for a charged particle, the gravitational attraction balancing the electrostatic repulsion. *C. W. Kilmister* (London)

9240:
Knapecz, Géza. Bemerkungen über die kanonischen Energie-Pseudotensoren der allgemein-kovarianten Wellenfeldtheorien. Z. Naturforschg. **15a** (1960), 467–470.

It is well known that in generally covariant theories, the canonical energy pseudotensor has undesirable transformation properties. The author shows that this pseudotensor has some additional defects, and thus infers that this cannot be the correct expression for energy. {In the reviewer's opinion, as long as we do not use such (or another) expression for some definite purpose, this problem is mainly one of semantics.} *A. Peres* (Haifa)

9241:
Forward, Robert L. General relativity for the experimentalist. Proc. IRE **49** (1961), 892–904.

Author's summary: "Einstein's general theory of relativity is broken down and simplified under limitations usually satisfied in experimentally realizable situations. Following the work of Møller [*The theory of relativity*, Clarendon Press, London, 1952; MR **14**, 212], an analogy between electromagnetism and gravitation is presented which allows calculation of various gravitational forces by considering the equivalent electromagnetic problem. A number of examples are included. Tensor formulation is not used except in the appendix, where justification for the analogies is given."

9242:
Debever, R; Hubaut, X. Tenseur de Weyl et directions isotopes. Acad. Roy. Belg. Bull. Cl. Sci. (5) **46** (1960), 355–362.

A Weyl tensor (-field) $C_{\alpha\beta\gamma\delta}$ in a space-time of general relativity is by definition such that

$$C_{\alpha\beta\lambda\mu} = -C_{\beta\alpha\lambda\mu} = -C_{\alpha\beta\mu\lambda} = C_{\lambda\mu\alpha\beta}$$

and also such that the contracted tensors $C^{\rho}{}_{\alpha\beta\rho}$ and ${}^*C^{\rho}{}_{\alpha\beta\rho}$ are identically zero, the latter being obtained from the tensor

$${}^*C_{\alpha\beta\lambda\mu} = \det \tfrac{1}{2}\eta_{\alpha\beta\rho\sigma}C^{\rho\sigma}{}_{\lambda\mu},$$

where $\eta_{\alpha\beta\rho\sigma}$ is the tensor volume-element. It has long been known that such a tensor in general determines four real isotropic (null) characteristic vectors. In the present paper the authors deal with the converse problem: supposing four null vectors $l_{(0)}{}^{\alpha}$ to be given initially, they find the most general Weyl tensor having $l_{(0)}{}^{\alpha}$ as characteristic null vectors. They also deal with the special cases obtained by supposing that two or more of the null vectors coincide, and finally give for each case an example metric of the form

$$ds^2 = f^2\,dx^2 + dy^2 + dz^2 - dt^2,$$

where $f = a_{ij}x_ix_j$ is a quadratic function of the three variables $(x_i) = (y, z, t)$. *H. S. Ruse* (Leeds)

9243:
Eisenhart, Luther P. The paths of rays of light in general relativity. Proc. Nat. Acad. Sci. U.S.A. **46** (1960), 1093–1097.

Certain types of solutions of the differential equations of families of geodesics and null-geodesics of a 4-dimensional Riemannian space with indefinite metric are derived subject to certain algebraic conditions. These would impose certain restrictions on the Ricci tensor, and these are compared with Einstein's field equations.

H. Rund (Durban)

9244:
Eisenhart, Luther P. Fields of unit vectors in the four-space of general relativity. Proc. Nat. Acad. Sci. U.S.A. **46** (1960), 1598–1601.

The present paper is a direct continuation of the author's article on paths of rays of light in general relativity [#9243]. The properties and the conditions imposed on the tangent vectors of null-geodesics are studied by means of a specially chosen set of mutually orthogonal basis vectors. These conditions would again imply that the Ricci tensor assumes special forms. [See also #8453 above.] *H. Rund* (Durban)

9245:
Watanabe, Ichie. On the quantization of physical space-time operators. Progr. Theoret. Phys. **24** (1960), 465–483.

The background of this paper is the circumstance that in the parametrization of general relativity, i.e., the introduction of a system of auxiliary coordinates [Dirac, Canad. J. Math. **2** (1950), 129–148; MR **13**, 306; Bergmann and Brunings, Rev. Mod. Phys. **21** (1949), 480–487; MR **11**, 299], the original coordinates appear as new field variables and the total energy and linear momentum of the field as their canonical conjugates. The application of the theory of constraints and of observables (called the "true" theory by the author) leads to the eventual elimination of these additional canonical variables. As an alternative, the author examines the possibility of introducing auxiliary

conditions, which in this case are coordinate conditions, and then applying Fermi's method well known in quantum electrodynamics ("altered" theory, Belinfante, Caplan, and Kennedy [ibid. **29** (1957), 518–546; MR **19**, 810]). By combining this approach with an indefinite metric in Hilbert space, the author produces a quantum theory in which the space-time coordinates are q-numbers subject to uncertainty relations.

P. G. Bergmann (Syracuse, N.Y.)

9246:

Le-Thanh-Phong. Sur le comportement asymptotique d'une classe de métriques de type II. C. R. Acad. Sci. Paris **251** (1960), 210–212.

A result of Sachs is generalized for a metric of type II. From the Bianchi identities, the condition $R_{\alpha\beta}=0$ and the symmetry of the curvature tensor, 16 scalar equations result. From one of these the equation of Robinson follows. Under certain additional hypotheses, the curvature tensor may be decomposed into the form $R=\mathrm{III}_b+\mathrm{III}_a+\mathrm{II}$, where the latter are respectively $O(r^{-1})$, $O(r^{-2})$ and $O(r^{-3})$.

D. E. Spencer (Storrs, Conn.)

9247:

Debever, Robert. Tenseur de super-énergie, tenseur de Riemann: cas singuliers. C. R. Acad. Sci. Paris **249** (1959), 1744–1746.

Author's summary: "La considération des quatre vecteurs isotropes associés au tenseur de Riemann des espaces à tenseur de Ricci nul, conduit aux résultats suivants: les trajectoires des vecteurs isotropes multiples sont des géodésiques isotropes; il existe des espaces remarquables dans le cas où ils sont distincts; on peut donner des formes réduites du tenseur de Riemann dans les cas singuliers."

H. Kiesow (Oberhausen)

9248:

Buchdahl, H. A. On the nonexistence of a class of static Einstein spaces asymptotic at infinity to a space of constant curvature. J. Mathematical Phys. **1** (1960), 537–541.

It is shown that $\mathfrak{R}=(-g)^{1/2}(R^2-4R_{st}R^{si}+R_{klst}R^{klst})$ can be written as an ordinary divergence $\mathfrak{R}=\mathfrak{B}^i{}_{,i}$, so that if the metric is static and regular, one has $\iiint \mathfrak{R}d^{(3)}x=\iint \mathfrak{B}^a n_a d^{(2)}x$. If furthermore $R_{kl}=0$, and if g_{kl} is asymptotically flat at infinity, then the surface integral tends to zero for sufficiently large r. It follows that $\mathfrak{R}$, which is positive semidefinite, must vanish everywhere, as is well known [A. Lichnerowicz, *Théories relativistes de la gravitation et de l'électromagnétisme*, Masson, Paris, 1955; MR **17**, 199; Chap. 8].

If $R_{kl}=\lambda g_{kl}$, one can define $\mathfrak{R}'=(-g)^{1/2}J_{klmn}J^{klmn}$, where $J_{klmn}=R_{klmn}-\frac{1}{3}\lambda g_{k[n}g_{m]l}$. It is shown that if g_{kl} is static, then $\mathfrak{R}'$ is positive semidefinite and can also be written as an ordinary divergence. An argument similar to the above then leads to the conclusion that there is no static solution of the equation $R_{kl}=\lambda g_{kl}$ $(\lambda<0)$ which is regular at all finite points and asymptotic to a space of constant curvature at infinity.

A. Peres (Haifa)

9249:

Schiff, L. I. Motion of a gyroscope according to Einstein's theory of gravitation. Proc. Nat. Acad. Sci. U.S.A. **46** (1960), 871–882.

General relativistic equations of motion are derived for a spinning test particle subject to a constraining force. It is shown that the angular momentum of a torque-free gyroscope in the Earth's gravitational field, measured by a co-moving observer, precesses with an angular velocity

$$\boldsymbol{\Omega} = \tfrac{1}{2}(\mathbf{f}\times\mathbf{v})+(3m/2r^3)(\mathbf{r}\times\mathbf{v})+[3(\mathbf{r}/r^2)(\mathbf{L}\cdot\mathbf{r})-\mathbf{L}]/r^3,$$

where $\mathbf{f}$ is the acceleration due to the nongravitational constraint, m and $\mathbf{L}$ are the Earth's mass and angular momentum, $\mathbf{r}$ is the vector from the Earth's center to the gyroscope, and $\mathbf{v}=d\mathbf{r}/dt$.

This result may be obtained either from the standard or the isotropic form of the metric. One must only take care to use different coordinate transformations in order to obtain the rate of precession as measured by the co-moving observer.

It is also shown that the supplementary condition $S^{\mu\nu}u_\nu=0$ [F. A. E. Pirani, Acta Phys. Polon. **15** (1956), 389–405; MR **19**, 509] corresponds to the absence of torque in a co-moving coordinate system, while the condition $S^{i4}=0$ [E. Corinaldesi and A. Papapetrou, Proc. Roy. Soc. London. Ser. A **209** (1951), 259–268; MR **13**, 695] corresponds to the absence of torque in the rest-frame of the Earth. Thus, if the latter condition is used, one must take into account the additional precession caused by an instrumental torque, because the position of the center of mass of the spinning particle (which is also the point at which the constraining force is applied) is not Lorentz-invariant, and measurements are made by a co-moving observer. If this torque is taken into account, the final result is independent of the choice of the supplementary condition (as it should be).

Finally, the author examines the feasibility of experimental arrangements which would display the above effect and thereby really test general relativity, and not only the equivalence principle.

A. Peres (Haifa)

9250:

Dehnen, H.; Hönl, H.; Westpfahl, K. Ein heuristischer Zugang zur allgemeinen Relativitätstheorie. Ann. Physik (7) **6** (1960), 370-406.

The authors resign from the covariant formulation of the equations for the gravitational field and derive all the three tests of general relativity theory from simple heuristic principles. These lead to the Schwarzschild's line element. Under the influence of the metric field (characterized by γ), the velocity of light (c), electronic charge (e), inertial mass of an electron (m_e), known from special relativity theory, change to: $c^*=c(1-2\gamma)$; $e^*=e(1-\gamma)$; $m_e^*=m_e(1+3\gamma)$.

Thus the change of the inertial mass is different from that considered by Einstein ($me=me(I+\gamma)$). The arguments, like so many of this type, seem to the reviewer a step backward from Einstein's general relativity theory.

The paper ends with an interesting note on Mach's principle.

L. Infeld (Warsaw)

9251:

Hély, Jean. Modèles d'univers en état de radiation pure. C. R. Acad. Sci. Paris **249** (1959), 1867–1868.

This paper gives a solution of the Einstein-Maxwell equations which represents a null electromagnetic-gravita-

tional field admitting a covariantly constant null vector field. *D. W. Sciama* (Ithaca, N.Y.)

9252:

Leutwyler, Heinrich. Sur une modification des théories pentadimensionnelles destinée à éviter certaines difficultés de la théorie de Jordan-Thiry. C. R. Acad. Sci. Paris **251** (1960), 2292–2294.

It has been discovered that the 5-dimensional, unified field theory of Jordan and Thiry leads to a wrong expression for the Lorentz force and to an excessive value for the advance of the perihelion of Mercury. To avoid these difficulties, the author proposes to modify Thiry's field equations by amending the Lagrangian using a supplementary condition in the form of the ordinary, Riemannian metric. It is found the resulting field equations admit the solutions of general relativity in the case of pure gravitation and yield the correct form of the equations of motion for charged particles.

A. H. Klotz (Newcastle-upon-Tyne)

9253:

Surin Aline. Sur les identités et les conditions de conservation en théorie de Jordan-Thiry. C. R. Acad. Sci. Paris **251** (1960), 2295–2296.

With the help of a lemma that the covariant derivatives of a tensor invariant under the isometric group of transformation, in the 5-dimensional space V_5, can be written in terms of covariant derivatives in V_4, and of the field equations in V_5, the conservation laws in V_5 are expressed in the form of V_4-relations.

A. H. Klotz (Newcastle-upon-Tyne)

9254:

Pachner, Jaroslav. Die unitäre Feldtheorie der Schwere und Elektrizität. Ann. Physik (7) **5** (1959), 70–106.

From author's summary: "Nach kurzer kritischer Übersicht über die verschiedenen unitären Feldtheorien, die eine Weiterentwicklung der allgemeinen Relativitätstheorie von Einstein darstellen, werden die Grundlagen der vierdimensionalen unitären Theorie der Schwere und Elektrizität erörtert. Die Feldgleichungen, die eine der modifizierten Delta-Funktion proportionale Materietensordichte und elektrische Stromvektordichte enthalten, werden von einen heterogenen Hamiltonian abgeleitet, ihre Kompatibilität und Vollständigkeit nachgewiesen und auf Grund der Maxwellschen Näherung ihre physikalische Interpretation gegeben. Die 1+1+4− Identitäten, die die Anzahl der unabhängigen Feldgleichungen erniedrigen, werden als Erhaltungssätze interpretiert. Die Bewegungsgleichungen werden aus den Feldgleichungen mit der neuen Methode von Infeld abgeleitet, für der Fall der Bewegung von zwei Körpern in niedrigster Näherung explizite angegeben und für den Fall der Bewegung eines elektrisch geladenen Probekörper integriert. Zum Schluss der Arbeit wird ein Fundamental system von Einheiten gebildet, in dem all physikalischen Grössen mit dimensionslosen Zahlen gemessen werden, der Zusammenhang der Naturkonstanten untersucht und eine neue Hypothese aufgestellt, nach der die von Null verschiedene Grösse der Kopplungskonstante zwischen dem Gravitations—und Elektromagnetischen Anteil des unitären Feldes durch den diskontinuierlichen Charakter der elektrischen Ladung bedingt ist." *K. Yano* (Seattle, Wash.)

9255:

Surin, Aline. Étude de la théorie de Jordan-Thiry dans le cas purement gravitationnel où la matière n'est pas chargée. C. R. Acad. Sci. Paris **251** (1960), 1270–1272.

L'A. montre qu'il est impossible en théorie unitaire de Jordan-Thiry, à cinq dimensions, de munir la variété V_4, quotient de V_5 par les trajectoires du groupe d'isométries, d'une métrique conforme $g_{ij}^* = \lambda g_{ij}$ ($g_{ij} = \gamma_{ij} - \gamma_{i0}\gamma_{j0}/\gamma_{00}$, $\gamma_{\alpha\beta}$ métrique de V_5) telle que l'on ait globalement, dans un repére orthonormé de V_5, $S_{ij} = S_{ij}^*$ en deuxiéme approximation quasi galiléenne. L'impossibilité est établie en l'absence de changes. *Y. Fourès-Bruhat* (Paris)

ASTRONOMY

See also 8721, 8931, 8937, 9277.

9256:

Kopal, Zdeněk. Numerical problems of contemporary celestial mechanics. Frontiers of numerical mathematics, pp. 45–58. Univ. of Wisconsin Press, Madison, Wis., 1960.

Review paper. *D. Brouwer* (New Haven, Conn.)

9257:

Aiton, E. J. The vortex theory of the planetary motions. II, III. Ann. of Sci. **14** (1958), 132–147, 157–172 (1960).

9258:

Petty, C. M.; Breakwell, J. V. Satellite orbits about a planet with rotational symmetry. J. Franklin Inst. **270** (1960), 259–282.

The gravitational field is assumed to be expressible in zonal spherical harmonics of degrees 0, 2, 3, 4. The variables $\rho = 1/(r\cos\phi)$, $\sigma = \tan\phi$, $\tau = \theta\rho^{-2}$, are taken (where r is the radius vector, θ the true anomaly, and ϕ the angular deviation seen from the origin of coordinates) for the planet from a steadily moving reference plane. The solution gives ρ, σ, τ as closed functions of the true anomaly and the steadily moving portion of the argument of perigee. Secular and long-period perturbations in the right ascension of the node, the inclination, the argument of perigee are given both in the general case and near the critical inclination of 63°.4.

J. A. O'Keefe (Chevy Chase, Md.)

9259:

Aarseth, S. The rotation of a barred galaxy under gravitational forces. Monthly Not. Roy. Astr. Soc. **121** (1960), 525–529.

The gravitational stability of a galaxy, with a central spherical nucleus and two cylindrical bars emerging from opposite sides of the nucleus symmetrically, is discussed.

The author attempts to determine whether there are density distributions which will be secularly stable over long time intervals for the case of rotation normal to the major axis.

He succeeds in deriving a density function which can satisfy the requirement of rigid-body rotation at five points on the bar, and he estimates a time interval during which the system will remain stable by the maximum

deviation from the required force for rigid rotation at points of the bar. *Y. Kozai* (Cambridge, Mass.)

9260:

Fieber, H. Zu den Lagrange'schen Lösungen des n-Körperproblems. Monatsh. Math. **64** (1960), 1–14.

A characterization of those solutions of the n-body problem, where the configuration of the n-bodies remains similar to its initial configuration. If $q(t)$ is the scalar function which describes the stretching of the configuration it is assumed that $\int dt/q(t)$ diverges. Let r denote the minimal dimension of the configuration. It is shown that for odd r all possible solutions are trivial, namely, all particles move along straight lines through the center of gravity. For even $r<n$ nontrivial solutions are possible, in which the particles move on conic sections (Lagrange's case) or on spheres. This generalizes known results to higher dimensions. *J. Moser* (New Rochelle, N.Y.)

9261:

Pines, Samuel. Variation of parameters for elliptic and near circular orbits. Astronom. J. **66** (1961), 5–7.

The classical parameters of Kepler used in the variation of constants for the special perturbation techniques in celestial mechanics present basic difficulties for orbits of small eccentricity and low inclination.

The author suggests a new method by using parameters a set of initial position and velocity vectors in the osculating orbit plane to avoid the difficulties.

Y. Kozai (Cambridge, Mass.)

9262:

Groves, Gordon W. On tidal torque and eccentricity of a satellite's orbit. Monthly Not. Roy. Astr. Soc. **121** (1960), 497–502.

From the author's summary: "It has long been known, by a simple and general argument, that tidal dissipation within the Earth must slow its rotation, slow the Moon's orbital motion and increase the Moon's distance. A hypothesis is made whereby it can be shown, under almost as general conditions, that tidal dissipation must also increase the eccentricity of the Moon's orbit."

The eccentricity of the Moon's orbit is obtained as a function of the semi-major axis. If the Moon originated close to the Earth, it is shown that its eccentricity would have been of the order 2×10^{-6}. An initially circular orbit would have remained circular. *G. E. Cook* (Farnborough)

9263:

Lyttleton, R. A. Dynamical calculations relating to the origin of the solar system. Monthly Not. Roy. Astr. Soc. **121** (1960), 551–569.

The dynamical aspects of certain hypotheses for the formation of planets are shown to be capable of close representation by a restricted three-body system, and accurate machine integrations of the equations of motion have been carried out on such a basis to test the hypotheses. It is first shown that the collisional and tidal mechanisms as applied to the Sun could not result in any particles going into orbital motion outside the Sun at however small a distance. The calculations also show that in a two-body collision of stars only a negligible quantity of material is likely to escape from both stars and be available for capture by the Sun; the amount could rise to planetary order only if the relative speed of the colliding stars when widely separated were several hundred kms per sec. Rotational break-up of a companion star to the Sun might fulfil such a condition.

The rotational instability of a primitive planet growing gradually from a disk of material in motion round the Sun would probably result in disruption into two main pieces with high mass-ratio and separating with hyperbolic speed. The motion of small particles formed in the stream of material drawn out between the separating main masses can also be investigated by means of a restricted three-body analogy, and in this way it is shown that satellite orbits are possible as a result of the great elongation that the last Jacobi figure possesses together with the high relative velocity of the separating pieces. It is suggested that the rotational stability of the great planets has been reached in this way and by the same means their principal satellites produced. The smaller main pieces resulting from the break-up would have sufficient terminal speed to escape entirely from the solar system.

D. W. Sciama (Ithaca, N.Y.)

9264:

Öpik, E. J.; Singer, S. F. Distribution of density in a planetary exosphere. II. Phys. Fluids **4** (1961), 221–233.

Authors' summary: "In an isolated neutral exosphere three components can be distinguished: (1) elliptic-ballistic; (2) hyperbolic-ballistic; (3) bound-elliptic. Component (2) forms the escape flux which cannot be defined simply. The velocity distribution of all components is calculated as a function of altitude in the exosphere; one can thereby assess the importance of the bound orbits. Even with the bound orbits completely filled, according to detailed balancing, the velocity distribution is non-Maxwellian; hence the barometric formula cannot be applied to determine the distribution of density. The absence of a Maxwellian distribution is shown to exist at all levels above the reference level (base of the exosphere) and the effective kinetic temperature of the gas is shown to decrease with altitude. The 'base of the exosphere' forms the dividing level between the barosphere and exosphere; its conventional definition has been re-examined and it is redefined to correspond to the level from which one-half of the escaping molecules may escape without suffering any collisions. Numerical results for all components versus altitude are given for neutral hydrogen and oxygen, applicable to the terrestrial exosphere. The contribution of bound orbits is estimated and shown to be minor at all levels. Bound orbits may be important for the exospheres of the outer planets. The interplanetary gas may be considered as an extension of the solar atmosphere; however caution must be exercised in treating it as an exosphere."

9265:

Öpik, E. J.; Singer, S. F. Distribution of density in a planetary exosphere. Phys. Fluids **3** (1960), 486–488.

9266:

Brandt, John C.; Chamberlain, Joseph W. Density of neutral gas in a planetary exosphere. Phys. Fluids **3** (1960), 485–486.

9267:

Das Gupta, Santi Ranjan. On the exact solution of transfer equation in the Milne-Eddington model: scattering according to Rayleigh phase function. Indian J. Theoret. Phys. **6** (1958), 77–84.

The following transfer equation is considered:

$$(1)\quad \frac{\mu\, dI(t,\mu)}{dt} = I(t,\mu) - \tfrac{3}{8}(3-\mu^2)J_0(t) - \tfrac{3}{8}(3\mu^2-1)J_2(t),$$

where $J_n(t) = \frac{1}{2}\int_{-1}^{1} I(t,\mu)\mu^n d\mu$. Here I is the intensity at t, travelling in a direction $\theta = \cos^{-1}\mu$. Taking the Laplace transform of (1) and inserting the boundary conditions converts (1) into an integral equation of the Wiener-Hopf type. The solution is obtained and shown to yield Chandrasekhar characteristic function H as an integral which of course forms a basis for obtaining numerical or analytical results. *H. Feshbach* (Cambridge, Mass.)

9268:

Kaplan, S. A.; Klimišin, I. A.; Sivers, V. N. The scattering of light in a medium with a moving boundary. Astronom. Ž. **37** (1960), 9–15 (Russian. English summary); translated as Soviet Astronom. AJ **4**, 7–12.

Expressions for the probability of escape of a photon from a medium with a moving boundary are developed with an eye toward studying the influence of radiative transfer on gas dynamics. The moving boundary separates fully ionized from neutral gas and the medium is plane parallel. Escape of a photon means passage from the neutral to the ionized (hence transparent) regions.

Two characteristic times are introduced: t_1, the average time spent by a photon in an absorbed state; and t_2, the average time between scatterings. Integral equations for the escape probability are written down for the cases $t_1/t_2 \gg 1$, $t_1/t_2 \ll 1$. The equations are for strictly monochromatic radiation, and no ready generalization to include noncoherence is apparent. Formal solutions for the equations are found but their discussion is reserved for a later paper. *E. A. Spiegel* (New York)

9269:

Kaplan, S. A.; Sivers, V. N. The general problem of the scattering of light in a one-dimensional medium with a moving boundary. Astronom. Ž. **37** (1960), 824–827 (Russian); translated as Soviet Astronom. AJ **4** (1961), 776–779.

This paper extends the range of validity of the work in the preceding paper [#9268] to arbitrary values of the two characteristic times which enter the problem.

E. A. Spiegel (New York)

9270:

Hoyle, F.; Ireland, J. G. On the magnetic field of the galaxy. Monthly Not. Roy. Astr. Soc. **120** (1960), 173–186.

Étude qualitative du champ magnétique de la galaxie. L'auteur suppose que les bras ont leur champ magnétique orienté alternativement vers l'intérieur et l'extérieur. Le champ magnétique est pressé par la rotation différentielle, et des instabilités apparaissent, se traduisant par la formation de boucles de champ magnétique qui viennent former le champ du halo galactique.

L'auteur donne diverses considérations d'ordres de grandeur. *E. Schatzman* (Paris)

9271:

Barton, D. E.; David, F. N. Models of functional relationship illustrated on astronomical data. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 9–33. (1 plate)

9272:

Avez, André. Propriétés globales des espaces-temps périodiques clos. C. R. Acad. Sci. Paris **250** (1960), 3585–3587.

Defining a regular periodic and closed space-time V_4, a quotient space $W_4 = V_4/\mathscr{R}$, $\mathscr{R}$ being a relation of equivalence in V_4, and a subspace Σ of V_4 of three dimensions, the author has shown that there exists a space $\Sigma_t \subset V_4$ of type Σ such that $\lim \text{area } p(\Sigma_t) = A$, the upper limit which the area $p(\Sigma)$ has and that there also exists a minimal space Σ_A such that $\Sigma_A = \text{area } p(\Sigma_A) = A$. If V is a part of V_4 and Σ_A' is a subspace of V_4 such that $p(\Sigma_A) = p(\Sigma_A')$, then it has been shown that $\int_V H dv \leqq 0$, where

$$H \equiv \int_0^s [R_{\alpha\beta} v^\alpha v^\beta + \nabla_\alpha v_\beta \nabla^\alpha v^\beta]\, ds.$$

Two cases have been considered: the exterior case $R_{\alpha\beta} = 0$ and the interior case k (the cosmological constant) is not 0. In the latter case $k > 0$. *R. S. Mishra* (Gorkhpur)

9273:

Scarf, F. L. Quantum electrodynamics in an expanding universe. Nuovo Cimento (10) **17** (1960), 375–383. (Italian summary)

The author treats quantum electrodynamics by standard renormalization procedures, but in an expanding cosmological manifold. He conjectures that though bare charge and bare mass of the electron are universal constants, phenomenological charge and mass need not be. {Note by reviewer: If this point of view is to be taken seriously, it would follow that the invariance principles on which the conservation laws are based—gauge invariance and coordinate invariance—hold for the bare theory but not the theory with renormalized interactions.}

P. G. Bergmann (Syracuse, N.Y.)

9274:

Ginzburg, V. L.; Syrovatskiĭ, S. I. Present status of the question of the origin of cosmic rays. Uspehi Fiz. Nauk **71** (1960), 411–469 (Russian); translated as Soviet Physics. Uspekhi **3** (1961), 504–541.

The question of the origin of cosmic rays has been receiving an increasing amount of attention from both astronomers and physicists during the past few years. The author presents a concise and authoritative general review on the problem. *H. Messel* (Sydney)

9275:

Reddish, V. C. On determining the ages of galaxies. Observatory **81** (1961), 19–24.

9276:

Salpeter, E. E. Matter at high densities. Ann. Physics **11** (1960), 393–413.

In this interesting paper the equation of state of matter in the most stable form is studied for all values of the

density ρ at zero temperature. Using effective-range theory, the energy and pressure of a nutron gas are calculated for low densities when the interparticle distance is large compared with the range of nuclear forces. Energy and pressure of a nutron gas at densities comparable to the nuclear densities are estimated by using a simple analytical form for the effective potential energy, in which the parameters are determined by using the experimental values for the equilibrium density, binding energy and symmetry energy for known complex nuclei. Two cases when all the nucleons are neutrons are considered. But most probably neutron matter is not bound at any density.

A completely ionized gas consisting of electrons and nuclei of atomic weight A and charge Z at densities large compared with 10^7 g/cc is then considered. If Am and Zm are the values of A and Z for the most stable nucleus for a given electron Fermi energy E_F, then Am increases with E_F. It is shown that free neutrons are at equilibrium with ionized nuclei like Tc^{145} at a density of about 3×10^{11} g/cc when the electrons Fermi energy is 23 Mev.

In the unlikely case of bound neutron matter an 'electron skin' surrounding a large neutron drop contributes a negative surface tension and then neutron drops of radius of about 100 fermis would be most stable. At densities of the order of 10^{15} g/cc hyperons coexist with protons and neutrons. At still higher densities the hard repulsive core of the nucleons becomes effective, and one can then use the 'cell method' for the calculation of energy and pressure.

F. C. Auluck (Delhi)

GEOPHYSICS

See also 8915, 8963, 8988.

9277:

Munk, Walter H.; MacDonald, Gordon J. F. ★The rotation of the Earth: A geophysical discussion. Cambridge Monographs on Mechanics and Applied Mathematics. Cambridge University Press, New York, 1960. xix + 323 pp. (1 plate) $13.50.

In writing this book, two noted mathematical geophysicists have filled a major gap in the literature by discussing with clarity the great developments in knowledge of the Earth's rotation that have taken place during the past several decades.

There are brief introductory chapters on the dynamical and deformation theory needed, and on such topics as precession and mutation. Love's numbers are introduced very early, and are followed by formal solutions of the relevant dynamical equations.

Throughout the book, it is made very clear that it is now far from sufficient to treat the Earth as a wholly rigid body. There are chapters on observations of the variation of latitude, the length of the day, and on seasonal and other variations. Attention is given to the Chandler and other periods and to their connection with varying rigidity and viscosity in the Earth's interior.

There are sections on bodily, ocean and atmospheric tides, on the dissipation of tidal energy, and on a number of other aspects of the dynamics of the Earth-Moon system. Relevant geomagnetic considerations are included, with special reference to currents in the Earth's outer core.

There is useful reference to the dynamics of polar wandering. A very readable chapter brings important principles of scientific method to bear in a discussion of palaeomagnetic observations, continental drift and related topics.

An appendix contains relevant theory of spherical harmonics and items of statistical theory bearing on the scientific method followed.

The book exhibits the real spirit of applied mathematics in its nice balance between observational evidence and mathematical argument. Much of its contents can be read with profit by geophysicists of limited mathematical training. For the fullest appreciation, knowledge of at least senior undergraduate mathematics is desirable, together with some measure of geophysical sophistication.

K. E. Bullen (Sydney)

9278:

Proudman, J. The condition that a long-period tide shall follow the equilibrium-law. Geophys. J. **3** (1960), 244–249.

The limiting form of a long-period tidal constituent will only follow the equilibrium law if friction is taken into account. By considering friction as directly proportional to the current (constant of proportionality k), the author shows that the constituents will approximate the equilibrium form if the period is large compared with the frictional period $2\pi/k$. From estimates of the frictional period, it appears that (i) the constituent whose period is nearly 19 years will certainly follow the equilibrium-law; (ii) the semiannual and annual constituents will probably follow the equilibrium-law; (iii) the fortnightly and monthly constituents will probably not follow the equilibrium law.

D. C. Gilles (Glasgow)

9279:

Hankó, G. Über Radialtriangulation. Acta Tech. Acad. Sci. Hungar. **30** (1960), 231–257. (2 inserts) (English, French, and Russian summaries)

Radial triangulation is a special case of photogrammetric triangulation in which only planimetric coordinates are obtained. Because of its relative simplicity, it is easily adaptable to solution by mechanical or graphical means, and these methods have been largely employed in the past. The author develops an analytical method of solution based on the adjustment of directions between the centers and selected conjugate image points of the photographs. Specific analyses are made of error propagation, the case of various weights, and the removal of large photographic tilts. Finally, the implication of these results to solution by the graphical method is discussed. Several numerical examples are displayed.

B. Chovitz (Washington, D.C.)

9280:

Uotila, Urho A. Investigations on the gravity field and shape of the earth. Ann. Acad. Sci. Fenn. Ser. A III No. 55 (1960), 92 pp.

Techniques are described for the calculation of the gravitational potential of the earth, both at sea level and at high elevations, chiefly from measured values of the intensity of gravity. The variation of gravity with height is studied and found to fit the Bouguer relation in many places. The emphasis is on the evaluation of the Stokes

formula for geoid height using templates on maps, and on the programming of electronic calculators, for which flow diagrams are shown. The Vening-Meinesz method of determining the deflections of the vertical from gravity anomalies is also discussed. The use of spherical harmonics is not generally recommended; techniques are, however, described for making sure that the gravity field assumed does not contain the five forbidden harmonics.

J. A. O'Keefe (Chevy Chase, Md.)

9281:

Morrison, John; Pines, Samuel. The reduction from geocentric to geodetic coordinates. Astronom. J. **66** (1961), 15–16.

Authors' summary: "This paper outlines an explicit procedure for obtaining the geodetic (latitude and altitude) coordinates of a point given the geocentric equatorial polar coordinates. The equations relating these coordinate systems require the solution of an eighth-order polynomial. The method outlined herein avoids an iterative procedure for solving these equations by providing a closed-form explicit solution using the Lagrange expansion formula." *J. A. O'Keefe* (Chevy Chase, Md.)

OPERATIONS RESEARCH, ECONOMETRICS, GAMES

See also A7922, A7958.

9282:

Shubik, Martin. Bibliography on simulation, gaming, artificial intelligence and allied topics. J. Amer. Statist. Assoc. **55** (1960), 736–751.

A bibliography on "Simulation", "Gaming and allied topics", "Monte Carlo", "Systems", and "Artificial intelligence", from the point of view of a social scientist, but useful to any one interested in these fields.

9283:

Shubik, Martin. ★Strategy and market structure: Competition, oligopoly, and the theory of games. John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London; 1959. xix + 387 pp. $8.00.

"Almost all situations in an economy must be portrayed by nonzero-sum games [p. 330]." Economists have long been familiar with a variety of market models—those of Cournot, Bertrand, Edgeworth, Chamberlin, and Robinson especially—that are essentially special cases of such games in which the analyst has assumed that the firms follow specific, and usually non-optimal, strategies. These conventional models suffer particularly from neglecting three aspects of actual markets: (1) that firms differ in their financial strength to sustain the costs of vigorous competition; (2) that in competition firms must contemplate driving competitors out of the industry and fear being driven out themselves; (3) that competition is essentially dynamic, in that it may be characterized by a drawn-out struggle to attain a desired (perhaps monopolistic) market position, in the course of which costs and incomes accrue peridoically or continuously. The special contribution of this study is analysis in terms of economic games of survival, in which the participants in the market choose strategies in the sense of von Neumann and Morgenstern with a view to maximizing the discounted expected value of the sum of their net receipts during the period of struggle, plus the liquidation value of their assets in case they should be forced to withdraw from the market, plus the value that will be theirs if they succeed in achieving the market position that is optimal for them. The choice of strategy (which includes the choice of desired market position) can be analyzed by means of game theory fairly specifically in the case of two-firm markets and in more general terms in the more general case.

This volume is the most successful essay yet achieved in the direction of understanding market behavior by applying the concepts of game theory and, although it contains few substantive results, has advanced this difficult area of economic analysis by developing promising conceptual tools. It contains useful reviews and appreciations both of the received economic doctrines of market structure and of many aspects of game theory, including nonzero-sum games, games in extensive form, the problem of information in game theory, and the strategy of implicit bargaining. The author has succeeded in throwing the first tenuous bridge across the chasm that divides game theory from practical economic analysis.

R. Dorfman (Stanford, Calif.)

9284:

Chipman, John S. The foundations of utility. Econometrica **28** (1960), 193–224.

This paper presents a careful abstract analysis of the mathematical—algebraic, topological and group—foundations of economic utility theory. Of the results, only the most important will be stated. $\mathscr{X}$ is a set (of alternatives), R is a relation (preference or indifference) over $\mathscr{X}$, R' is the converse and $\tilde{R}$ the complement of R, $I = R \cap R'$ (indifference), $P = R \cap \tilde{R}'$ (preference), O is the null and X the universal relations, $\mathscr{L}$ is the real line, and if A and B are relations, $AB = \{\langle x, y\rangle \mid (\exists z) xAz, zBy\}$. Axiom 1: R is transitive, comparable, and non-symmetric. A number of consequences of axiom 1, not all familiar, are proved. Axiom 2 (density): $\mathscr{X}$ is dense in the sense that $P \subseteq P^2$ and doubly directed in the sense that $X \subseteq P'P \cap PP'$. Theorem: When axioms 1 and 2 hold, the set of relations generated by R under union, intersection, conversion, complementation, and composition is $\{O, I, X, R, P, R', P', \tilde{I}\}$, which is a commutative associative algebra under composition, with zero O, identity I, and infinity X, all of whose elements except the diversity relation $\tilde{I}$ are idempotent. Axiom 3 (axiom of choice): There is a 1:1 correspondence between $\dot{\mathscr{X}} = \mathscr{X}/I$ and some ordinal number Ω. Theorem: Let $\langle \mathscr{L}^\Omega, \geq^L \rangle$ denote the Ω-ordered power of $\mathscr{L}$ under the lexicographic ordering. When axioms 1 and 3 hold, there exists an order-homomorphism $f: \mathscr{X} \to \mathscr{L}^\Omega$ for some ordinal number Ω. Axiom 4 (substitution): Suppose axioms 1, 2, and 3 hold. There exists a topology (an exact description is omitted) induced by the order-homomorphism of the previous theorem such that if H is a neighborhood of I in the corresponding product topology for which $I \subset H \subset X$, then $H^2 \supset H$. Axiom 4' (Archimedean): Suppose $K \subseteq \mathscr{B} \times \mathscr{B}$, where $\mathscr{B}$ is an ordered-bounded subset of $\mathscr{X}$, and suppose axioms 1, 2, and 3 hold. For all neighborhoods H of I in the f-topology, there exists an integer n such that $H^n \supset K$. Theorem: Axioms 4 and 4' are equivalent. Theorem: When axiom 4 holds, there exists an order-homomorphism of $\mathscr{X}$ into $\mathscr{L}$. The paper concludes with an

axiom (intensity), too complex to reproduce here, that implies that the above order homomorphisms are unique up to transformations which preserve certain group operations. *R. D. Luce* (Philadelphia, Pa.)

9285:

Förstner, Karl. ★Über die Bestimmbarkeit wirtschaftlicher Kenngrössen. Wirtschaftswissenschaftliche Schriften, Heft 3. Verlag Anton Hain K.G., Meisenheim am Glan, 1960. 64 pp. DM 8.80.

The author surveys the development of econometric methodology with regard to the problem of verifying economic relations, especially the problem of identifying such relations by means of current statistical observations of economic market data. *T. Haavelmo* (Oslo)

9286:

Burkhardt, F. Input-output matrix. Les mathématiques de l'ingénieur, pp. 245–246. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

A brief summary of known results on the moduli of the Leontief matrix and the approximation methods of sectoral outputs by iteration. *S. Ichimura* (Osaka)

9287:

Howard, Ronald A. ★Dynamic programming and Markov processes. The Technology Press of M.I.T., Cambridge, Mass.; John Wiley & Sons, Inc., New York-London, 1960. viii + 136 pp. $5.75.

It is frequently necessary to make a sequence of decisions, the object of which is to maximize a return or minimize a cost. Only rarely will the decisions be made on the basis of complete information concerning cause and effect, the duration of the process, the object of the process, and so on, so that a theory for decision-making under conditions of uncertainty is of paramount importance in many branches of applied mathematics. These remarks obviously apply in operations research and industrial engineering, where human beings serve as the decision-makers. They also apply in the field of automatic control, where presently control devices that have the capability to "learn" to improve their own capabilities based on experience are contemplated. One of the great challenges to the creative mathematician working in these areas is that many of the significant problems have not yet even been formulated in mathematical language. Still other fascinating problems exist at the analytical and computational levels.

This monograph—essentially the author's doctoral thesis—provides a delightful introduction to the subject of multi-stage decision processes, a field pioneered and extensively cultivated by Richard Bellman, who coined the term "dynamic programming".

The reader is first introduced to the concept of discrete-time Markov processes and their treatment via matrix analysis and generating functions. Then the expected return from a Markov process is discussed, and finally a decision aspect is introduced which leads to the question of the determination of optimal policies. The emphasis is upon determining optimal policies for processes of infinite duration, and a solution based upon successive approximations in policy-space is proposed. The remaining chapters are devoted to the treatment of processes in which there is discounting of future returns and the treatment of continuous time Markovian decision processes. Throughout, Bellman's principle of optimality plays a key role.

Some of the carefully worked-out examples, concerning baseball, operation of a taxicab fleet, and maintenance of equipment, are interesting and suggestive. One, concerned with the operation of a business concern, demonstrates the advisability of engaging in research and development activities, no doubt intended for subliminal effects on managerial readers.

Concepts from probability theory, matrix theory, differential and difference equations, Laplace transform theory, and numerical analysis, as well as the functional equation technique of dynamic programming, are skillfully blended together to provide a framework for the formulation and treatment of a variety of significant and topical conundrums in this engaging work.

R. E. Kalaba (Santa Monica, Calif.)

9288:

Bellman, Richard. Dynamic programming, invariant imbedding, and two-point boundary value problems. Boundary problems in differential equations, pp. 257–272. Univ. of Wisconsin Press, Madison, Wis., 1960.

Expository. *J. Kiefer* (Ithaca, N.Y.)

9289:

Bellman, Richard. Dynamic programming approach to optimal inventory processes with delay in delivery. Quart. Appl. Math. **18** (1960/61), 399–403.

If the delay in delivery of ordered goods is d time periods, the "least attainable cost" function in the functional equation studied by Dvoretzky, Kiefer, and Wolfowitz [Econometrica **20** (1952), 187–222; MR **13**, 856] which yields the optimum ordering policy is a function of d real variables. The author shows how, for certain simple cost structures, this function reduces to a function of one real variable, thus simplifying computation.

J. Kiefer (Ithaca, N.Y.)

9290:

Karlin, Samuel; Fabens, Augustus J. A stationary inventory model with Markovian demand. Mathematical methods in the social sciences, 1959, pp. 159–175. Stanford Univ. Press, Stanford, Calif., 1960.

There are, generally speaking, two procedures which are used in the analysis of inventory problems and other dynamic maximization problems of a similar sort. The first procedure, which may be used either in computing optimal policies, or in more theoretical work concerned with the nature of optimal policies, is to convert the problem, by the appropriate identification of state variables, to a recursive calculation, or a dynamic programming problem. The second procedure, which is the one followed in the present paper, is to assume that certain simple policies are to be used (say (S, s) policies); then to compute the steady-state distributions of quantities of interest such as stock size and shortages; to calculate, for any policy, the average cost based on these limiting distributions; and finally to select the policy which minimizes average cost.

This latter procedure was carried out by the author for relatively simple inventory models in chapters 14 and 15

of *Studies in the mathematical theory of inventory and production*, by Arrow, Karlin and Scarf [Stanford Univ. Press, Stanford, Calif., 1958; MR **20** #767]. The present paper describes a very interesting series of generalizations. Instead of demands being governed by independent distributions from period to period, it is assumed that the system may be in several possible demand states i, each associated with a specific density of demand ϕ_i. The system undergoes transitions from period to period according to a Markov transition law $P=(p_{ij})$. In addition, the delivery lead time is assumed to have a general distribution. Again an (S, s) policy is adopted, with the assumption that there is at most one outstanding order at any moment of time, and that when delivery is made, the stock level will be brought up to S.

Using the technique of semi-Markov processes, the limiting distribution of stock size is determined, along with the limiting distributions of several other quantities of interest. Some examples are worked out in detail and two slightly modified models are considered.

H. Scarf (Stanford, Calif.)

9291:

Morse, Philip M. Statistics and Operations Research. Bull. Inst. Internat. Statist. **35** (1957), no. 2, 365–379. (French summary)

The relation of statistics and operations research is discussed from the premise that the former is a branch of mathematics which may be applied in many fields of science and engineering, the latter an experimental science which utilizes many branches of mathematics in its analyses. The common ground is explored with reference to many enlightening examples, particularly the flow of automobile traffic.

9292:

Eddison, R. T.; Rivett, B. H. P.; Williams, E. C. Statistics in Operational Research in the United Kingdom. Bull. Inst. Internat. Statist. **35** (1957), no. 2, 381–389. (French summary)

9293:

Sichel, H. S. Operational Research in South Africa. Bull. Inst. Internat. Statist. **35** (1957), no. 2, 391–392, 394. (French summary)

9294:

Guilbaud, G. Th. La Recherche Opérationnelle en France. Bull. Inst. Internat. Statist. **35** (1957), no. 2, 395–398.

9295:

Adler, F. P. Relationships between organization size and efficiency. Management. Sci. **7** (1960/61), 80–84.

Author's summary: "The relation between productivity, efficiency and size of a technical organization as affected by internally generated and circulated paperwork is analyzed. It is shown that there exists an upper bound to total productive output which is independent of the number of employees and that as the organization size is increased the efficiency generally first rises and then falls off inversely proportionally to the number of employees."

H. Scarf (Stanford, Calif.)

9296:

Inzinger, R. Verkehrsplanung und Mathematik. Math.-Tech.-Wirtschaft **7** (1960), 49.

9297:

Dorfwirth, Josef R. Verkehrsplanung und Mathematik. I. Zur Analyse und Prognose des städtischen Verkehrs. Math.-Tech.-Wirtschaft **7** (1960), 50–55.

9298:

Inzinger, Rudolf. Verkehrsplanung und Mathematik. II. Das mathematische Modell einer Verkehrserhebung. Math.-Tech.-Wirtschaft **7** (1960), 107–109.

9299:

Knödel, Walter. Verkehrsplanung und Mathematik. III. Netzmodelle. Math.-Tech.-Wirtschaft **7** (1960), 147–154.

9300:

Ammeter, Hans. Le problème de la ruine dans la couverture des excédents de sinistres. Mitt. Verein. Schweiz. Versich.-Math. **60** (1960), 21–36. (German, English and Italian summaries)

Die Arbeit untersucht das Ruinproblem für eine Ueberschadendeckung in Verallgemeinerung von Resultaten schwedischer Mathematiker sowohl vom Standpunkt des Erstversicherers als auch des Rückversicherers und behandelt auch den noch allgemeineren Fall einer Ueberschadendeckung mit Retrozession der Superexzedenten. Die theoretischen Untersuchungen werden durch numerische Beispiele über die Höhe der theoretischen Risikoprämien und der Sicherheitszuschläge ergänzt. Daraus ergibt sich zum Beispiel, dass bei einer Ueberschadendeckung der Selbstbehalt des Erstversicherers so gewählt werden kann, dass die Rückversicherung sowohl vom Standpunkt des Erstversicherers als auch des Rückversicherers optimal wird. Im Falle der Ueberschadendeckung mit Retrozession der Superexzedenten gibt es eine bestimmte Tranche des Selbstbehaltes, die, wenn sie vom Erstversicherer bzw. von den verschiedenen Rückversicherern gewählt wird, als besonders gefährlich betrachtet werden muss.

W. Saxer (Zürich)

9301:

Chuard, Philippe. Le calcul, par ajournement, des primes pour rentes différées. Mitt. Verein. Schweiz. Versich.-Math. **60** (1960), 49–68. (German, Italian and English summaries)

Der Verf. gibt ein gegenüber der üblichen Praxis verschiedenes Formelsystem, wie man aus den Prämien einer sofort beginnenden Leibrente diejenige von aufgeschobenen Renten berechnen kann. Die Bruttoprämien, die nach dieser Methode berechnet werden, unterscheiden sich von denjenigen der üblichen Methode wegen anderer Normierung der Verwaltungskosten.

W. Saxer (Zürich)

9302:

Urech, Aususte. La correspondance prospective-rétrospective et les méthodes de calcul par groupes de réserves mathématiques. Mitt. Verein. Schweiz. Versich.-Math. **60** (1960), 131–149. (German, Italian and English summaries)

The author studies systematically various grouping methods for calculating premium reserves. His considerations are based on the identity of prospective and retrospective reserves. *E. Lukacs* (Washington, D.C.)

9303:

Ederer, Fred. A parametric estimate of the standard error of the survival rate. J. Amer. Statist. Assoc. **56** (1961), 111–118.

The author proposes a simplification of Greenwood's estimate of the standard error of the survival rate. Controls with numerical examples drawn from cancer registry. A table is given for rapid estimation of the standard error of the survival rate. *W. Saxer* (Zürich)

9304:

Thompson, D. J.; Kedlin, D. A note on follow-up for survival in the presence of movement. J. Amer. Statist. Assoc. **56** (1961), 119–124.

Die Sterblichkeitskontrollen werden durch Mutationen erschwert und können bei Nichtberücksichtigung der Ausgeschiedenen zu Fehlern führen. Der Verf. beschreibt ein Modell, das solche Fehler möglichst verhindern soll. *W. Saxer* (Zürich)

9305:

Hammerton, J. C. Automatic machine scheduling. Computers and Automation **10** (1961), no. 5, 17–22.

9306:

Romer, B. Das Haupttheorem der linearen Programmierung. Mitt. Verein. Schweiz. Versich.-Math. **60** (1960), 247–257. (English, French and Italian summaries)

The conditions for the existence of a maximum in a linear programming problem with n unknowns and k inequalities are discussed for: (a) $n=2$, $k=1$; (b) n arbitrary, $k=1$; (c) n and k arbitrary. Some generalisations are indicated. *G. Tintner* (Ames, Iowa)

9307:

Matthys, Gaston. Sur les problèmes linéaires de flot dans un réseau. C. R. Acad. Sci. Paris **250** (1960), 2675–2676.

The note considers flow in a transportation network in the sense of Berge [*Théorie des graphes et ses applications*, Dunod, Paris, 1958; MR **21** #1608], i.e., flows are unidirectional and integral on each arc, and there is one source and one sink.

Let A be a tree containing all vertices of the network R, H the chain of A joining source to sink, and B the cycle basis obtained by orienting each cycle associated with A in the direction of its arc of closure. Then the flows from source to sink are precisely those linear combinations of H and cycles of the base B with non-negative integral coefficients which give rise to non-negative values in the branches of A. Any problem of optimising a linear functional of the flows in R, subject to linear constraints, corresponds to a set of linear programming problems, one for each choice of A. The example given is taken up in a later note [#9309 below]. *A. M. Duguid* (London)

9308:

Matthys, Gaston. Une application de la dualité. C. R. Acad. Sci. Paris **250** (1960), 2794–2795.

Consider the dual linear programs

$$\text{(I)} \qquad x \geqq 0, \quad Ax \geqq 0, \quad \text{Min } \langle b, x \rangle,$$

$$\text{(II)} \qquad y \geqq 0, \quad yA \leqq b, \quad \text{Max } \langle y, a \rangle.$$

Let y_1 be a feasible vector of (I) and define $\gamma_i = +1$ or 0 according as $(y_1)_i > 0$ or $(y_1)_i = 0$. Let A' be the matrix obtained from A by retaining only those columns for which $(y_1 A)_j = b_j$, and let U denote the unit matrix with the same number of rows as A. Let z_0, e_0, and t_0 be solutions of the dual linear programs

$$z \geqq 0, \quad e \geqq 0, \quad A'z - Ue = a, \quad \text{Min } \langle \gamma, e \rangle,$$

$$tA' \leqq 0, \quad tU \geqq -\gamma, \quad \text{Max } \langle t, a \rangle.$$

A short direct proof is given that $\langle \gamma, e_0 \rangle = 0$ implies that y_1 is a solution of (II) and that, if $\langle \gamma, e_0 \rangle \neq 0$, then, for some $\theta > 0$, $y_2 = y_1 + \theta t_0$ is a feasible vector for (II) and $\langle y_2, a \rangle > \langle y_1, a \rangle$. *A. M. Duguid* (London)

9309:

Matthys, Gaston. Sur les problèmes de flot dans un réseau à capacités de faisceaux. C. R. Acad. Sci. Paris **250** (1960), 2840–2842.

This note establishes an algorithm for finding a flow on a network, subject to linear constraints, $\sum_{u \in F_K} \phi_u \leqq S_K$, and maximising a linear form $\phi D = \sum_{u \in U} D_u \phi(u)$. The F_K are subsets of the collection U of arcs of the network R. It is assumed that, for any circuit μ, $\sum_{u \in \mu} D_u \leqq 0$.

Regarding the D_u as distances, a flow is established on the longest path from source to sink, so as to satisfy one of the constraints, say K_1. Then the length, x_1, of that path is subtracted from the length of every arc in F_{K_1}. This procedure is repeated till there remains no path of positive length from source to sink. Let G be the subgraph of longest paths from the source to every vertex, at this stage. Let A be a tree of G including all the vertices, H the path from source to sink on A, B the cycle base associated with A, and M the incidence matrix of arcs and constraint sets F_K. Using results previously established [#9307], the flow ϕ so far constructed gives rise to a feasible vector for the linear program

$$\phi_c \geqq 0, \quad \nu > 0, \quad \phi_c BM + \nu HM \leqq S,$$

$$-\phi_c B_A - \nu H_A \leqq 0, \quad \text{Max}(\phi_c BD + \nu HD),$$

where the suffix A on a matrix denotes the retention of only those columns corresponding to branches of the tree A. Again applying earlier results [#9308], either ϕ is the required maximising flow, or one can construct a new flow ϕ' with $\phi' D > \phi D$. In the latter case, substitute $S - \phi' M$ for S in the original constraints and repeat the above process. *A. M. Duguid* (London)

9310:

Vajda, S. Inequalities in stochastic linear programming. Bull. Inst. Internat. Statist. **36** (1958), no. 357–363. (French summary)

Consider the linear programming problem $c'x=$ minimum, where $Ax=b$, $x\geq 0$. Let x_0 be the vector of solutions, and $c'x_0=M$ the minimum. Now consider the restrictions $Ax=b+b_t$, where b_t is a random variable with a known probability distribution and $Eb_t=0$. Let x_{0t} be the solution and $c'x_{0t}=M_t$ the value of the objective function. Then $EM_t\geq M$. The second problem considered is: Find min $E(c'x+f'y_t)$ under the conditions $Ax+y_t=b+b_t$, $Eb_t=0$, $x\geq 0$. Let the minimum now be M_f. It follows that $M_f\leq M$. If to the last problem $y_t\geq 0$ is added, no inequality can be established. Numerical examples are supplied, and the relation to parametric programming stressed.

G. Tintner (Ames, Iowa)

9311:

Kelley, J. E., Jr. The cutting-plane method for solving convex programs. J. Soc. Indust. Appl. Math. **8** (1960), 703–712.

This paper presents an algorithm for minimizing a linear form f on a region where some convex function $G(\mathbf{x})\leq 0$. (In practice this is likely to take the form $g_i(\mathbf{x})\leq 0$ for $i=1,\cdots,n$.) The basic idea of the algorithm is to approximate the inequalities by a set of weaker linear inequalities. Let $\mathbf{x}=\mathbf{t}_k$ minimize f subject to these linear inequalities. If $G(\mathbf{t}_k)\leq 0$ this is the required solution. Otherwise introduce the further linear inequality that the linear approximation to G in the neighbourhood of $\mathbf{x}=\mathbf{t}_k$ must be nonpositive, and reminimize f (using the dual simplex method). The author acknowledges that this method, which he first presented orally some years ago, has been developed independently by E. W. Cheney and A. A. Goldstein [Numer. Math. **1** (1959), 253–268; MR **22** #316]. He also points out that the cutting-plane concept is similar to that used in Gomory's methods for integer programming, and he shows how his ideas and Gomory's can in principle be combined to solve convex programming problems in integers.

E. M. L. Beale (London)

9312:

Shubik, M. Games, decisions and industrial organization. Management. Sci. **6** (1959/60), 455–474.

This paper consists of a review of the concepts of n-person game theory, and a discussion by means of the types of economic and industrial situations to which game theory might be applied. The author laments the fact that in the fifteen years since the appearance of game theory, relatively little has been done in the way of direct application of game theory to problems in business and social science.

H. Scarf (Stanford, Calif.)

9313:

Vorob'ev, N. N. Stable situations in coalition games. Dokl. Akad. Nauk SSSR **131** (1960), 493–495 (Russian); translated as Soviet Math. Dokl. **1**, 273–275.

The author reformulates and extends his theory of "coalitional games" [same Dokl. **124** (1959), 253–256; MR **21** #1240]. He states that a strengthening of the main theorem follows from the Nash equilibrium point theorem. After introducing a great bulk of auxiliary machinery (in which some of the notation is not explained) he concludes (theorem 5): Every coalition game, for every mapping of the form φ^*, has a φ^*-stable situation in its mixed extension. The notion of φ^*-stability appears to be analogous to Luce's ψ-stability, but more special.

J. R. Isbell (Seattle, Wash.)

9314:

Isbell, J. R. Homogeneous games. II. Proc. Amer. Math. Soc. **11** (1960), 159–161.

This paper describes "fair games" for n players, where $n=2^k(2^l-1)$, $l>k$, for the (Mersenne) primes 2^l-1 and for $l=4, 6$. A fair game is a strong, simple game whose group of permutations taking winning coalitions into winning coalitions is transitive. The author uses a result of his previous paper on this subject [Math. Student **25** (1957), 123–128; MR **20** #262] to reduce the problem of constructing a fair game for n players to the problem of constructing a transitive group of permutations for $(1,\cdots,n)$, each element of which has an odd cycle. He develops a lemma concerning a transitive collineation group of PG$(2, l-1)$, and from this constructs the desired group for $n=2^k(2^l-1)$ for the values of l and k of interest.

H. M. Gurk (Princeton, N.J.)

BIOLOGY AND SOCIOLOGY

See also A7957, A7958, 8539, 8541.

9315:

Rashevsky, N. ★Mathematical biophysics: Physico-mathematical foundations of biology. 3rd revised ed. 2 vols. Dover Publications, Inc., New York, 1960. Vol. I, xxvi+488 pp. $2.50; Vol. II, xii+462 pp. $2.50.

This third edition of the author's major opus is almost half again as large as the second (1948) and almost three times as large as the first (1938). Some of the expansion is due to amplified treatment of old problems, particularly by approximation methods, and discussions of additional special cases. For the most part, however, the additions contain new material, largely the result of research by the author and several others who at one time or another had been his co-workers.

Like the previous edition, the present has four major parts, now two in each volume: I. Mathematical biophysics of vegetative cells and cellular aggregates; II. Mathematical biophysics of excitation and conduction in peripheral nerves; III. Mathematical biophysics of the central nervous system; IV. General mathematical principles in biology (formerly, The organism as a whole and the organic world as a whole).

The most conspicuous additions to part I are the two chapters dealing with a molecular theory of cell division (including a physico-mathematical theory of mitosis) which have been juxtaposed to the older gross ("systemic") theories based on general energy considerations and on the operation of diffusion drag forces.

Part II has been left largely unchanged.

Part III has been amplified by a chapter on color vision and flicker phenomena and another on random nets, which includes some early work on that topic.

The most radical innovations are those in part IV dealing with the author's pioneer work on the general mathematical principles of biology (as against the working out of specific mathematical models of biological structures and events). He develops these general principles by considering an organism as a process which can be represented by a set of logical, spatial, or temporal relations, hence, for example, by a linear graph. In this way topology and the calculus of relations become the appropriate tools in this approach. This marks it off distinctly from the earlier approaches in which specific models were described in the language of classical mathematical physics.

Because of the rapid growth of mathematical biology in the last twenty years, the third edition of *Mathematical biophysics* represents a less adequate survey of the field than did the very much shorter first edition, as the author himself acknowledges. Nevertheless for breadth of area and for diversity of approaches and methods represented, this book still remains the most informative single source available. *A. Rapoport* (Ann Arbor, Mich.)

9316:

Gel'fand, I. M.; Cetlin, M. L. Continuous models of control systems. Dokl. Akad. Nauk SSSR **131** (1960), 1242–1245 (Russian); translated as Soviet Math. Dokl. **1**, 409–412.

The authors study the spread of an excited state in a continuous medium, characterized by the following properties. (1) Each point of the medium is capable of excitation. Excitation is instantaneous and is followed by an absolute refractory period R. (2) Excitation spreads through the medium with speed $c(x, t)$, which is a function of the "phase," i.e., the time elapsed since point x was last excited, denoted by $\tau(x, t)$. Thus $c(x, t) = \varphi[\tau(x, t)]$, where $\varphi(\tau)$ is defined for all $\tau \geqq R$. Excitation cannot spread through refractory regions. (3) There is periodic spontaneous activity at each point with constant period T.

This model is applied to continuous media under various "work regimes", i.e., imposed excitations and/or spontaneous activity. In particular, in a thin piece of tissue of length l, if the speed of excitation is a monotone increasing function of phase and if the point x is stimulated periodically with period T (spontaneous activity being absent), it is shown that the time of passage of the kth impulse through point x, denoted by $y_k(x)$ $(k = 1, 2, \cdots)$, is described by a system of differential equations

$$y_k'(x) = c^{-1}(T + y_k - y_{k-1}),$$

with initial conditions $y_k(0) = 0$, $y_0(x) = \varphi(x)$. Asymptotically one obtains

$$\lim_{k \to \infty} y_k(x) = xc^{-1}(T).$$

That is, for an arbitrary function $\varphi(x)$ the spread of excitation periodically activated within limits occurs with a constant speed. This result allows for the experimental determination of $c(x, t)$. *A. Rapoport* (Ann Arbor, Mich.)

9317:

Bellman, Richard; Jacquez, John A.; Kalaba, Robert. Some mathematical aspects of chemotherapy. I. One-organ models. Bull. Math. Biophys. **22** (1960), 181–198.

A drug injected into the blood stream diffuses first from the capillaries into the extracellular fluid and thence may enter the cells themselves and react with substances within the cells to form new compounds. Such processes may be represented by a set of differential-difference equations together with an integro-differential-difference equation. The time lag furnished by the circulation of the blood appears in the boundary conditions. A simplifying approximation applicable under certain conditions results in a set of ordinary differential equations. The application of Laplace transform technique to the solution of such systems is discussed. *I. M. H. Etherington* (Edinburgh)

9318:

Jacquez, John A.; Bellman, Richard; Kalaba, Robert. Some mathematical aspects of chemotherapy. II. The distribution of a drug in the body. Bull. Math. Biophys. **22** (1960), 309–322.

Compare the preceding review. The authors now set up a more realistic model of the body, in which each organ is treated as a single capillary bed and is linked via the circulation to other organs in the parallel or series arrangements found in the body. The concentration of an injected drug reaching any one capillary bed is determined by the concentrations leaving all the others, and by the time lags and mixing involved in the circulation. These processes and those which occur within each capillary bed are described by means of a large set of differential-difference and integro-differential-difference equations.

I. M. H. Etherington (Edinburgh)

9319:

Gittelsohn, A. M. A model for the analysis of the distribution of qualitative characters in sibships. Biometrics **16** (1960), 534–546.

Assume that in a sibship of T sibs each sib has a chance p of being affected by a genetical condition, and each affected sib has a chance p' of being "ascertained", independently of the other sibs. A sibship enters the records of an investigator if and only if it contains at least one ascertained member. Assume further that the distribution of T is (i) geometric, or (ii) Poisson. The problem is considered of the simultaneous estimation by maximum likelihood of p, p' and the parameter of the T distribution; the estimation is also done assuming "complete ascertainment", i.e., that $p' = 1$. The author also considers the effect of a simple scheme of family limitation, whereby once r affected have been born the parents terminate reproduction. r is here taken to be either constant or to have a geometric distribution. *C. A. B. Smith* (London)

9320:

Morley, F. H. W. An effect of genotype × environment interactions on estimation of genotypic covariances. Austral. J. Statist. **2** (1960), 97–101.

The estimation of components of genotypic variance is considered in cases in which the available data consists of measurements on the progeny of a number of sires, each mated to a number of randomly selected dams, this number possibly varying from sire to sire, and the progeny of each cross is tested in r replicates in a randomized block experiment. It is assumed that genotype × replicate interactions may be present. If replicates are random, it is

possible to get unbiased estimates of most of the variance components (but two are confounded); if replicates are fixed, the situation is a little more complicated, in that only certain combinations of the variance components are estimable, and some of these have a different intepretation to those in the random replicate case.

C. A. B. Smith (London)

9321:

Malecot, G. Sur l'estimation des taux de mutation et de sélection, compte tenu des migrations. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 121–129. (English summary)

9322:

Bartlett, M. S.; Gower, J. C.; Leslie, P. H. A comparison of theoretical and empirical results for some stochastic population models. Biometrika **47** (1960), 1–11.

The authors discuss a number of asymptotic approximations for the first three central moments of the distribution of population size in single-species and two-species models of biological systems. The approximations are carried out for continuous and discrete time models. Special attention is paid to the case in which the infinitesimal transition mechanism (continuous time) is linear or quadratic in population size. The theoretical results are compared with numerical results obtained from artificially generated series.

M. Rosenblatt (Providence, R.I.)

9323:

Bartlett, M. S. ★Stochastic population models in ecology and epidemiology. Methuen's Monographs on Applied Probability and Statistics. Methuen & Co., Ltd., London; John Wiley & Sons, Inc., New York; 1960. x+90 pp. $2.00.

This stimulating little work directs attention to two very important fields of application of the theory of stochastic processes (especially Markov processes): the growth of competing (or otherwise interacting) populations, and the growth and spread of epidemic diseases. The development of stochastic models in ecology is particularly timely both because of the availability of extensive series of observations by Park and others on such organisms as the Tribolium beetle, where care has been taken to record individual fluctuations as well as mean values, and because of the readiness of ecologists today to reformulate some of their problems in this way. The first five chapters of Bartlett's book are devoted to ecological problems, and form a valuable supplement to earlier contributions by the author and others [Bartlett, Biometrika **44** (1957), 27–42; MR **19**, 233; and #9322 above]. The last three chapters are concerned with epidemics. Chapter VII contains an account of the author's stochastic version of the Hamer-Soper model for recurrent measles epidemics, and chapter VIII deals with the extremely difficult problem of the spread of an epidemic in space. Here (and elsewhere in the book) the value of the discussion is much enhanced by the inclusion of diagrams illustrating the author's extensive Monte Carlo experiments (one of which simulates the spread and recurrence of an epidemic in a population of 100,000). The book is worth buying for these alone.

D. G. Kendall (Oxford)

9324:

Leslie, P. H. A note on some approximations to the variance in discrete-time stochastic models for biological systems. Biometrika **47** (1960), 196–197.

The author proposes an approximation in the computation of conditional variances for constant birth-rate and constant death-rate stochastic models for biological systems. The approximation is said to be more effective than those suggested in an earlier paper [see Leslie, Biometrika **45** (1958), 16–31; MR **19**, 1245].

M. Rosenblatt (Providence, R.I.)

9325:

Richardson, Lewis F. ★Arms and insecurity: A mathematical study of the causes and origins of war. Edited by Nicolas Rashevsky and Ernesto Trucco. The Boxwood Press, Pittsburgh, Pa.; Quadrangle Books, Chicago, Ill.; 1960. xxv+307 pp. $10.00.

This is one of the two posthumously published volumes comprising over thirty years of research which the author, a physicist and meteorologist by profession and a pacifist by persuasion, dedicated to the study of war from the point of view of interaction of quasi-physical systems.

The mathematical approach stems from a philosophical view that war and the attendant social and economic activities are primarily large-scale phenomena and so are determined more by the massive influence of systemic forces than by strategically calculated decisions. The principal model is a pair of differential equations

$$\frac{dx}{dt} = ky - ax + g, \quad \frac{dy}{dt} = lx - by + h \quad (a, b, k, l > 0),$$

where x and y are the respective armament budgets of two rival states. As is apparent from the equations, the expenditures of one enhance the rate of increase of expenditures of the other, while inhibiting one's own rate. The constants g and h, which may be of either sign, represent the "grievances" or the reservoirs of "good will", which contribute positively or negatively to the rates of expenditure increase. As is well known, the coefficients determine the existence of an equilibrium in (x, y)-space and the stability of this equilibrium. The unstable case is interpreted by Richardson as either a runaway arms race leading to war or a progressive disarmament, passing to an unlimited growth of cooperation (negative values of x and y), which he interprets in terms of international trade.

Thus the mathematical formulation is extremely simple. Aside from some modifications of the model (e.g., introduction of a "submission" term, which denotes inhibition of armament expenditures of one state by those of the other) and a generalization of the linear system to n equations, no further essential mathematical ideas are offered in the book. However the principal problem, that of application of the theory to arms races, is developed at great length, often ingeniously, and this development constitutes the bulk of the book.

The problems of applying the model are largely problems of measurement, of estimating the parameters, and of interpretation. One straightforward interpretation gave a good fit, namely, the joint armament expenditures of the two rival camps in the arms race preceding World War I, when expressed in gold-standard budgets. The disappearance of the gold standard following that war and the use

of camouflaged war budgets by some states made this simple approach inapplicable to the second great arms race (1933–1939). Accordingly, Richardson was led to exert tremendous efforts of data gathering and of painstaking analysis, interpretation, and reinterpretation in his attempt to derive quantitative measures of war preparations of the powers and their interactions. While the underlying differential equation model may give the impression of facile flippancy in a theoretical approach to an extremely complex problem, the total effort resulting from this point of departure appears herculean in scope, profoundness, and devotion. Richardson died in the midst of the present arms race (1953) without venturing a prediction of its outcome.

As is often the case with pioneering research attempts, the chief value of Richardson's work is not in the results attained, which are meager, but in the questions brought to the forefront, which are challenging. Particularly intriguing is the principal methodological question which should occur to every serious student of conflict. To what extent are the rational theories of conflict, represented in our day by game theory and associated decision theories, of which Richardson was apparently not cognizant, relevant to the understanding of large-scale conflicts, which sweep across the panorama of human history? Certainly some apparently calculated strategic decisions appear in retrospect as rationalizations of gross systemic trends, sometimes even as ritually or emotionally stimulated reactions, results of self-fulfilling assumptions which initiate vicious cycles of mounting hostility. If this is so, it appears that these phenomena, ignored in the formal schemes of rational conflict theories but emphasized in Richardson's formulation, should be more seriously considered. *A. Rapoport* (Ann Arbor, Mich.)

9326:

Luce, R. Duncan (Editor). ★Developments in mathematical psychology: Information, learning, and tracking. The Free Press, Glencoe, Ill., 1961. vii + 294 pp. $7.50.

The third volume issued by the Behavioral Models Project of the Bureau of Applied Social Research, Columbia University, presenting surveys of mathematical developments in psychology. The subtitle indicates the topics of its three essays, by R. D. Luce, R. R. Bush, and J. C. R. Licklider, respectively, which will be reviewed individually.

9327:

Anderson, T. W. Some stochastic process models for intelligence test scores. Mathematical methods in the social sciences, 1959, pp. 205–220. Stanford Univ. Press, Stanford, Calif., 1960.

A subject's scores (measured from the group means) on n intelligence tests are regarded as the values at "times" $t_1 < t_2 < \cdots < t_n$ of a realisation of a Gaussian process $X(t)$ with $EX(t) = 0$. The cases analysed include the Wiener process and a new process defined by $t = \theta$, $0 \leq \theta \leq 2\pi$; $X(0) = X(2\pi)$; $EX(\theta)X(\phi) = \cosh[\lambda(|\phi - \theta| - \pi)]/\cosh \lambda\pi$, $0 \leq |\phi - \theta| \leq 2\pi$. For the latter, the test correlations may firstly decrease and then increase with distance from the main diagonal of the correlation matrix; results are found when one and two values of $X(\theta)$ are fixed.

M. Stone (Princeton, N.J.)

9328:

Galanter, Eugene; Miller, George A. Some comments on stochastic models and psychological theories. Mathematical methods in the social sciences, 1959, pp. 277–297. Stanford Univ. Press, Stanford, Calif., 1960.

The authors point out that stochastic models of behavior rely heavily upon the basic assumptions of reflex theory. They raise doubts about the adequacy of that theory and rather tentatively suggest alternative lines of theory construction. No mathematical content.

Kenneth May (Northfield, Minn.)

9329:

Marschak, Jacob. Binary-choice constraints and random utility indicators. Mathematical methods in the social sciences, 1959, pp. 312–329. Stanford Univ. Press, Stanford, Calif., 1960.

In this paper the author gives a summary of many new and old results obtained by means of a stochastic approach to the theory of choice. Starting from the postulate that there exists a probability $p_{xy} = 1 - p_{yx}$ that the alternative x will be chosen when the choice has to be either x or y, he develops the stochastic approach by making suggestions for further conditions (C) to be fulfilled by the probabilities that a certain alternative will be chosen among a set of admissible alternatives. In this way he determines certain classes of interesting combinations of probabilities of choices. He then studies the logical interrelations between these conditions. Hereby he elegantly condenses his findings by means of arrow-schemes of implications. The perhaps most interesting condition (u) is one where the probability that x_i is chosen among the admissible alternatives x_j ($j \in M$) is $p_i(M) = u_i / \sum_{j \in M} u_j$ for every finite subset M of a certain set of subscripts for the alternatives x_j. In a still unpublished similar study the reviewer has called probabilities of choice fulfilling this condition a relatively stable decision field, because $p_i(M)/p_k(M) = u_i/u_k$ independent of exclusions of other alternatives than x_i and x_k from the set of alternatives considered.

The author shows that the condition (u) implies all earlier introduced conditions; among them those called "weak, mild and strong transitivity". A relatively stable decision field may thus give a good model for decision-makers who have reached a modest level of consistency and rationality in their choices and a stable level of skill in evaluating the preference values of different alternatives of choices. He does not take into account any dynamic features of the models for stochastic choices; there is no learning process nor any changes in the decision fields depending on different amounts of effort to concentrate the probabilities of choice towards those alternatives which somehow seem to increase in relative value when they are more carefully evaluated. The "degree of randomness of the choices" is kept fixed. The author gives a few hints for applications of the models to classical problems of consumption patterns.

The mathematical results found in the stochastic theory of choices can also be considered as contributions to the theory of multivariate distribution functions, especially with respect to the ranking order of the different coordinates and the compatibility of certain combinations of marginal distributions with the existence of a multivariate distribution having these marginal distributions in spaces of lower dimensionality.

L. Törnqvist (Helsinki)

9330:

Solomon, Herbert. Measures of worth in item analysis and test design. Mathematical methods in the social sciences, 1959, pp. 330–347. Stanford Univ. Press, Stanford, Calif., 1960.

9331:

Overall, John E. A cognitive probability model for learning. Psychometrika **25** (1960), 159–172.

The author develops a model based upon a cognitive conceptualization of the learning process. The central idea is to make response probabilities depend upon the probabilities of a hypothetical search mechanism selecting deposited traces of past behavior. The two parameters of the model are the probability a of recalling the most recent trace and a constant of decay k. When $a=1-k$ the model reduces to a standard linear operator model.

P. Suppes (Stanford, Calif.)

9332:

Suppes, Patrick. Some open problems in the foundations of subjective probability. Information and decision processes, pp. 162–169. McGraw-Hill, New York, 1960.

A brief survey of (i) some unsolved problems in the axiomatization of rational behaviour, and (ii) of the interpretation of utility in stimulus-response models.

I. J. Good (Teddington)

9333:

Martin, Léopold. Modèle probabiliste de la récupération de la sensibilité lumineuse après éblouissement. Acad. Roy. Belg. Bull. Cl. Sci. (5) **46** (1960), 396–410.

Immediately following a flash illumination, the sensitivity of the retina to light is impaired, then gradually recovers. The recovery process can be expressed as the trend of the threshold intensity to the initial value.

The author assumes that the gross recovery of the retina is a reflection of a stochastic process, in which the individual retinal units recover spontaneously and independently, the probability density of retinal unit recovering at time t being (1) $p(t)=\lambda/t$. From this assumption he derives the probability of finding an element of area in the retina with exactly n retinal units recovered:

$$(2) \qquad P_n(t) = \left(\frac{t}{t_0}\right)^{-\lambda} \frac{(\lambda \log (t/t_0))^n}{n!},$$

which corresponds to a Poisson distribution with mean $\mu(t)=\lambda \log (t/t_0)$. In particular, the probability of finding no retinal unit recovered in an element of area is (3) $P_0(t)=(t/t_0)^{-\lambda}$. If we now interpret threshold intensity $I(t)$ in terms of $P_0(t)$, namely

$$(4) \qquad \log \frac{I(t)}{I_0} = \log \left(\frac{I(t_0)}{I_0}\right) P_0(t),$$

we obtain from (3) and (4)

$$(5) \qquad \log \log I(t) = A - B \log t,$$

where A and B are constants. Equation (5) agrees well with observed data.

{Actually, equation 1 is meaningless as it stands, since both the zero point and the unit of the time scale must be arbitrary. In fact, we see that we cannot assign the value 0 to t_0 in (3), whereas we should be able to take the origin of time arbitrarily. This difficulty is removed if we modify the author's assumption in (1) to read $p(t)=\lambda/(a+t)$, where a is a constant with dimension of time, and $t_0=0$. In that case, (3) becomes $p_0(t)=((a+t)/a)^{-\lambda}$. If we interpret $I(t)$ as before, the theoretical equation assumes the form

$$\log \log I(t) = A - B \log (a+t),$$

which may account for the same experimental data if a is sufficiently small.} *A. Rapoport* (Ann Arbor, Mich.)

INFORMATION AND COMMUNICATION THEORY

See also 9419.

9334:

Middleton, David. ★An introduction to statistical communication theory. International Series in Pure and Applied Physics. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1960. xix+1140 pp. $25.00.

The term "statistical communication theory" used in the title is commonly interpreted in radio engineering to include information theory in the sense of Shannon's basic theorems, the related topics of error-correcting codes and the design of signals, and the statistical analysis of the reception of signals in the presence of noise and random distortions. Customarily, information and coding theory is separated in the literature from the statistical reception theory, and this book is no exception, although there is a chapter on information theory. The real emphasis, however, is on reception: first in a detailed analysis of signals and noise in receivers (both AM and FM), and then in an elaborate treatment of various problems in the design of radio receivers according to modern notions of statistical inference. Roughly the first half of the book covers general background material on probability and stochastic processes to be used later in the problems just mentioned. The mathematical handling of the underlying theory of stochastic processes is mostly heuristic, with a fair amount of accompanying discussion. The formulations of the engineering problems are usually stated in very general terms, followed by a methodical consideration of special cases. A mild criticism is that this generality of approach coupled, presumably, with a desire by the author to be consistent throughout the considerable length of the text has led to some complicated notation.

The book has four major divisions: Part 1, An introduction to statistical communication theory; part 2, Random noise processes; part 3; Applications to special systems; and part 4, A statistical theory of reception. Part 1 includes various topics: introductory material on random processes, covariance functions and spectra of stationary processes, second-moment theory of the response of zero-memory non-linear devices (such as rectifiers) to stochastic inputs, some discussion of sampling and interpolation, and an introduction to information theory. Part 2 is chiefly a discussion of Gaussian processes and processes derived from the Gaussian, with special attention to those with narrow-band spectrum. There is a chapter on the Langevin and Fokker-Planck equations, and a chapter on thermal, shot and impulse noise. Part 3 contains the analysis of noise and signals plus noise in particular circuits. Covariance functions and spectra are calculated for

amplitude-modulated signals and noise after frequency conversion and detection. They are also calculated for phase and frequency-modulated signals and noise, after modulation and after having been passed through limiter and discriminator. There are, in addition, chapters on linear prediction and optimum linear filtering, and on the calculation of probability densities of certain non-linear functionals of Gaussian processes. Part 3 is a particularly useful and valuable compilation of material which appears in no other one place. Part 4 consists of a (formal) introduction to statistical decision theory, and applications of this to various signal detection and extraction problems. The two types of problem are, first, to specify a receiver which is optimum for prescribed conditions of modulation and noise, according to some criterion such as, say, minimum average risk; and, second, to evaluate the performance of a given receiver according to some such criterion. There are appendices on particular special functions and on integral equations whose kernels have rational Fourier transforms. There is an extensive bibliography, and a large collection of problems, many containing results supplementing the text.

The word "introduction" appearing in the title is not to be taken seriously; the book is by all odds the most exhaustive treatment of its subject to appear in English.

W. L. Root (Lexington, Mass.)

9335:

Goldstine, H. H. Information theory. Science **133** (1961), 1395–1399.

Survey, with references.

9336:

Pinsker, M. S. The extrapolation of random vector processes and the quantity of information contained in a stationary random vector process relative to another one stationarily connected with it. Dokl. Akad. Nauk SSSR **121** (1958), 49–51. (Russian)

Let $\xi(t)=\{\xi_1(t), \cdots, \xi_n(t)\}$ be a stationary n-dimensional stochastic process, $\mathbf{M}\xi_i(t)=0$, $\mathbf{D}\xi_i(t)<\infty$ $(1\leq i\leq n$; t is an integer- or real-valued parameter); H_t the closed linear space spanned by $\xi_i(\tau)$ $(\tau\leq t,\ 1\leq i\leq n)$; $\hat{\xi}_i(s)$ $(s>0)$ the perpendiculars dropped from $\xi_i(t+s)$ on H_t. $\xi(t)$ is of rank $k\leq n$ if there are exactly k linearly independent random variables in the family $\hat{\xi}_i(s)$, $1\leq i\leq n$. If t is integer-valued and $\hat{r}_{ij}=\mathbf{M}\hat{\xi}_i(1)\hat{\xi}_j(1)$, then

$$(*)\qquad (\det\|\hat{r}_{ij}\|_{i,j=1,\cdots,n})^{1/2}=(2\pi)^{n/2}\exp(4\pi)^{-1}\int_{-\pi}^{\pi}\log\det\|f_{t_it_j}(\lambda)\|_{i,j=1,\cdots,n}\,d\lambda,$$

where $f_{t_it_j}(\lambda)$ is the joint spectral density of ξ_t and ξ_j. If t is real-valued, $\xi(t)$ is of rank n if and only if the integral of $(*)$ is convergent; this fact yields a regularity criterion for processes of rank n. Further, let $\xi(t)=\{\xi_1(t), \cdots, \xi_n(t)\}$ and $\eta(t)=\{\eta_1(t), \cdots, \eta_n(t)\}$ be two stationarily connected stochastic processes. The following two variants of rate of creating information are considered:

$$\vec{I}(\xi,\eta)=T^{-1}\mathbf{M}I(\xi_T,\eta\,|\,\xi_-),\quad \vec{I}(\xi,\eta)=\lim_{T\to\infty}T^{-1}I(\xi_T,\eta_T),$$

where ξ_T and ξ_- are the parts of ξ corresponding, respectively, to $[0, T]$ and $(-\infty, 0]$. Now, using the discrete processes $\xi^{(h)}(l)=\xi(hl)$, $\eta^{(h)}(l)=\eta(hl)$, $l=0, \pm1, \cdots$, the following two expressions are also considered:

$$\vec{I}^{(d)}(\xi,\eta)=\lim_{h\to+0}h^{-1}\vec{I}(\xi^{(h)},\eta^{(h)}),$$

$$\bar{I}^{(d)}(\xi,\eta)=\lim_{h\to+0}\bar{I}(\xi^{(h)},\eta^{(h)}).$$

It is shown that $\bar{I}(\xi,\eta)\geq\bar{I}^{(d)}(\xi,\eta)\geq\vec{I}^{(d)}(\xi,\eta)\geq\vec{I}(\xi,\eta)$. If (ξ,η) has a joint normal distribution, the effective expression M of $\vec{I}(\xi,\eta)$ is given in terms of the spectral densities; if $\xi(t)$ is singular, $\vec{I}(\xi,\eta)=0$. Moreover, if $\xi(t)$ is regular of rank n, then $\bar{I}^{(d)}(\xi,\eta)=\vec{I}^{(d)}(\xi,\eta)=\vec{I}(\xi,\eta)=M$. Finally, for rational spectral densities, $\bar{I}(\xi,\eta)=M$.

R. Theodorescu (Bucharest)

9337:

Watanabe, Satosi; Abraham, Chacko T. Loss and recovery of information by coarse observation of stochastic chain. Information and Control **3** (1960), 248–278.

Consider a microscopic chain with a finite set of states $1, \cdots, n$ and denote by $p^{(k)}(x_1, \cdots, x_k)$ a unique probability defined for a sequence of k consecutive states $x_1, \cdots, x_k$; it is assumed that this probability is independent of the location of the segment $x_1, \cdots, x_k$ in the infinite sequence of states and satisfies the conditions

$$p^{(k)}(x_1, \cdots, x_k)\geq 0,$$

$$\sum_{x_1=1}^{n}p^{(k)}(x_1, \cdots, x_k)=p^{(k-1)}(x_2, \cdots, x_k),$$

$$\sum_{x_k=1}^{n}p^{(k)}(x_1, \cdots, x_k)=p^{(k-1)}(x_1, \cdots, x_{k-1}).$$

For such a chain the following information content of a segment of length k is considered:

$$S^{(k)}=-\sum_{1\leq j\leq k}^{x_j=1,\,n}p^{(k)}(x_1, \cdots, x_k)\log p^{(k)}(x_1, \cdots, x_k),$$

$$S^{(0)}=0,\quad G^{(k)}=(1/k)S^{(k)}.$$

Suppose now that the n microscopic states are grouped in $\nu\leq n$ disjoint non-empty macrostates and consequently for a segment of macrostates $\xi_1, \cdots, \xi_k$ of length k the corresponding probability

$$\pi^{(k)}(\xi_1, \cdots, \xi_k)=\sum_{1\leq j\leq k}^{x_j\neq i_j}p^{(k)}(x_1, \cdots, x_k)$$

is defined (in the following, Greek letters refer to the "macroscopic" chain). Firstly, it is shown that the coarseness $\mathfrak{R}=S^{(1)}-\Sigma^{(1)}$ of the macroscopic observation is nonnegative; more generally, the information loss $\mathfrak{L}^{(k)}=G^{(k)}-\Gamma^{(k)}$ per symbol in a sequence of length k is also nonnegative and $\mathfrak{R}\geq\mathfrak{L}^{(k)}$. The theory is then illustrated by several examples.

R. Theodorescu (Bucharest)

9338:

Wolfowitz, J. The coding of messages subject to chance errors. Illinois J. Math. **1** (1957), 591–606.

New simple proofs for some theorems concerning information and communication theory are given. The following result, stated for general memory m by Shannon [Bell System Tech. J. **27** (1948), 379–423, 623–656; MR **10**,

133], is also proved: let $m=0$, and let λ, $1>\lambda>0$, be any given number; there exists a $K'>0$ such that, for any n, any code with the property that the probability of transmitting any word incorrectly is $<\lambda$, cannot have a length greater than $2^{nC_0+K'n^{1/2}}$, where C_0 is the capacity of the channel. *R. Theodorescu* (Zbl 78, 325)

9339:

Wolfowitz, J. An upper bound on the rate of transmission of messages. Illinois J. Math. **2** (1958), 137–141.

The present paper is a continuation of a previous one [#9338] and contains an analogous result for any given memory m. This result implies for $m=0$ the theorem given by the author in the quoted paper.

R. Theodorescu (Zbl 81, 369)

9340:

Wolfowitz, J. Information theory for mathematicians. Ann. Math. Statist. **29** (1958), 351–356.

The author describes some of his results on coding theorems for the stationary discrete communication channel. The articles reviewed above [#9338, 9339] give a more detailed version, with proofs.

S. P. Lloyd (Murray Hill, N.J.)

9341:

Wolfowitz, J. The maximum achievable length of an error correcting code. Illinois J. Math. **2** (1958), 454–458.

The author uses results described in previous articles [#9338, 9339] to obtain bounds for the size of an error-correcting block code in a discrete channel with memory m. The bounds do not involve the statistical nature of a source feeding to the channel. That is, there exists a functional J of the channel error probabilities which has the following property. Let $\varepsilon>0$ and $\lambda>0$ be arbitrary, and let n be sufficiently large. Then there exists a block code of block length n which handles $2^{n(J-\varepsilon)}$ code sequences with probability of error less than λ each, but no code of block length n and error bound λ contains more than $2^{n(J+\varepsilon)}$ code sequences. If the channel capacities for independent [Markov, ergodic stationary] sources are C_0 [C_1, C_2, respectively], then $C_0\leqq C_1\leqq C_2\leqq J$. The author conjectures that $C_1<J$ when $m>1$. He reduces the problem for $m>0$ to the memoryless case ($m=0$) by considering subblocks of suitable length and discarding the small fraction (essentially, ε in $J\pm\varepsilon$) of received symbols at the end of each subblock which produce memory between subblocks. Thus the spread $o(1)$ of bounds for the case $m>0$ is larger than the corresponding spread $O(n^{-1/2})$ the author has obtained in the case $m=0$ [above references]. *S. P. Lloyd* (Murray Hill, N.J.)

9342:

Wolfowitz, J. ★La plus grande longueur réalisable d'un code correcteur d'erreurs. Le calcul des probabilités et ses applications. Paris, 15–20 juillet 1958, pp. 139–141. Colloques Internationaux du Centre National de la Recherche Scientifique, LXXXVII. Centre National de la Recherche Scientifique, Paris, 1959. 196 pp.

In dieser Note ohne Beweise beschreibt der Verf. Ausdehnungen seiner Ergebnisse [#9338], die insbesondere obere Schranken für die Länge eines Code im Fall eines Kanals, dessen Gedächtnis nicht notwendig Null ist, betreffen. Eine ausführlichere Darstellung erschien inzwischen [#9339, 9341]. *K. Krickeberg* (Aarhus)

9343:

Blackwell, David; Breiman, Leo; Thomasian, A. J. Proof of Shannon's transmission theorem for finite-state indecomposable channels. Ann. Math. Statist. **29** (1958), 1209–1220.

The channel of the authors is a sequence of A Markov matrices, $C(1), \cdots, C(A)$, each of size $R\times R$. The transmitted [received] alphabet contains A [R] symbols, $1, \cdots, A$ [$1, \cdots, R$]. Transmission is performed as follows. At the beginning let the channel be in the initial state (symbol) r. The symbol a_1 is sent. The probability that the symbol r_1 is received is the probability of a transition from r to r_1 according to $C(a_1)$. Let a_2 be the next symbol sent. The probability that the next received symbol will be r_2 is the probability of a transition from r_1 to r_2 according to $C(a_2)$, etc. If the word $a_1, \cdots, a_n$ of length n is sent, the probability that the word $r_1, \cdots, r_n$ will be received is equal to the product of the probabilities that $r_1, \cdots, r_n$ will be received.

A source is any pair (M, φ), where M is a (say) $D\times D$ indecomposable Markov matrix (i.e., there is only one irreducible class), and φ is a function from $1, \cdots, D$ to $1, \cdots, A$. The source drives the channel as follows: If d and r are, respectively, the initial states of the source and channel, the source moves from d to a state e with probability $M(d, e)$ given by M. The source then emits $\varphi(e)$, which is the symbol sent, etc. The channel is called indecomposable if, for every source, the matrix of transition probabilities from (d, r) to (e, s) (where s is the symbol received when $\varphi(e)$ is sent) is indecomposable.

For any n, a code of length N and probability of error $\leqq\lambda$ is a set $\{(u_1, A_1), \cdots, (u_N, A_N)\}$, where the u_i are words of length n in the transmitted alphabet, the A_i are sets of words of length n in the received alphabet, the A_i are disjoint, and, when the word u_i is sent, the probability that the word received will be in A_i is $\geqq 1-\lambda$, no matter what the initial state of the channel.

The authors define the capacity H of the channel by maximizing the usual difference of entropies over all sources. (Actually, it is sufficient [see p. 1216] to maximize over matrices M which give a stochastic input of long, independent, identically distributed blocks of symbols in the transmitted alphabet.) They then prove the coding theorem: For any $\varepsilon>0$ and $0<\lambda\leqq 1$, there exists, for all n sufficiently large, a code of length $2^{n(H-\varepsilon)}$ and probability of error $\leqq\lambda$. {Herewith a shorter proof: Let $x_1, x_2, \cdots$ be a source process for which the difference of entropies is within $\varepsilon/2$ of H and for which the process $\varphi(x_1), \varphi(x_2), \cdots$ is one of independent, identically distributed blocks. From the latter property, $-n^{-1}\log p(\varphi(x_1), \cdots, \varphi(x_n))\to H(X)$, with probability 1. Since $(\varphi(x_1), r_1), (\varphi(x_2), r_2), \cdots$, and and $r_1, r_2, \cdots$ are ergodic Markovian in the blocks, $-n^{-1}\log p((\varphi(x_1), r_1), \cdots, (\varphi(x_n), r_n))\to H(X, Y)$ and $-n^{-1}\log p(r_1, \cdots, r_n)\to H(Y)$, with probability 1. But this suffices for the proof of Feinstein [Trans. I.R.E. PGIT-4 (1954), 2–22; MR **19**, 516] to go through.}

There are two types of converses to the coding theorem. The weak converse asserts that, for any $\varepsilon>0$ and λ sufficiently small, there does not exist, for n sufficiently large, a code of length $2^{n(H+\varepsilon)}$ and probability of error $\leqq\lambda$. The strong converse asserts the same conclusion as

the weak converse, but for any $\lambda<1$, not only for λ sufficiently small. The authors prove the weak converse of the coding theorem for their channel. The specific claim of the authors to extend a result of the reviewer [#9338; Theorem 2] is therefore not correct, because the latter result is an improvement on the strong converse for channels without memory. Moreover, the authors' result is already implicit in the result of Fano [*Statistical theory of communication*, notes of a course at Mass. Inst. of Technology, 1953; (*) A. Feinstein, *Foundations of information theory*, McGraw-Hill, New York, 1958; MR **20** #1594; p. 44; see also Fano, M.I.T. Res. Lab. of Electronics Tech. Rep. No. 169 (1950)]. Fano's proof does not use the particular structure of the channel, but only that $1/n$ times the capacity of an n-fold extension of the channel is arbitrarily close to H for large n. In fact, on p. 1217 the authors rediscover Fano's lemma [(*), p. 35], from which the weak converse can be proved expeditiously in a few lines [(*), p. 44]. *J. Wolfowitz* (Ithaca, N.Y.)

9344:

Feinstein, A. On the coding theorem and its converse for finite-memory channels. Information and Control **2** (1959), 25–44.

The author considers the channel with finite duration of memory studied by the reviewer [see #9338]. He proves a coding theorem and a strong converse of the latter, thus determining the capacity of the channel. [The same result was obtained by the reviewer, #9341 above. The problem for the general finite-memory channel was solved by the reviewer, #9346 below.]

For the general finite-memory channel let C_s [resp. C_e] be the supremum over all stationary inputs [resp. stationary ergodic inputs] of the rate of transmission of the input. The author proves that $C_s=C_e$ [this result was also proved by I. P. Caregradskiĭ, Teor. Veroyatnost. i Primenen. **3** (1958), 84–96; MR **20** #5703].

J. Wolfowitz (Ithaca, N.Y.)

9345:

Feinstein, A. On the coding theorem and its converse for finite-memory channels. Nuovo Cimento (10) **13** (1959), supplemento, 560–575.

Identical with the paper of the same title reviewed above.

9346:

Wolfowitz, J. A note on the strong converse of the coding theorem for the general discrete finite-memory channel. Information and Control **3** (1960), 80–93.

The strong converse of the coding theorem for a special discrete finite-memory channel was established independently by the author [#9341] and by the reviewer [#9344], as well as results concerning the computation of the channel capacity. In the present paper these results are extended to the general discrete finite-memory channel by a modification of the author's earlier methods. The semi-continuous case is not considered.

A. Feinstein (Urbana, Ill.)

9347:

Breiman, Leo. On achieving channel capacity in finite-memory channels. Illinois J. Math. **4** (1960), 246–252.

Let Γ be a discrete finite-memory channel with input alphabet A and let $R(p)$ be its rate with respect to the stationary probability measure p on $A^I=\prod_{-\infty}^{\infty}\otimes A_i$, where $A_i=A$ for all i. The author shows that (1) $C_s=C_e$, where $C_s=\sup R(p)$ over all stationary p and $C_e=\sup R(p)$ over all ergodic p, by proving actually the much stronger assertion (2): there exists at least one ergodic p' such that $C_s=R(p')$. While, as the author acknowledges, (1) has already appeared in the literature [Caregradskiĭ, Teor. Veroyatnost. i Primenen. **3** (1958), 84–96; MR **20** #5703; Carleson, Math. Scand. **6** (1958), 175–180 (which, however, seems to contain a gap); MR **21** #3281; Feinstein, #9344], (2) is definitely new. The proof is accomplished by showing that $R(p)$ is linear in p and that it is upper-semicontinuous on the set $\mathfrak{S}$ of stationary probability measures on A^I, $\mathfrak{S}$ being given the weak dual topology as a subset of the adjoint space of $C(A^I)$, the set of all continuous functions on A^I. It follows that C_s is achieved, and by an application of the Kreĭn-Mil'man theorem and the linearity of $R(p)$ it is shown that at least one maximizing p' must be an extreme point of $\mathfrak{S}$, and therefore ergodic.

A. Feinstein (Urbana, Ill.)

9348:

Bose, R. C.; Ray-Chaudhuri, D. K. Further results on error correcting binary group codes. Information and Control **3** (1960), 279–290.

A continuation and generalization of an earlier paper by the same authors [Information and Control **3** (1960), 68–79; MR **22** #3619]. Methods for constructing certain classes of error-correcting codes are given, including a generalization of some codes first found by Prange.

R. W. Hamming (Stanford, Calif.)

9349:

Peterson, W. W. Encoding and error-correction procedures for the Bose-Chaudhuri codes. Trans. IRE **IT-6** (1960), 459–470.

This paper describes methods for encoding Bose-Chaudhuri codes [see review above]. The Bose-Chaudhuri codes are generalizations of the Hamming codes, and for any positive integers m, t there exists a code of block length 2^m-1 that corrects t errors. The codes are shown to be cyclic and hence they can be encoded with a shift register. The number of operations for decoding rises as a small power of the block length. Thus this paper brings much recent theory into the domain of possible application.

R. W. Hamming (Stanford, Calif.)

9350:

Gorenstein, Daniel; Peterson, W. Wesley; Zierler, Neal. Two-error correcting Bose-Chaudhuri codes are quasi-perfect. Information and Control **3** (1960), 291–294.

This paper shows that every 2-error correcting Bose-Chaudhuri code of block length 2^m-1 is quasi-perfect and hence optimal, and suggests that there are no other non-trivial Bose-Chaudhuri codes with this property.

R. W. Hamming (Stanford, Calif.)

9351:

Yoshida, Shoji; Tomaru, Keisuke. A channel selector applying error-correcting codes. Rev. Elec. Comm. Lab. **8** (1960), 540–548.

This paper discusses the application of error-correcting codes to the problem of the design of a channel selector

using magnetic cores driven by parametrons. The experimental aspects are also discussed. Such a channel selector has been in actual telephone use for more than one year. *R. W. Hamming* (Stanford, Calif.)

9352:

Ostianu, V. M. On binary signal corrector schemes. Avtomat. i Telemeh. **21** (1960), 615–623 (Russian. English summary); translated as Automat. Remote Control **21**, 426–431.

Author's summary: "Brief descriptions are given of the methods of constructing and correcting signal systems which were developed by M. A. Gavrilov, R. R. Varšamov and R. W. Hamming. The structural formulas of the corrector scheme for each of the correction methods described are found. Corrector schemes are contrived."

9353:

Capon, Jack. A probabilistic model for run-length coding of pictures. Trans. IRE **IT-5** (1959), 157–163.

From author's summary: "A first-order Markoff process representation for pictures is proposed in order to study the picture coding system known as run-length coding (differential-coordinate encoding). A lower bound for the saving in channel capacity is calculated on the basis of this model, and is compared with the results obtained by previous investigators." *S. W. Golomb* (Pasadena, Calif.)

9354:

Radčenko, A. N. Methods of synthesizing code rings. Radiotehn. i Èlektron. **4** (1959), 1783–1795 (Russian); translated as Radio Engrg. and Electronics **4**, no. 11, 38–56.

Author's summary: "Methods for condensing the notation of a set of code combinations in the form of code rings are considered. The possibility is established of representing a complete code set by a system of code rings of a new type (type *B*). An efficient method of constructing the rings is proposed, based on transformation of a type *A* ring to a system of type *B* code rings of higher order with subsequent joining of the rings of the system in a single type *A* rings. Other methods of ring synthesis are also considered briefly."

9355:

Gutkin, L. S. On the significance of optimum criteria and a priori distributions in the theory of signal reception. Radiotehn. i Èlektron. **4** (1959), 1575–1584 (Russian); translated as Radio Engrg. and Electronics **4**, no. 10, 1–18.

Author's summary: "An analysis is given of the significance of optimum criteria for receivers and the form of a priori distributions of messages in the reception of signals with a background of fluctuation noise. It is proved that with high requirements on the accuracy (reliability) of message reproduction the structure of the optimum receiver and its properties are practically independent of the form of a priori distribution and of the selected optimum criterion for a broad class of these distributions and criteria. Therefore, in seeking a precise optimum receiver and investigation of its properties, it is permissible in many cases to start from the simplest (uniform) a priori message distribution and one of the simplest optimum criteria. From this it also follows that the broad results obtained for high signal-to-noise ratio by a number of authors, in particular V. A. Kotel'nikov, starting from a uniform a priori distribution and the simplest optimum criterion, may be extended to broad classes of more complicated a priori distributions and optimum criteria."

9356:

Bunimovič, V. I.; Morozov, V. A. On the probability of distortion caused by spurious currents and Gaussian noise. Radiotehn. i Èlektron. **4** (1959), 1585–1593 (Russian); translated as Radio Engrg. and Electronics **4**, no. 10, 19–33.

Authors' summary: "Signal distortions caused by the excitation of spurious currents and receiver noise in pulse-signal transmission over communications lines are considered. The line inhomogeneities leading to the spurious currents are assumed lumped and located at approximately equal distances from each other. Expressions are obtained for the mean probability of distortion in pulse-signal reception and the dispersion of this probability. A knowledge of the mean probability of distortion is found sufficient for practical estimation of the probability that the distortion probability does not exceed a preassigned value."

9357:

Cybakov, B. S. Capacity of certain multi-path communications channels. Radiotehn. i Èlektron. **4** (1959), 1602–1608 (Russian); translated as Radio Engrg. and Electronics. **4**, no. 10, 48–59.

Author's summary: "The capacity of communications channels in which the wave arriving at the receiving antenna consists of two components, the 'direct' wave and a wave due to scattering, is studied. An expression is obtained for the channel capacity in this case when the additive noise has a uniform spectrum. Certain particular cases are considered."

9358:

Širman, Ya. D. Theory of detecting the useful signal in the background of Gaussian noise and an arbitrary number of interfering signals with random amplitudes and initial phases. Radiotehn. i Èlektron. **4** (1959), 1957–1966 (Russian); translated as Radio Engrg. and Electronics **4**, no. 12, 9–21.

The author considers the problem of detecting a signal on which there is superimposed white Gaussian noise and m interfering signals with random amplitudes and initial phases. It is shown that the addition of interfering signals changes the pulse characteristic of the optimum filter and the energy of the threshold signal, but not the character of the optimum process and the form of the detection curve. Iterative formulae are given for the determination of the pulse characteristic of the optimum filter. The energy utilization factor for composite detection is derived. *S. Kullback* (Washington, D.C.)

9359:

Slepian, D. Some comments on the detection of Gaussian signals in Gaussian noise. Trans. IRE **IT-4** (1958), 65–68.

"Let $x(t)$, $0 \leq t \leq T$, be a sample from a stationary Gaussian ensemble with mean zero and power-density spectrum $\phi_A(f)$ or $\phi_B(f)$. If (a) $\phi_A(f) \not\equiv \phi_B(f)$, (b) $\phi_A(f)$ is either bandlimited or rational in f, (c) $\phi_B(f)$ is either bandlimited or rational in f, and if, in the case where ϕ_A and ϕ_B are both rational, $\lim_{f\to\infty}(\phi_A(f)/\phi_B(f)) \neq 1$, then there exist tests based on $x(t)$, $0 \leq t \leq T$, such that $p_A < \varepsilon$ and $p_B < \varepsilon$ for any preassigned $\varepsilon > 0$. This result holds for arbitrarily small $T > 0$." Some comments are made showing why, from the engineering point of view, the result is not as exciting as one would imagine.

S. Sherman (Detroit, Mich.)

9360:

Pickard, T. B. The effect of noise upon a method of frequency measurement. Trans. IRE **IT-4** (1958), 83–88.

Let a signal be coded as the frequency shift of a carrier. It is assumed that it is possible to receive the transmitted signal in two channels which have a 90° phase difference. Then under the assumption that the noise sources in the two channels are uncorrelated, the author describes a method of frequency measurement which can be used to recover the transmitted information.

K. S. Miller (New York)

9361:

Marcus, Michael B. The utility of a communication channel and applications to suboptimal information handling procedures. Trans. IRE **IT-4** (1958), 147–151.

The application of dynamic programming, i.e., a multistage decision process carried out on a computer, to the optimization of a communication channel, is discussed with particular reference to meeting external constraints that are difficult to include in more conventional information-theory analyses. Two examples are given of the functional equation approach used in dynamic programming, the first involving the Kelly betting model [Bell System Tech. J. **35** (1956), 917–926; MR **19**, 823] and the second extending this model to the null-zone reception system of Bloom, Chang, Harris, Hauptschein and Morgan [Proc. IRE **45** (1957), 963–975; MR **19**, 622]. The results of these examples are not new, but indicate that dynamic programming is a valid procedure for dealing with problems, such as suboptimization, that are nearly intractable by other methods of information theory.

R. Price (Lexington, Mass.)

9362:

Lerner, Robert M. A matched filter detection system for complicated Doppler shifted signals. Trans. IRE **IT-6** (1960), 373–385.

The author considers the problem of detecting signals subject to a wide range of possible Doppler shifts (i.e., translation in the frequency domain of the spectrum of the signal). A matched-filter detection system employing a tapped delay line is utilized. The performance of the physical system is claimed to be within 5 db of that of an ideal theoretical model. *K. S. Miller* (New York)

9363:

Titsworth, R. C. Coherent detection by quasi-orthogonal square-wave pulse functions. Trans. IRE **IT-6** (1960), 410–411.

It is sometimes desirable to code the input to a digital system in terms of a set of orthogonal functions (or almost orthogonal functions provided the error due to nonorthogonality be small). The author suggests square-wave pulse functions $\{\phi_i(t)\}$ of differing repetition frequencies. If the $\{\phi_i(t)\}$ are properly chosen, they can be shown to be quasi-orthogonal in the sense that the inner product of ϕ_i and ϕ_j is "small" for $i \neq j$. *K. S. Miller* (New York)

9364:

Boguslavskiĭ, I. A. On unbiased estimates of signals which depend nonlinearly on unknown parameters. Avtomat. i Telemeh. **21** (1960), 42–47 (Russian. English summary); translated as Automat. Remote Control **21**, 27–30.

Author's summary: "A method is presented which, in many cases, allows one to use linear theory for obtaining, in the presence of noise, an unbiased estimate of a signal which depends nonlinearly on unknown parameters."

9365:

Glaser, E. M.; Park, J. H., Jr. On signal parameter estimation. Trans. IRE **IT-4** (1958), 173–174.

A signal $S(t)$, $0 \leq t \leq T$, is written in the form $S(t) = \sum a_j \phi_j(t)$, where $\phi_j(t)$ are known orthonormal functions over $(0, T)$ and a_j are unknown constants. The signal $V(t) = S(t) + N(t)$ is observed, where $N(t)$ is white noise of spectral density N_0, $N(t) = \sum c_j \phi_j(t)$, where c_j are independent $N(\mu_j, N_0)$. The Bayes estimates of a_j are obtained, leading to an estimate $\gamma(t)$ of $S(t)$, when the a_j are a priori independent and normally distributed and the loss functions (a) $L(S, \gamma) = \|S - \gamma\|^2$, (b) $L(S, \gamma) = A - \delta(S - \gamma)$ are used. In both cases the result is $\hat{a}_j = \int_0^T V(t)\phi_k(t)dt$. In statistical language these are standard results on Bayes estimates of normal means.

D. V. Lindley (Aberystwyth)

9366:

Sherman, Seymour. Non-mean-square error criteria. Trans. IRE **IT-4** (1958), 125–126.

Let $x(t)$ be a stationary Gaussian process and let $\check{x}(t_0)$, for some $t_0 > 0$, be the best estimate of $x(t_0)$, given $x(t)$ for all $t \leq 0$, in the sense that the expected value of $\phi[x(t_0) - \check{x}(t_0)]$ is minimized. The author shows that if $\phi(\cdot)$ satisfies: $0 \leq \phi(e) = \phi(-e)$; $0 \leq e_1 \leq e_2 \Rightarrow \phi(e_1) \leq \phi(e_2)$; then $\check{x}(t_0)$ is the usual Wiener estimate.

D. V. Lindley (Aberystwyth)

9367:

McDonnell, D.; Perkins, R. W. On the interpolation and prediction of signals plus noise for infinite and finite smoothing times. Proc. Inst. Elec. Engrs. C **106** (1959), no. 9, 47–54.

Wiener [*Extrapolation, interpolation, and smoothing of stationary time series*, Wiley, New York, 1949; MR **11**, 118] considered the problem of predicting the future of $S(t)$, given $S(t) + N(t)$ for all negative values of t, where S and N are Gaussian and of known spectra. Zadeh and Ragazzini [J. Appl. Phys. **21** (1950), 645–655; MR **12**, 347] extended the theory to the case where the observation interval is finite, and S is the sum of an unknown polynomial and a Gaussian function, but with the restriction that the random part of $S(t) + N(t)$ has a rational spectral density. See also Davenport and Root [*An*

introduction to the theory of random signals and noise, McGraw-Hill, New York, 1958; MR **19**, 1090; pp. 240–242]. The author treats the problem by a somewhat different method, but without the polynomial component.

I. J. Good (Teddington)

9368:

Zakai, Moshe. On a property of Wiener filters. Trans. IRE **IT-5** (1959), 15–17.

A Hilbert-space argument is used to show that the solution to the general Wiener filtering problem can be obtained from the solution for the special case of pure prediction. Let $Y(\omega, \alpha)$ be the transfer function of the physically realizable linear filter that, operating on the mixture $s(t)+n(t)$ of stationary-signal-plus-stationary-noise, yields the least-mean-square approximation to $s(t+\alpha)$. Then the physically realizable filter $Y_L(\omega)$ that gives the best approximation to $L_\alpha[s(t+\alpha)]$, where L_α is an arbitrary linear operation, is given by $Y_L(\omega)=L_\alpha[Y(\omega, \alpha)]$.

R. Price (Lexington, Mass.)

9369:

Thaler, S.; Meltzer, S. A. The amplitude distribution and false alarm rate of noise after post-detection filtering. Proc. IRE **49** (1961), 479–485.

Authors' summary: "A digital computer has been used to simulate the passage of white, Gaussian noise through a narrow-band filter, followed by a detector and a post-detection filter. The amplitude distribution of the output of the post-detection filter has been obtained for several different detectors. In addition, the variation of false-alarm rates with detector law, post-detection filtering, time constants, and threshold setting has been investigated. Not only the numerical results but also the approach and the new detectors described are believed of interest."

9370:

Blum, Marvin. On the mean-square noise power of an optimum continuous filter for correlated noise. Trans. IRE **IT-6** (1960), 426–434.

Author's summary: "This paper presents the equations for the mean square error of the output of a continuous finite memory filter. The filter-output error is unbiased for arbitrary input polynomials up to degree n, and has minimum variance. The input is taken as a polynomial of degree n plus random stationary noise. Noise processes are considered, (1) where the noise is exponentially correlated, and (2) in the white-noise case. The solution for a desired output which is an arbitrary fixed linear operation on the input polynomial is given. Tables and graphs of the mean-square error for the derivative and prediction operator for the 0th, 1st, and 2nd derivatives are presented, and for input polynomials up to the 6th degree."

S. Kullback (Washington, D.C.)

9371:

Keilson, J.; Mermin, N. D.; Bello, P. A theorem on cross correlation between noisy channels. Trans. IRE **IT-5** (1959), 77–79.

Each of a pair of partially dependent noise processes, $N_0(t)+N_1(t)$ and $N_0(t)+N_2(t)$, is applied to a detector, $N_0(t)$, $N_1(t)$ and $N_2(t)$ being mutually independent gaussian processes that are stationary with zero mean and have narrow-band power spectra that are symmetric about a frequency ω_0. Each detector is of the form $F(x(t), y(t))$, where its narrow-band input is $x(t)\cos\omega_0 t + y(t)\sin\omega_0 t$. It is assumed that F is homogeneous, so that $F(\lambda x, \lambda y)=\lambda^\nu F(x, y)$ for all real λ and for a particular real constant ν. It is then established that the normalized cross-correlation function of the detector outputs has the same dependence on the normalized cross-correlation function of their input processes, as the normalized auto-correlation function of the ouput of either detector has on the normalized auto-correlation function of its input. Thus, let $R_0(\tau)=G(\sigma_0^2, \rho(\tau))$ be the normalized auto-correlation function of the output of a detector whose input has variance σ_0^2 {actually, there is no direct dependence of $R_0(\tau)$ on σ_0^2 if the detector is homogeneous}, and whose input components $x(t)$ and $y(t)$ have the normalized auto-correlation function $\rho(\tau)$. Then the normalized cross-correlation function of the pair of detector outputs whose inputs are $N_0(t)+N_1(t)$ and $N_0(t)+N_2(t)$ is $R_{12}(\tau)=G(\sigma_0^2, Z\rho(\tau))$, where $Z\rho(\tau)=[\{1+(\sigma_1^2/\sigma_0^2)\}\{1+(\sigma_2^2/\sigma_0^2)\}]^{-1/2}\rho(\tau)$ is the normalized cross-correlation function of the $x(t)$ and $y(t)$ components, taken from one input to the other. The variances of $N_1(t)$ and $N_2(t)$ are σ_1^2 and σ_2^2, respectively, and σ_0^2 and $\rho(\tau)$ are the variance and normalized component auto-correlation function of the common-noise process $N_0(t)$.

R. Price (Lexington, Mass.)

9372:

Jacobson, Melvin J. Correlation and delay line attenuation. Trans. IRE **IT-5** (1959), 4-9.

From the author's summary: "The effect of delay line attenuation to the output of a correlator is studied for the case where the attenuation varies linearly with delay and as the square root of frequency. It is shown how attenuation affects the output signal-noise ratio of the correlator, increasing it for some signal spectrum shapes and decreasing it for others."

G. Weiss (Washington, D.C.)

9373:

Karr, Philip R. An inequality concerning the envelope of a correlation function. Trans. IRE **IT-5** (1959), 33.

Let $K(t)=\int_0^\infty w(f)e^{2\pi ift}df$, $w(f)$ real, and call its real part, $R(t)$, a correlation function. This note consists of the following two obvious statements: (1) $|K(t)|\leq K(0)\equiv R(0)$; (2) this inequality is stronger than simply $|R(t)|\leq R(0)$. {Reviewer's remark: The author gives the impression that his notion of calling $|K(t)|$ the envelope of $R(t)$ is novel; however, noting that the Fourier transform of K vanishes for $f<0$, K is the pre-envelope of its real part R, and so $|K|$ is the usual definition of envelope of R after all.}

J. Dugundji (Los Angeles, Calif.)

9374:

Dugundji, J. Envelopes and pre-envelopes of real waveforms. Trans. IRE **IT-4** (1958), 53–57.

Author's summary: "Rice's formula [Bell System Tech. J. **23** (1944), 282–332; **24** (1945), 46–156; MR **6**, 89, 233] for the 'envelope' of a given signal is very cumbersome; in any case where the signal is not a single sine wave, the

analytical use and explicit calculation of the envelope is practically prohibitive. A different formula for the envelope is given herein which is much simpler and easier to handle analytically. We show precisely that if $\hat{u}(t)$ is the Hilbert transform of $u(t)$, then Rice's envelope of $u(t)$ is the absolute value of the complex-valued function $u(t)+i\hat{u}(t)$. The function $u+i\hat{u}$ is called the pre-envelope of u and is shown to be involved implicitly in some other usual engineering practices.

"The Hilbert transform $\hat{u}$ is then studied; it is shown that $\hat{u}$ has the same power spectrum as u and is uncorrelated with u at the same time instant. Further, the autocorrelation of the pre-envelope of u is twice the pre-envelope of the autocorrelation of u.

"By using the pre-envelope, the envelope of the output of a linear filter is easily calculated, and this is used to compute the first probability density for the envelope of the output of an arbitrary linear filter when the input is an arbitrary signal plus Gaussian noise. An application of pre-envelopes to the frequency modulation of an arbitrary waveform by another arbitrary waveform is also given." *S. Kullback* (Washington, D.C.)

9375:

Parker, Winston E.; East, Larry V. Automatic processing system for acoustical data. J. Acoust. Soc. Amer. **33** (1961), 1–6.

9376:

Foulkes, J. D. Computer identification of vowel types. J. Acoust. Soc. Amer. **33** (1961), 7–11.

9377:

Scheinberg, Stephen. Note on the Boolean properties of context free languages. Information and Control **3** (1960), 372–375.

For a finite state language of Chomsky type 2 (context-free) the paper establishes the theorem: The set of context-free languages is closed under the operation of set union, but not under difference or intersection.

A. D. Booth (London)

9378:

Freudenthal, Hans. ★Lincos: Design of a language for cosmic intercourse. Part I. Studies in Logic and the Foundations of Mathematics. North-Holland Publishing Co., Amsterdam, 1960. v+224 pp. $6.00.

One may assume, with unascertainable loss of generality, that other creatures exist beyond the earth with whom it may ultimately be possible to communicate. Having made this assumption, three classes of problems involving this communication may be distinguished, viz., technological, linguistic, and diplomatic. The technological problem is to provide a physical communication channel (e.g., a radio link) between the two cultures. The linguistic problem is to evolve a "language" for this communication which will be mutually intelligible. The diplomatic problem is to determine what type of information to give out, and what type to seek. In LINCOS (derived from *lingua cosmica*) no serious attempt is made to discuss either the technological or diplomatic aspects of the subject. The purported objective is to develop an unambiguous language which could be taught to intelligent creatures who could receive messages from us only in the form of sequences of symbols from a finite alphabet. While the freedom from ambiguity is to be comparable with that of formal mathematical logic, the range of discoursable subject matter is to be broadened to the entire range of human experience. The author claims merely to have taken some first tentative steps, both with respect to elimination of ambiguity and broadening the list of discussable subjects. Succeeding volumes may take him closer to the stated goal.

The "practical" uses of LINCOS, if indeed there be any, are not likely to be in the area of communication with extra-terrestrials. For one thing, the necessity of expanding "I know Caesar" to "there is an x with the properties that x is Caesar and that I know: 'is this x Caesar?' " [sic], would tax prohibitively the undoubtedly infinitesimal channel capacity (the technological problem) of the intercelestial communication link. More important, diplomacy has always required the highest degree of ambiguity in order to function successfully, so that LINCOS would tend to complicate the diplomatic problem intolerably. Finally, there is the objection that the Centaurians will be too busy trying to teach us the special language which they invented for the purpose to spend time listening to LINCOS.

A more intriguing possibility is that a language like LINCOS could have significant uses on earth, e.g., in the communication of ideas between humans and artificially intelligent devices (i.e., digital computers), between two such devices, or even among human beings, as in the writing of totally unambiguous scientific articles, and perhaps also the enactment of unmisinterpretable laws. (The latter use would be opposed by many legal experts, not only because it would pose a threat of technological unemployment, but also because one current school of jurisprudence holds that the cause of justice can only be served when the laws contain some degree of flexibility.)

As a purely mathematical attempt to extend to language generally the precision of the restricted usages of mathematical logic, LINCOS is, at very least, a noble experiment. In detail, many of the conventions adopted appear somewhat bizarre, but this was also the surface impression conveyed by *Principia Mathematica*. The fact that total freedom from ambiguity has not been attained is partly an indication that more effort along these lines is required. Finding and correcting the occasional misprints in the symbolic portion of the book serve in lieu of exercises for the ambitious reader.

The table of contents is: Introduction; Mathematics; Time; Behavior; Space, motion, mass. This choice of subject matter is equally appropriate for educating either extra-terrestrials or digital trajectory computers.

A severe test of LINCOS, and one which it would not pass in its present form, would be to program a computer to respond to "commands" given it in LINCOS. There is small chance that the computer will supply intended but incorrectly expressed meanings based on previous experience or extra-linguistic cues, and where ambiguities do exist, the computer will either stop entirely or take the wrong branch. Although the author makes no mention of it, the teleological evolution of compiler programs for computers is aimed at the same mathematical-linguistic goal as is LINCOS.

S. W. Golomb (Pasadena, Calif.)

SERVOMECHANISMS AND CONTROL

See also A8167, 8674, 8706.

9379:

Bibbero, Robert J. ★Dictionary of automatic control. With a foreword by Theodore K. Steele. Automatic Control Library. Reinhold Publishing Corporation, New York; Chapman & Hall, Ltd., London; 1960. xiii+282 pp. $6.00.

This dictionary is intended to help those engineers who enter the field of automatic control in the present "industrial revolution", which introduces automatic control into engineering branches where it was not used earlier. A classified index (I. Control theory and basic concepts, II. Computers and data processing, III. Industrial machine and process control, IV. Aircraft and missile control and telemetering, V. Control components and design factors) will assist in finding a term for which the precise word or phrase is not known to the user.

The book is oriented primarily toward the practising engineer; it gives comprehensible definitions of commonly occurring terms, and wherever possible, gives concise descriptions of the mechanisms and processes wherein the terms gain usage.

{Some remarks: p. 29, line 1 "continuous" should be replaced by "piecewise continuous"; p. 115, the definition of impedance is rather narrow; p. 267, Z-Transform, line 8, it should read $z=e^{+Ts}$ instead of e^{-Ts}.}

I. Flügge-Lotz (Stanford, Calif.)

9380:

Mesarović, Mihajlo D. ★The control of multivariable systems. Technology Press Research Monographs. The Technology Press of M.I.T., Cambridge, Mass.; John Wiley & Sons, Inc., New York-London; 1960. xi+112 pp. $3.50.

9381:

Goodman, N. R.; Katz, S. Calculating open loop transfer functions from closed loop measurements. J. Assoc. Comput. Mach. **5** (1958), 289–297.

The transfer function of a linear, stationary system can be estimated by calculating the spectra and cross-spectra between input and output when the system is running in open loop (uncontrolled). This paper is concerned with the problem when open-loop measurements are unrealistic and the system must be observed operating under normal load (in closed loop). The procedure is illustrated by an actual example. *W. Freiberger* (Providence, R.I.)

9382:

Hohn, Franz E. ★Applied Boolean algebra: An elementary introduction. The Macmillan Co., New York, 1960. xx+137 pp. Paperbound: $2.50.

"The purpose of this little book is to give an elementary, intuitive introduction to Boolean algebra as a model of a class of simple switching circuits, then to show that the same algebra provides a theory of propositional functions and the logic circuits of computers, and finally to show that the same algebra may also be interpreted as the algebra of the subsets of a set. Thus the emphasis is on Boolean algebra as applied mathematics rather than as abstract algebra."

Contents: Introduction (pp. ix–xx, chapter 1); Boolean algebra as a model of combinatorial relay circuitry (pp. 1–45, chapter 2); Boolean algebra as a model of propositional logic (pp. 46–77, chapter 3); Boolean algebra of the subset of a set (pp. 78–93, chapter 4); The minimization problem (pp. 94–112, appendix I); the binary system of numeration (pp. 113–121, appendix II); Semiconductor logic elements (pp. 122–139).

S. Sherman (Detroit, Mich.)

9383:

Heijn, H. J. Representations of switching functions and their application to computers. Philips Res. Rep. **15** (1960), 305–341, 448–491. (French and German summaries)

The purpose of this paper is to derive principles for the design of a digital computer using magnetic cores and associated gates as logic elements. A way of representing switching functions which corresponds directly to a magnetic-core realization of the function is given. This representation consists of writing the functions as a sum of products of positive or negative decimal numbers and binary variables. A method for deciding if an arbitrary function can be represented in this way is given but no effective test is presented. It is shown how functions which cannot be represented directly in the desired fashion can be represented by using implicit functions, that is, by defining new variables which are equal to a function of some of the original variables, which can be represented in the desired fashion, and then representing the original function in terms of the original variables and the new variables. An alternative representation of a switching function as a sum of products of positive or negative decimal numbers and logical sums of binary variables is also presented. It is indicated that every switching function has such a representation. It is shown that any switching function can be written as an arithmetic sum of functions of non-negated variables represented in the manner discussed. Tables showing ways of representing functions of 1, 2, 3, and 4 variables are given. A discussion of magnetic core switching and the realization of standard computer circuits using such cores is given. A study of average length of carry propagation for a standard adder is made and an already known upper bound is derived. A lower bound of $\log_2 n-1$ where n is the number of digits in each of the numbers being added is given. The design of what is sometimes called a distant-carry adder, i.e., an adder in which the numbers to be added are divided into segments, is discussed and the average length of carry propagation, which is significantly less than for the standard adder, is calculated.

B. Hazeltine (Providence, R.I.)

9384:

Sun, Czyan'. Optimal control of a nonlinear system. Avtomat. i Telemeh. **21** (1960), 3–14 (Russian. English summary); translated as Automat. Remote Control **21**, 2–8.

Author's summary: "The paper considers transient responses, optimal as concerns speed of response, in control systems containing a rotary amplifier and a dc motor. The motor's excitation voltage is considered as a second independent control parameter. Methods are

suggested for the synthesis of an optimal controlling device. An example is given of the synthesis for simplified second- and third-order systems."

9385:

Rozenvasser, E. N. The reduction of nonlinear control system equations to their simplest form. Avtomat. i Telemeh. **21** (1960), 15–19 (Russian. English summary); translated as Automat. Remote Control **21**, 9–11.

Author's summary: "Formulas are given for linear transformations which take the equations of direct control systems to single nth-order differential equations. Use of this transformation allows one to shorten the computations required in carrying out certain calculations for control systems, for example, for determining the auto-oscillations in the case of piecewise-linear characteristics."

9386:

Kocanov, N. S. The relationship between linear systems' transient responses and their Laplace transforms. Avtomat. i Telemeh. **21** (1960), 20–28 (Russian. English summary); translated as Automat. Remote Control **21**, 12–17.

Author's summary: "Methods are considered for the determination of discrete values of a linear system's transient response from its given Laplace transform, and for finding the Laplace transform from equally spaced given values of the transient response. These methods do not require that the roots of algebraic equations be computed."

9387:

Matyaš, I. [Matyáš, J.]; Šilhanek, Ya. [Šilhánek, J.]. A generator of random processes from their given spectral density matrices. Avtomat. i Telemeh. **21** (1960), 29–35 (Russian. English summary); translated as Automat. Remote Control **21**, 18–22.

Authors' summary: "A method is described for constructing a generator of n stationary random processes with arbitrary matrices of rational spectral densities, using a minimal number of uncorrelated white-noise generators and stable linear filters. The transfer functions of these filters are determined by a very simple method from the given spectral densities. The method is illustrated by an example for the case when $n=3$."

9388:

Landa, P. S. On the stability of servo-controlled systems with random stimuli. Avtomat. i Telemeh. **21** (1960), 36–41 (Russian. English summary); translated as Automat. Remote Control **21**, 23–26.

Author's summary: "Excitation conditions are found for servo systems containing nonlinear backlash-type elements. The probability of excitation of such a system when noise is present is computed as a function of time."

9389:

Rubinovič, Ya. I. Synthesizing control systems with monotonically decreasing gains by the root-locus method. Avtomat. i Telemeh. **21** (1960), 48–55 (Russian. English summary); translated as Automat. Remote Control **21**, 31–36.

Author's summary: "We consider a periodically connected control system which contains a link whose gain decreases monotonically with time. We introduce the concept for the quasi-majorant closed control system for which, using the root-locus method, we choose a correcting circuit."

9390:

Emel'yanov, S. V.; Fedotova, A. I. Constructing optimal second-order automatic control systems using limiting values of control loop element gains. Avtomat. i Telemeh. **21** (1960), 56–63 (Russian. English summary); translated as Automat. Remote Control **21**, 37–41.

Authors' summary: "The possibility is considered of obtaining optimal control processes in second-order automatic control systems with the use of limiting values of control loop element gains by means of nonlinear 'key'-type correcting devices."

9391:

Teverovskiĭ, V. I. On one particular case of a sampled-data system with variable parameters which change by jumps. Avtomat. i Telemeh. **21** (1960), 64–71 (Russian. English summary); translated as Automat. Remote Control **21**, 42–46.

Author's summary: "A method is given for determining the transfer function of a sampled-data system with periodically varying parameters in the particular case when the system is comprised of a series connection of a first-order link with variable parameters and links with constant parameters."

9392:

Kogan, B. Ya. On methods of simulating rational-fraction functions without the use of differentiating elements. Avtomat. i Telemeh. **21** (1960), 72–81 (Russian. English summary); translated as Automat. Remote Control **21**, 47–53.

Author's summary: "Methods of simulating rational-fraction transfer functions without the use of differentiating elements when the original differential equations contain constant and variable coefficients are compared. It is asserted that the minimum number of operational amplifiers required is $n+3$, where n is the order of the differential equation. For simplicity of the circuit and for the minimum amount of preliminary calculation, the method of combining derivatives is preferable for constant coefficients, whereas the method of going over to the equivalent system of first-order equations is preferable when the coefficients are variable."

9393:

Fel'dbaum, A. A. Statistical theory of gradient systems of automatic optimization for objects with quadratic characteristics. Avtomat. i Telemeh. **21** (1960), 167–179; erratum, 1098 (Russian. English summary); translated as Automat. Remote Control **21**, 111–118; erratum, 775.

Author's summary: "The paper formulates the problem

of investigating gradient systems of automatic optimization in the presence of random factors. Methods are given for determining the steady-state error and estimating the duration of the transient response for gradient systems with several input variables. The results are illustrated by examples of systems with one and with two variables."

9394:

Pupkov, K. A. Method of investigating the accuracy of essentially nonlinear automatic-control systems by means of equivalent-transfer functions. Avtomat. i Telemeh. **21** (1960), 191–200 (Russian. English summary); translated as Automat. Remote Control **21**, 126–140.

Author's summary: "A method is considered for approximating essentially nonlinear functions by equivalent frequency characteristics on the basis of a comparison of the spectra of the random process at the input and at the output of the nonlinear element."

9395:

Pyškin, I. V. Processes of finite duration in pulse-width systems. Avtomat. i Telemeh. **21** (1960), 201–208 (Russian. English summary); translated as Automat. Remote Control **21**, 141–145.

Author's summary: "Equations are derived for pulse-width systems which determine the form of the transient responses which terminate in a finite time. For second-order systems, methods of realizing such processes are given. It is suggested that the complicated nonlinearities necessary for the realization of such processes be replaced by linear elements. Results of experiments on a model are given, confirming the possibility of such replacements."

9396:

Matveev, P. S. Determination of the optimal impulsive response function in the presence of internal noise. Avtomat. i Telemeh. **21** (1960), 286–292 (Russian. English summary); translated as Automat. Remote Control **21**, 198–203.

Author's summary: "The problem of determining the optimal transfer function is generalized to the cases when the input stimuli are applied to n different elements of a servosystem and to systems with variable parameters."

9397:

Zakrevskiĭ, A. D. A visual-matrix method for minimizing Boolean functions. Avtomat. i Telemeh. **21** (1960), 369–373 (Russian. English summary); translated as Automat. Remote Control **21**, 255–258.

Author's summary: "A semi-intuitive method is presented for the minimization of Boolean functions which is based on the use of certain properties of visually perceived information. The method is applicable to the minimization of arbitrary Boolean functions with up to 10 to 12 variables, and is particularly effective for the minimization of incompletely defined functions."

9398:

Kislyakov, V. S. Application of the Krylov-Bogolyubov method of constructing asymptotic approximations to the investigations of systems with lags. Avtomat. i Telemeh. **21** (1960), 442–455 (Russian. English summary); translated as Automat. Remote Control **21**, 307–315.

Author's summary: "The present paper treats of the application of the Krylov-Bogolyubov method of constructing asymptotic approximations to the investigation of linear and nonlinear systems with lags. The effectiveness of this application of the method is illustrated by the example of the investigation of the Minorsky equation."

9399:

Širokorad, B. V. On steady-state modes in automatic control systems. Avtomat. i Telemeh. **21** (1960), 456–464 (Russian. English summary); translated as Automat. Remote Control **21**, 316–321.

Author's summary: "The present paper discusses certain basic concepts and methods of automatic control system (ACS) theory which are related to the existence of steady-state modes of autonomous systems with the coexistence of several independent frequencies."

9400:

Andreev, N. I. Determining pulse-system optimum weighting function which ensures the extreme of a functional. Avtomat. i Telemeh. **21** (1960), 465–473 (Russian. English summary); translated as Automat. Remote Control **21**, 322–327.

Author's summary: "A method is described for finding the optimum pulse system which ensures an extreme of a sufficiently general criterion in the form of a functional $I=\Phi(I_1, \cdots, I_{n+1})$, where Φ is a known function, and I_i are quadratic functionals of the weighting function of the system."

9401:

Pyatnickiĭ, G. I. The effect of stationary random processes on automatic control systems containing essentially nonlinear elements. Avtomat. i Telemeh. **21** (1960), 474–480 (Russian. English summary); translated as Automat. Remote Control **21**, 328–332.

Author's summary: "A method of calculating nonlinear automatic control systems under the action of stationary random processes is proposed. The method presented is a generalization of the method of statistical linearization and allows one to take into account the distortion of the spectrum of a random process by nonlinear elements."

9402:

Černeckiĭ, V. I. An interpolation method for analyzing automatic control system accuracy under random stimuli. Avtomat. i Telemeh. **21** (1960), 481–488 (Russian. English summary); translated as Automat. Remote Control **21**, 333–337.

Author's summary: "An approximate method is suggested for constructing the solution of a system of ordinary differential equations containing random functions and random variables, and formulas are derived for the computation of the moments. The method is a development of certain ideas of Chebyshev."

9403:

Tovstuha, T. I. The effect of random noise on the steady-state operation of a step-type extremal system for an object with a parabolic characteristic. Avtomat. i Telemeh. **21** (1960), 575–584 (Russian. English summary); translated as Automat. Remote Control **21**, 398–404.

Author's summary: "The optimal parameters of the controlling portion of the extremal system under consideration are calculated. These parameters must be determined from estimates of the deviations from the minimum characteristics in the steady state. It is shown that, in the particular case of uniformly distributed noise increments, it is most advantageous to operate with small steps, to be determined below."

9404:

Pyatnickiĭ, G. I. The action of random processes on discontinuous control systems. Avtomat. i Telemeh. **21** (1960), 585–594 (Russian. English summary); translated as Automat. Remote Control **21**, 405–411.

Author's summary: "Questions of the statistical calculations of discontinuous (sampled-data) control systems are considered, these systems being acted upon by stationary random processes. We also consider one particular case when a nonstationary random process acts. It is shown that the method of statistical linearization can be used for the approximate calculation of nonlinear systems in the same form in which it is used for the calculation of continuous nonlinear systems."

9405:

Gorskaya, N. S. Effect of rate feedback on loaded-relay servomechanism dynamics. Avtomat. i Telemeh. **21** (1960), 601–614 (Russian. English summary); translated as Automat. Remote Control **21**, 416–425.

Author's summary: "The effect of velocity feedback on the free oscillations of a loaded-relay servomechanism is considered. The servomechanism's linear portion is described by a complete second-order equation. The investigation is carried out by the point method, using the theory of point transformations. The dependence of the amplitude and time of the auto-oscillations' half periods on the system's parameters, including the velocity-feedback coefficient, is determined."

9406:

Šilhanek, Ya. [Šilhánek, J.]. On the simulation of linear systems with n stationary random input processes without the use of random-function generators. Avtomat. i Telemeh. **21** (1960), 652–654 (Russian. English summary); translated as Automat. Remote Control **21**, 451–452.

Author's summary: "A method is described for determining the mean value of the square of the output quantity of a linear system with n stationary random input processes by means of a simulating device without the use of random function generators. This method is a generalization to systems with n inputs of the method described in J. H. Laning, Jr. and R. H. Batten, *Random processes in automatic control* [McGraw-Hill, New York, 1956; MR **18**, 74]."

9407:

Leonov, Yu. P. The problem of shaping filters and optimal linear systems. Avtomat. i Telemeh. **21** (1960), 674–681 (Russian. English summary); translated as Automat. Remote Control **21**, 467–471.

Author's summary: "It is shown that, under sufficiently general conditions, a nonstationary random process can be formed from 'white noise' by means of linear filters. The weight functions of the shaping filters are determined."

9408:

Butkovskiĭ, A. G.; Lerner, A. Ya. The optimal control of systems with distributed parameters. Avtomat. i Telemeh. **21** (1960), 682–691 (Russian. English summary); translated as Automat. Remote Control **21**, 472–477.

Authors' summary: "The paper poses the problem of the optimal control of systems with distributed parameters for certain classes of objects described by systems of first-order partial differential equations and by the heat-conduction equation. The solution is adduced for one optimal control problem for an object described by a first-order partial differential equation, and the functional block schematic is given for an optimal control system for objects of this type."

9409:

Bloh, Z. Š. Transient responses in control systems with lags. Avtomat. i Telemeh. **21** (1960), 710–719 (Russian. English summary); translated as Automat. Remote Control **21**, 493–499.

Author's summary: "A method is given for constructing the transient response in systems with lags. Transfer functions are found for systems with lags, where these lags may be in the forward circuit, in the basic feedback path and in auxiliary feedback paths. For the transient response under consideration, an integral equation is set up and solved, its solution being given in the form of a series which converges absolutely and uniformly on any given finite interval of time. Estimates of overshoot and control time (duration of the transient response) are given, based on the parameters of the real and imaginary frequency characteristics of the system with lags. Relationships are derived which permit one to compare different integral criteria of control quality for systems with, and without, lags."

9410:

Alimov, Yu. I. Lyapunov functions for relay control systems. Avtomat. i Telemeh. **21** (1960), 720–728 (Russian. English summary); translated as Automat. Remote Control **21**, 500–506.

Author's summary: "The idea of a trajectory is introduced, and the stability in the large is investigated of the equilibrium state of one class of relay control systems."

9411:

Ryabov, Yu. A. The application of the method of small parameters in the investigation of systems of automatic

control with delay. Avtomat. i Telemeh. **21** (1960), 729–739 (Russian. English summary); translated as Automat. Remote Control **21**, 507–514.

Author's summary: "A method of investigation of systems of automatic regulation (SAR) with retardation is presented, based on the consideration of retardation as a small parameter and the application of the Lyapunov-Poincaré method of small parameters. The case is considered of an SAR expressed as a linear equation with constant coefficients and with a single constant retardation."

9412:

Razumihin, B. S. The application of Lyapunov's method to problems in the stability of systems with delay. Avtomat. i Telemeh. **21** (1960), 740–748 (Russian. English summary); translated as Automat. Remote Control **21**, 515–520.

Author's summary: "We consider the application of the direct Lyapunov method to problems in the stability of systems with delay. Certain criteria are given for the stability and asymptotic stability of such systems. The possibility is shown of applying quadratic forms in stability problems in a first approximation, and estimates are proposed for the perturbations."

9413:

Syuzyumova, E. M. Some questions concerning coordinated control systems. Avtomat. i Telemeh. **21** (1960), 749–760 (Russian. English summary); translated as Automat. Remote Control **21**, 521–528.

Author's summary: "A theorem concerning inverse transfer matrices is considered, which can be used to formulate the conditions of static independence of the control of several variables in a short form. Also considered are the conditions for dynamic independence of several nonlinear coordinated control systems."

9414:

Činaev, P. I. On the principles of synthesizing automatic systems with many controlled quantities. Avtomat. i Telemeh. **21** (1960), 761–771 (Russian. English summary); translated as Automat. Remote Control **21**, 529–537.

Author's summary: "The paper suggests principles for synthesizing systems with many controlled variables. The basic mathematical relationships for combined-type automatic systems are derived in matrix form. It is shown that the use of the principles of physical realizability, invariance, and optimality permit one to select a controller's operator coefficients (elements of matrices B and C) for multiloop systems."

9415:

Gluškov, V. M.; Daševskiĭ. L. N.; Nikitin, A. I. Use of electronic computers for Bessemer-process automation. Avtomat. i Telemeh. **21** (1960), 877–883 (Russian. English summary); translated as Automat. Remote Control **21**, 617–621.

The intent to control a Bessemer converter with a digital computer in order to produce steel of a given carbon content is announced. Data sources and the method of data reduction are discussed. Realization of the data-gathering device and progress toward realization of the complete system are reported.

J. J. Eachus (Cambridge, Mass.)

9416:

Pervozvanskiĭ, A. A. Continuous extremum control systems in the presence of random noise. Avtomat. i Telemeh. **21** (1960), 958–963 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 673–677.

Author's summary: "The dynamics of one class of continuous systems of extremum control is investigated. The drift in the position of the extremum, and also the high-frequency interference, are regarded as random functions of time."

9417:

Litovčenko, I. A. On a problem of optimum control. Avtomat. i Telemeh. **21** (1960), 1122–1133 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 791–798.

Author's summary: "The problem of optimization (in the specified meaning of the term) is considered in tracking a trajectory of a body in accordance with the rule of proportional search. The control function related to the position of the direction control is given on an interval of the real axis. The method of parametric representation of the control function is used in solving the problem; this enables one to use the methods of classical calculus of variations. The optimum control is found and its sufficient conditions are analyzed."

9418:

Volgin, L. N.; Smolyar, L. I. Servomechanism-compensation by using discrete calculating devices. Avtomat. i Telemeh. **21** (1960), 1158–1164 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 816–820.

Authors' summary: "A method is proposed for calculating a program of discrete calculating devices for linear servomechanisms, based on the use of statistical results on possible external influences. The method ensures that the systems obtained will satisfy the 'roughness' condition in the presence of unstable elements in the control circuit."

9419:

Fel'dbaum, A. A. Dual-control theory. I. Avtomat. i Telemeh. **21** (1960), 1240–1249 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 874–880.

Author's summary: "Some fundamental problems in communication theory and control theory are compared. The problem of designing an optimum (in the statistical sense) closed-loop dual-control system, is formulated. Its solution, as well as examples and some generalizations, will be given in parts II, III, and IV."

9420:

Morosanov, I. S. Effect of fluctuations on extremal relay systems under self-oscillation conditions. Avtomat. i Telemeh. **21** (1960), 1254–1263 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 884–890.

Author's summary: "This article presents an analysis of extremal relay-control systems under self-oscillation conditions with fluctuations at the regulator input. A method of statistical linearization of such systems, based on the reduction of external random disturbances to parametric action on the regulator, is proposed. This method makes it possible to obtain an expression for the equivalent gain in the form of a random step-function of time according to the describing-function method. The average value of this function is taken as the statistically equivalent transfer coefficient of the nonlinear component. This method can be applied to ordinary nonlinear self-oscillation systems."

9421:

Čžan, Žèn'-vèĭ [Chang, Jên-wei]. Stability of a controlled system with two nonlinear elements. Avtomat. i Telemeh. **21** (1960), 1272–1278 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 897–901.

Author's summary: "This article presents a study of the second B. V. Bulgakov problem where the nonlinearity of one of the measuring units is taken into account and a method is described for solving the stability problem, which makes it possible, on the one hand, to find acceptable regulator parameters and, on the other hand, to determine the controlled-variables range, containing the coordinate origin, for which the system stability is guaranteed."

9422:

Rozenvasser, E. N. On the accurate determination of periodic regimes in sectionally linear automatic control systems. Avtomat. i Telemeh. **21** (1960), 1279–1292 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 902–910.

Author's summary: "An exact method for determining periodic regimes in arbitrary sectionally linear systems is considered. The solution of the problem is reduced to the determination of forced oscillations of a linear system with periodically changing parameters. These forced oscillations are found in an exact manner by means of a new approach, which is based on the solution of an auxiliary Fredholm integral equation of the second kind. The thus obtained periodic solution depends on the unknown times of motion along sections of nonlinear characteristics. By using this solution and the conditions for the transition from one section of the nonlinear characteristic to another, it is possible to determine the transcendental equations of periods."

9423:

Kazakov, V. D.; Kuznecov, O. P. List of foreign works published in 1958 on the theory of relay actions and finite automata. Avtomat. i Telemeh. **21** (1960), 1332–1338 (Russian. English summary); translated as Automat. Remote Control **21** (1961), 941–944.

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